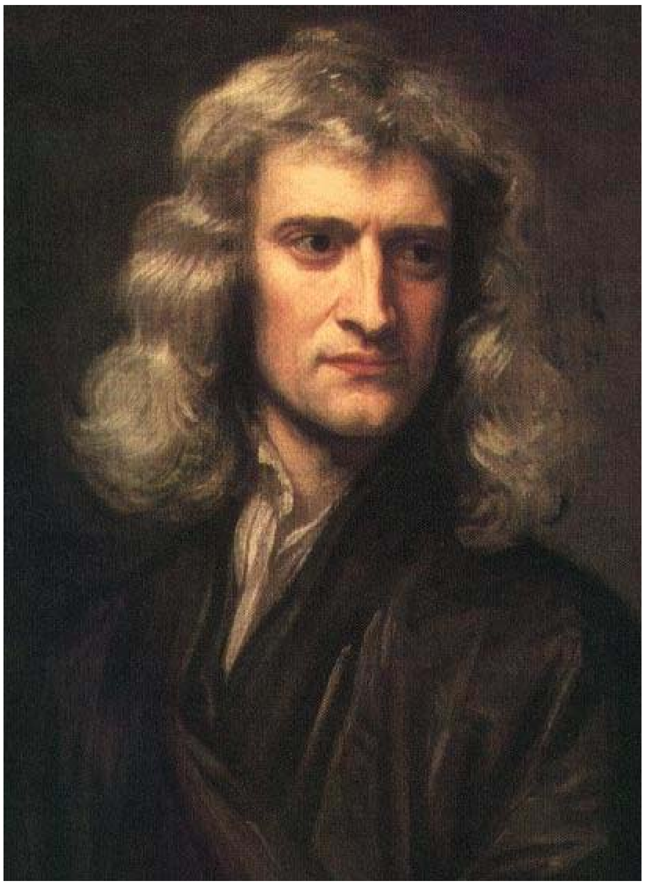
Lecture Note
Chapter 5

1 Newton's Law

1.1 Newton's First Law

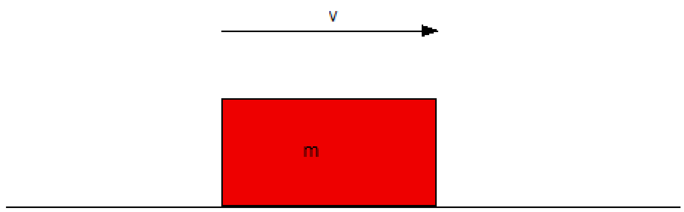
Sir Isaac Newton, FRS (Wikipedia) (4 January 1643 - 31 March 1727)
Link: http://en.wikipedia.org/wiki/Isaac_Newton

Newton was an English physicist, mathematician, astronomer, natural philosopher, alchemist, and theologian and one of the most influential men in human history. His *Philosophiæ Naturalis Principia Mathematica*, published in 1687, is by itself considered to be among the most influential books in the history of science, laying the groundwork for most of classical mechanics. In this work, Newton described universal gravitation and the three laws of motion which dominated the scientific view of the physical universe for the next three centuries.



A body acted on by no net force moves with constant velocity (which may be zero) and zero acceleration.

If an object does not interact with other objects, it is possible to identify a reference frame in which the object has zero acceleration. This is also called the law of inertia It defines a special set of reference frames called inertial frames. We call this an inertial frame of reference



((**Newton's First Law – Alternative Statement**))

In the absence of external forces, when viewed from an inertial reference frame, an object at rest remains at rest and an object in motion continues in motion with a constant velocity. Newton's First Law describes what happens in the absence of a force. Also tells us that when no force acts on an object, the acceleration of the object is zero

((**Inertial frame of reference**))

A frame of reference in which Newton's first law is valid, is called a inertial frame of reference.

Any reference frame that moves with constant velocity relative to an inertial frame is itself an inertial frame. A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame. We can consider the Earth to be such an inertial frame although it has a small centripetal acceleration associated with its motion

1.2 Newton's Second Law

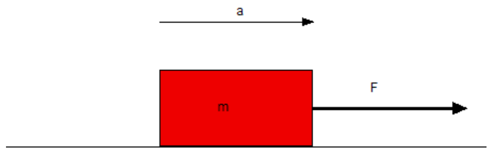
If a net force acts on a body, the body accelerates. The direction of acceleration is the same ad the direction of the net force. The net force vector is equal to the mass of the body times the acceleration of the body

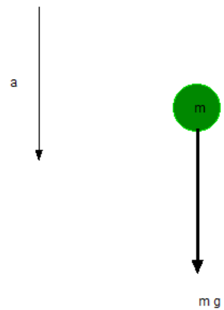
$$\boldsymbol{F} = m\boldsymbol{a}$$

More generally

$$\boldsymbol{F} = \frac{d\boldsymbol{p}}{dt}$$

where m is the mass, $\boldsymbol{a}$ is the acceleration, and $\boldsymbol{p}$ (= $m\boldsymbol{v}$) is the linear momentum.





((Note))
One newton is the amount of net force that gives an acceleration of one meter per second squared to a body with a mass of one kilogram.

SI units

Mass (kg)
Force (N):
$N = kg\ m/s^2$

cgs units

Mass = g
Force (dyne)
dyne=g cm/s^2

$$N = \frac{kgm}{s^2} = \frac{10^3 g 10^2 cm}{s^2} = 10^5 dyne$$

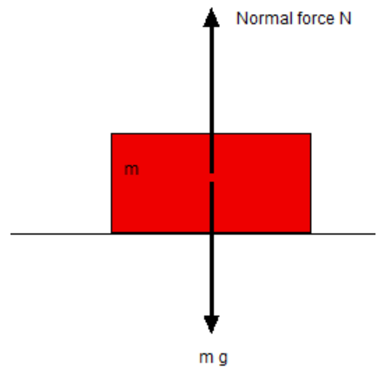
1.3. Newton's Third Law

If body A exerts a force on body B (an action), then the body B exerts a force on body A (a reaction). These two forces have the same magnitude but are opposite in direction. These two forces act on different bodies.

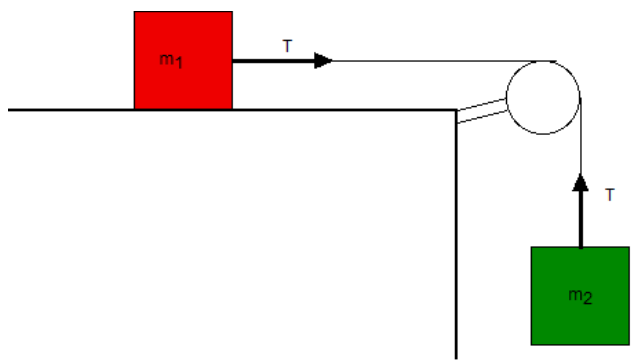
((**Newton's Third Law, Alternative Statements**))

Forces always occur in pairs. A single isolated force cannot exist. The action force is equal in magnitude to the reaction force and opposite in direction. One of the forces is the action force, the other is the reaction force. It doesn't matter which is considered the action and which the reaction. The action and reaction forces must act on different objects and be of the same type.

(a) Normal force



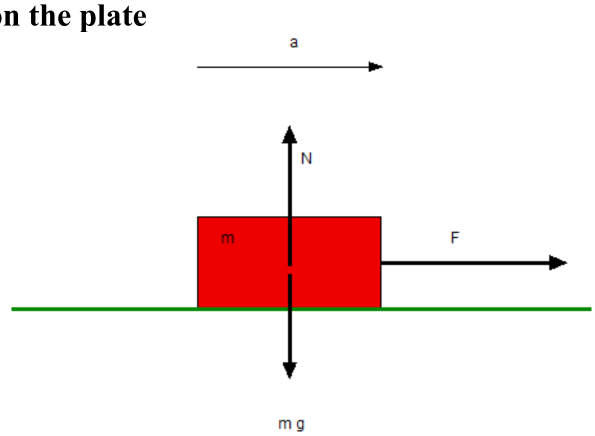
(b) Tension



2 Free body diagram

It is a diagram showing the chosen body by itself, “free” of its surroundings.

2.1 The motion on the plate

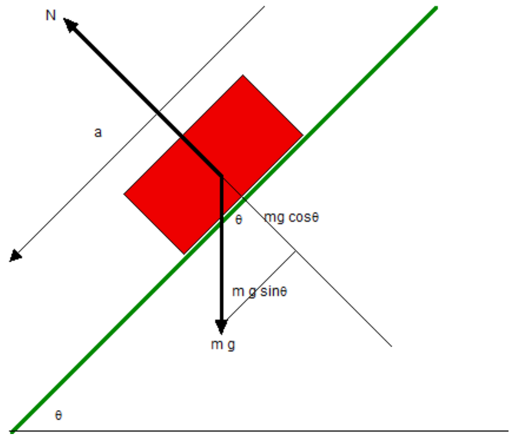


Newton’s second law

$$\sum F_x = F = ma$$
$$\sum F_y = N - mg = 0$$

$$a = \frac{F}{m}$$
$$N = mg$$

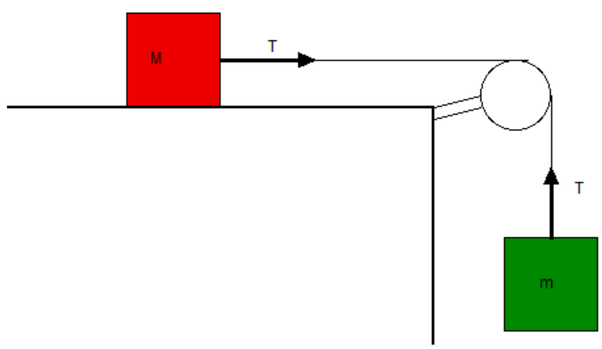
(b) Motion on the slide

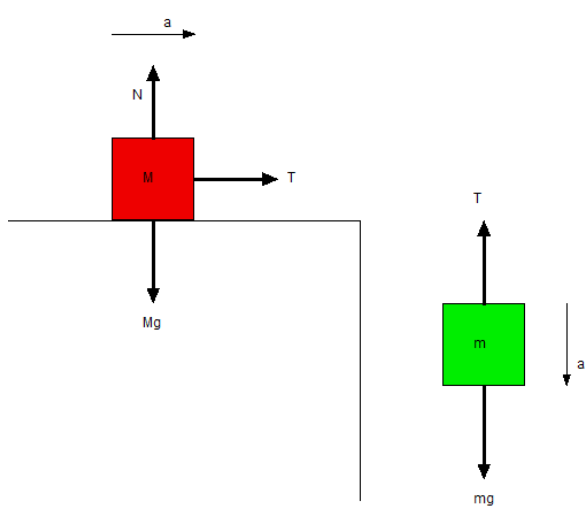


Newton's second law

$$\sum F_x = mg\sin\theta = ma \qquad a = g\sin\theta$$
$$\sum F_y = N - mg\cos\theta = 0 \qquad N = mg\cos\theta$$

(c) The Free body diagram for the two body systems





We apply the Newton's second law to free body diagrams (in this case the masses *m* and *M*)

$$\sum F_x = ma_x$$
$$\sum F_y = ma_y$$

For the mass *M*

$$\sum F_x = T = Ma$$
$$\sum F_y = N - Mg = 0$$

For the mass *m*,

$$\sum F_y = mg - T = ma$$

From the two equations

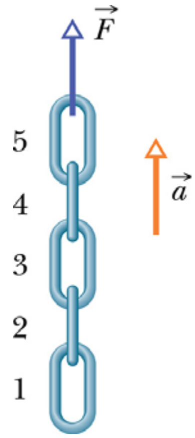
$$T = Ma \qquad (1)$$
$$mg - T = ma \qquad (2)$$

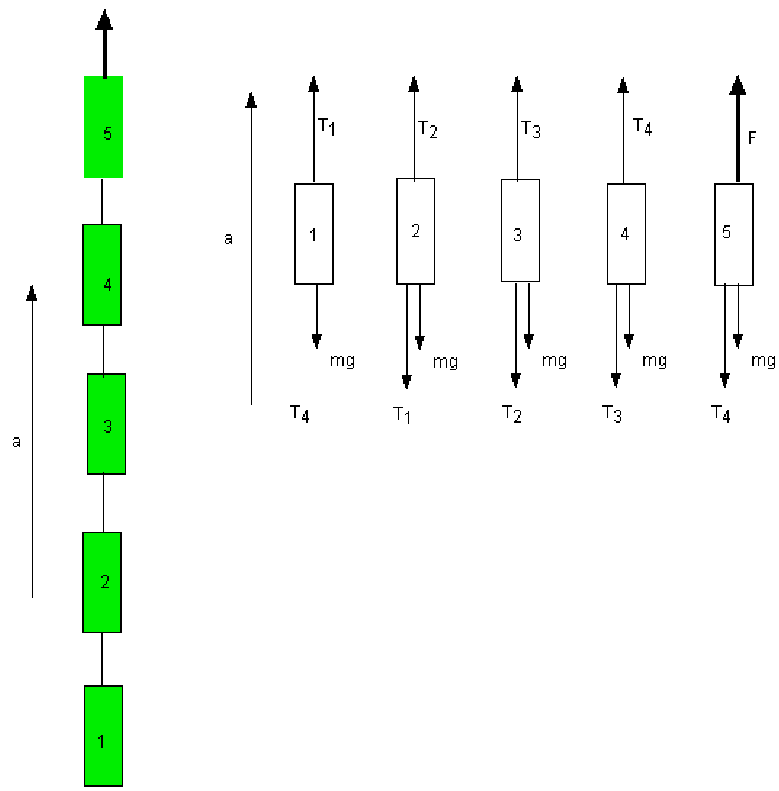
we get

$$a = \frac{mg}{M+m}$$
$$T = \frac{Nmg}{M+m}$$

(d) The Free body diagram for the five body systems

Problem 5-43 (SP-05, Hint) (10-th edition)**



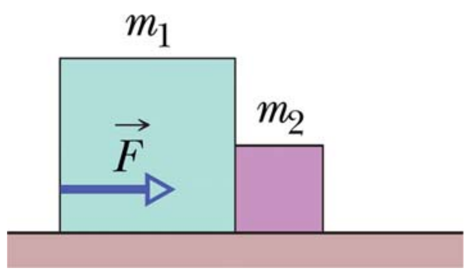


3 Applying Newton's law

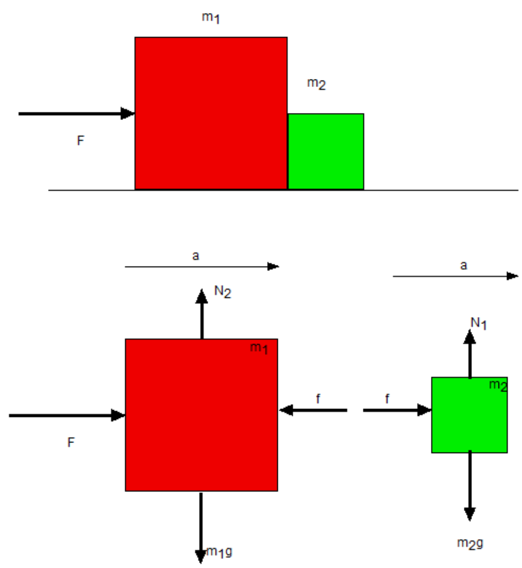
3.1

Problem 5-55 (SP-5) (10-th edition)

Two blocks are in contact on a frictionless table. A horizontal force is applied to the larger block, as shown in Fig. (a) If m_1 = 2.3 kg, m_2 = 1.2 kg, and F = 3.2 N, find the magnitude of the force between the two blocks. (b) Show that if a force of the same magnitude F is applied to the smaller block but in the opposite direction, the magnitude of the force between the blocks is 2.1 N, which is not the same value calculated in (a). (c) Explain the difference.



(a)
Free-body diagram



((**Solution**))
$m_1 = 2.3$ kg, $m_2 = 1.2$ kg, $F = 3.2$ N

(a) The Newton's second law

$$\sum F_x = F - f = m_1 a$$
$$\sum F_x = f = m_2 a$$

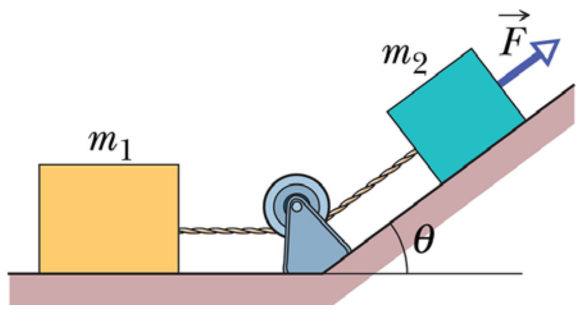
Then we have

$$a = \frac{F}{m_1 + m_2} = 0.91 m/s^2$$
$$f = \frac{m_2 F}{m_1 + m_2} = 1.1 N$$

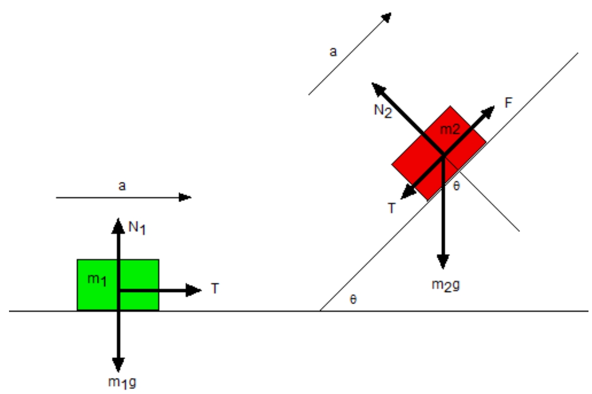
3.2

Problem 5-78 (Hint) (10-th edition)

In Fig, a force $\boldsymbol{F}$ of magnitude 12 N is applied to a FedEx box of mass m_2 = 1.0 kg. The force is directed up a plane tilted by θ = 37°. The box is connected by a cord to a UPS box of mass m_1 = 3.0 kg on the floor. The floor, plane, and pulley are frictionless, and the masses of the pulley and cord are negligible. What is the tension in the cord?



Free-body diagram

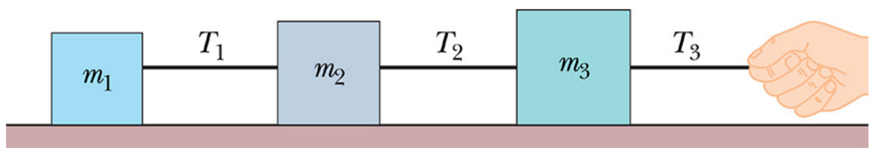


Free body diagram

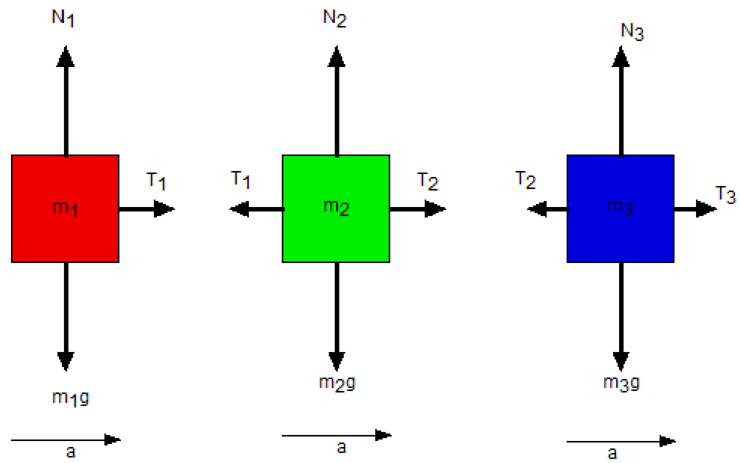
3.3

Problem 5-53 (From SP-05, Hint) (10-th edition)

In Fig., three connected blocks are pulled to the right on a horizontal frictionless by a force of magnitude T_3 = 65.0 N. If m_1 = 12.0 kg, m_2 = 24.0 kg, and m_3 = 31.0 kg, calculate (a) the magnitude of the system's acceleration, (b) the tension T_1, and (c) the tension T_2.



Free-body diagram



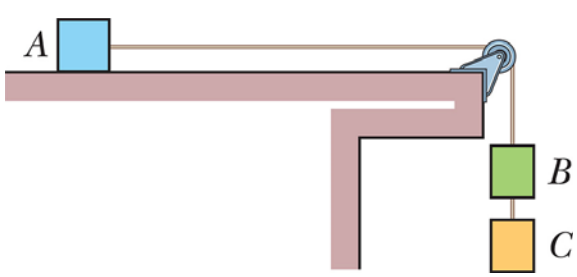
Newton's second law

$$T_1 = m_1 a$$
$$T_2 - T_1 = m_2 a$$
$$T_3 - T_2 = m_3 a$$

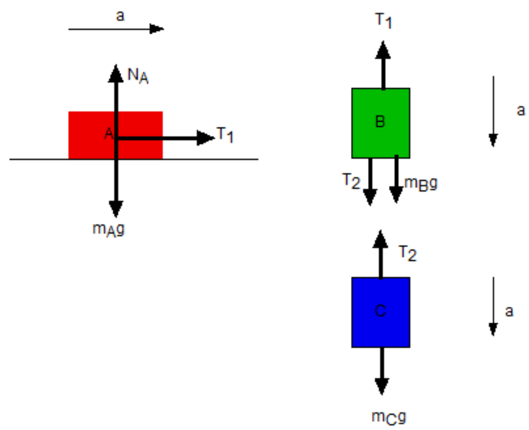
3.4

Problem 5-50 (SP-5) (10-th edition)

In Fig, three ballot boxes are connected by cords, one of which warps over a pulley having negligible friction on its axle and negligible mass. The three masses are m_A = 30.0 kg, m_B = 40.0 kg, and m_C = 10.0 kg. When the assembly is released from rest, (a) what is the tension in the cord connecting B and C, and (b) how far does A move in the first 0.250 s (assuming it does not reach the pulley)?



Free-body diagram



Newton's second law

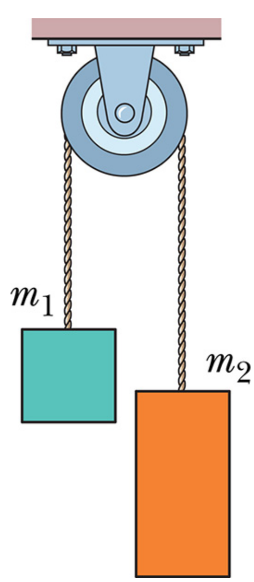
$$
\begin{aligned}
T_1 &= m_A a \\
T_2 + m_B g - T_1 &= m_B a \\
m_C g - T_2 &= m_C a
\end{aligned}
$$

3.5 Example-5

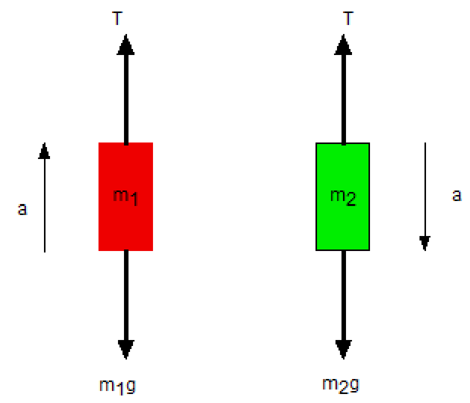
Problem 5-51 (HW-05, Hint) (10-th edition)

Atwood machine

Figure shows two blocks connected by a cord (of negligible mass) that passes over a frictionless pulley (also of negligible mass). The arrangement is known as Atwood's machine. One block has mass m_1 = 1.30 kg; the other has mass m_2 = 2.80 kg. What are (a) the magnitude of the blocks' acceleration and (b) the tension in the cord?



Free-body diagram



Newton's second law:

$$T - m_1 g = m_1 a$$
$$m_2 g - T = m_2 a$$

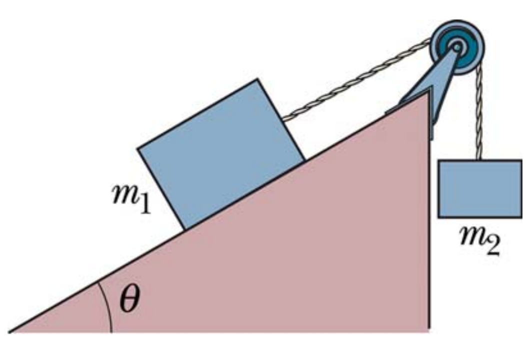
((Note)) **Atwood machine**

The Atwood machine (or Atwood's machine) was invented in 1784 by Rev. George Atwood as a laboratory experiment to verify the mechanical laws of motion with constant acceleration. Atwood's machine is a common classroom demonstration used to illustrate principles of classical mechanics.

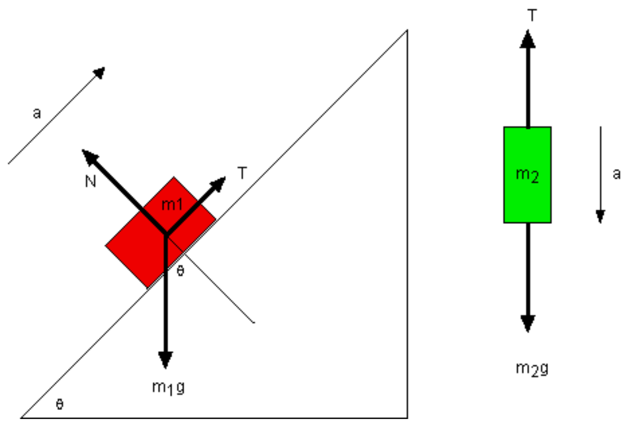
3.6

Problem 5-57 (SP-5, Hint) (10-th edition)

A block of mass m_1 = 3.70 kg on a frictionless plane inclined a angle θ = 30° is connected by a cord over a massless, frictionless pulley to a second block of mass m_2 = 2.30 kg (Fig.). What are (a) the magnitude of the acceleration of each block, (b) the direction of the acceleration of the hanging block, and (c) the tension in the cord?



Free body diagram



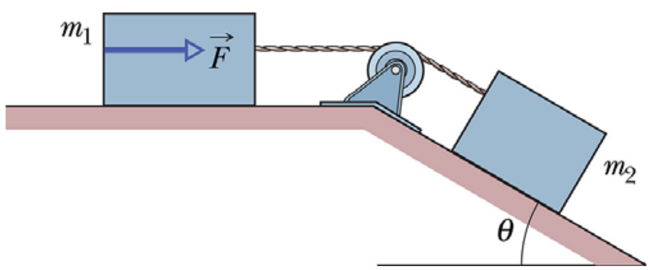
Newton's second law

$$m_2 g - T = m_2 a$$
$$T - m_1 g \sin\theta = m_1 a$$

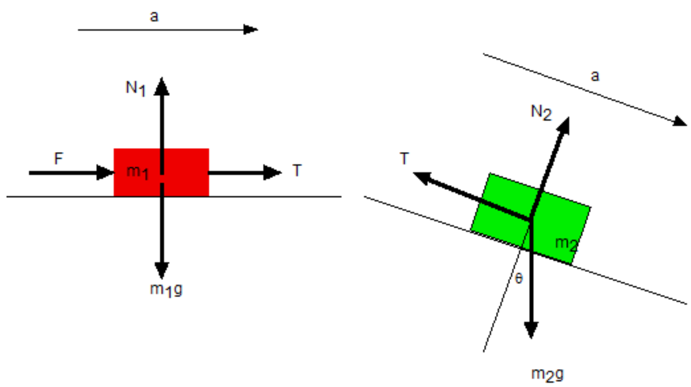
3.7

Problem 5-64* (Hint, HW-05) (10-th edition)**

Figure shows a box of mass m2 = 1.0 kg on a frictionless plane inclined at angle θ = 30°. It is connected by a cord of negligible mass to a box of mass m_1 = 3.0 kg on a horizontal frictionless surface. The pulley is frictionless and massless. (a) If the magnitude of the horizontal force ***F*** is 2.3 N, what is the tension in the connecting cord? (b) What is the largest value the magnitude of ***F*** may have without the cord becoming slack?



Free-body diagram



Newton's second law

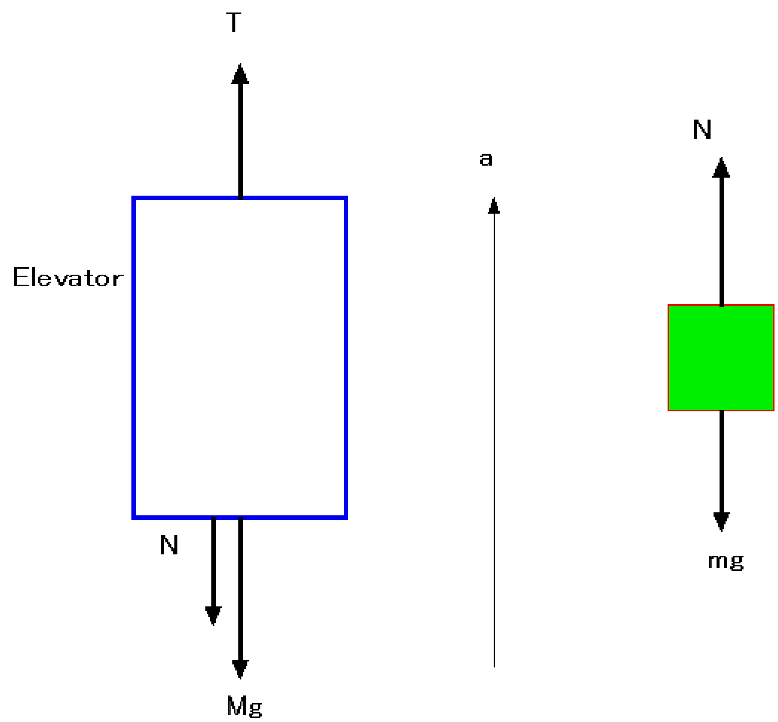
$$T + F = m_1 a$$
$$m_2 g \sin\theta - T = m_2 a$$

3.8

Problem 5-98 (SP-5) (8-th edition)
This problem was removed in the 9-th edition.

A 50 kg pasenger rides in a elevator cab that starts from rest on the ground floor of a building at t = 0 and rises to the floor during a 10 s interval. The cab's acceleration as a function of the time is shown in Fig., where positive values of the acceleration mean that it is directed upward. What are the (a) magnitude and (b) direction (up or down) of the maximum force on the passengaer from the floor, the (c) magnitude and (d) direction of the minimum force on the passenger from the floor, and the (e) magnitude and (f) direction of the maximum force on the floor from the passenger?

((**Solution**))
m = 50 kg.



$$N - mg = ma$$

$$N = m(g + a)$$

(a) and (b)
N takes maximum when $a = 2$ m/s^2.

$N = 50\ (9.8+2.0) = 590.0$ N (normal force is upward)

(c) and (d)
N takes minimum when $a = -3$ m/s^2.

$N = 50\ (9.8 - 3.0) = 340.0$ N (normal force is upward).

(e) and (f)
According to the Newton's third law (action-reaction), the force N is applied to the floor of the elevator downward.

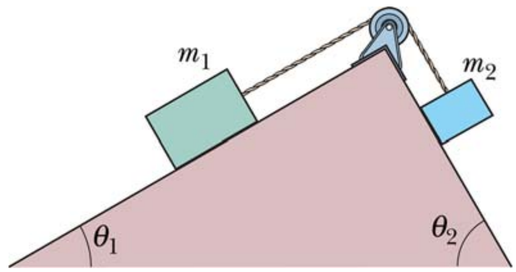
The maximum force on the floor is

$N = 590$ N (downward)

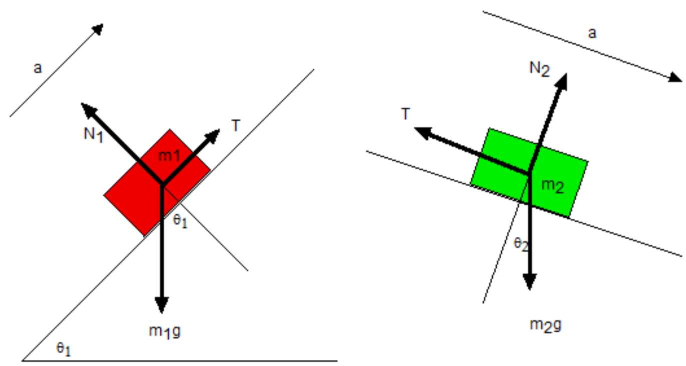
3.9

Problem 5-71 (HW-05, Hint) (10-th edition)

Figure shows a box of dirty money (mass m_1 = 3.0 kg) on a frictionless plane inclined at angle θ_1 = 30°. The box is connected via a cord of negligible mass to a box of laundered money (mass m_2 = 2.0 kg) on a frictionless plane inclined at angle θ_2 = 60°. The pulley is frictionless and has negligible mass. What is the tension in the cord?



Free-body diagram



Free body diagram

Newton's second law

$$T - m_1 g \sin\theta_1 = m_1 a$$
$$m_2 g \sin\theta_2 - T = m_2 a$$

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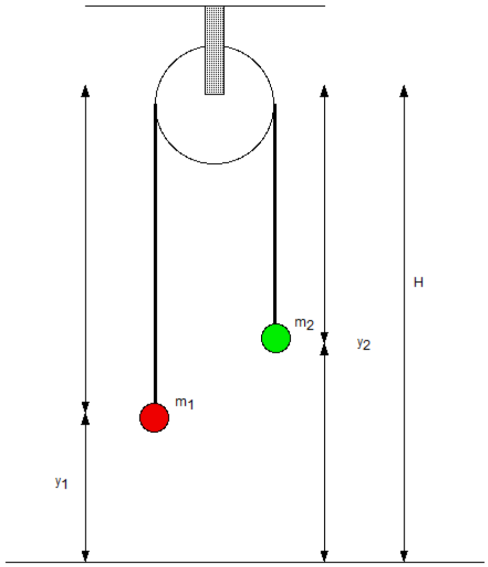
4. Comments and advanced problems

4.1 Atwood machine (reply to the student's question)

In the Atwood machine, we show that the magnitude of the acceleration of the mass m_1 is the same as that of the mass m_2. Of course, the directions are opposite.

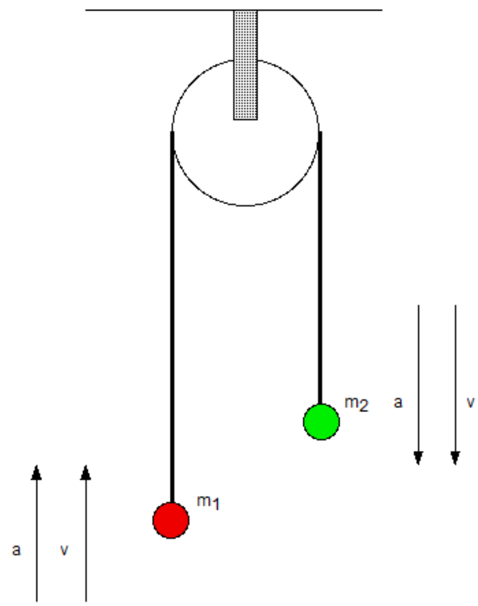


The total distance of the rope is constant.

$$H - y_1 + H - y_2 = const$$
$$-\frac{dy_1}{dt} - \frac{dy_2}{dt} = 0$$
$$-\frac{d^2 y_1}{dt^2} - \frac{d^2 y_2}{dt^2} = 0$$

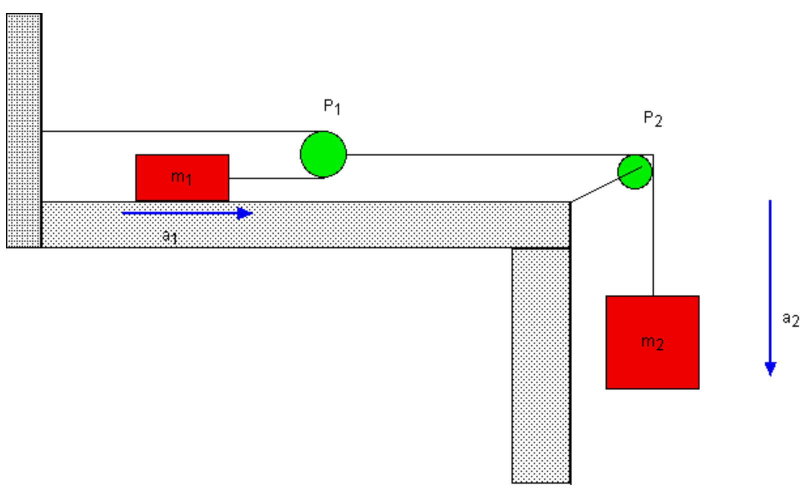
This means that

$$\boldsymbol{v}_2 = -\boldsymbol{v}_1$$
$$\boldsymbol{a}_2 = -\boldsymbol{a}_1$$



4.2 Serway Problem 4-38 (Advanced problem)

An object of mass m1 on a frictionless horizontal table is connected to an object of mass m_2 through a very light pulley P_1 and a light fixed pulley P_2 as shown in Fig. (a) If a_1 and a_2 are the accelerations of m_1 and m_2, respectively, what is the relation between these accelerations? Express (b) the tension in the string and (c) the accelerations a_1 and a_2 in terms of g and the masses m_1 and m_2.



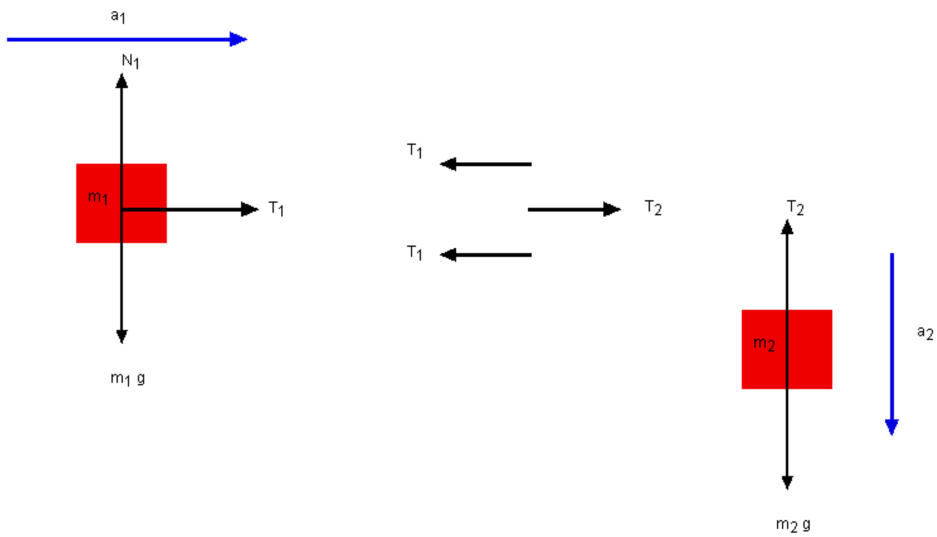
Suppose that the positions of P_1 and mass m_1 are expressed by x_p and x_1. The length of the string through the pulley P_1 is

$$x_p + (x_p - x_1) = \cos\tan t$$

When the acceleration of m_2 is given by a_2, we have

$$a_1 = 2a_p = 2a_2$$

Free-body diagram



$$T_1 = m_1 a_1$$
$$T_2 = 2T_1$$
$$a_1 = 2a_2$$
$$m_2 g - T_2 = m_2 a_2$$

Then we have

$$a_1 = \frac{2m_2 g}{4m_1 + m_2}$$
$$a_2 = \frac{m_2 g}{4m_1 + m_2}$$
$$T_1 = \frac{2m_1 m_2 g}{4m_1 + m_2}$$
$$T_2 = \frac{4m_1 m_2 g}{4m_1 + m_2}$$

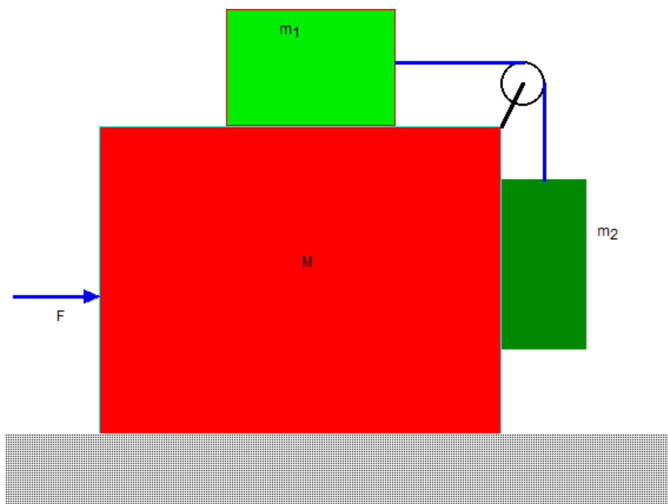
((**Mathematica**))

```
Clear["Global`*"]; eq1 = m2 g - T2 == m2 a2;
eq2 = T2 - 2 T1 == 0; eq3 = a1 == 2 a2; eq4 = T1 == m1 a1;
Solve[{eq1, eq2, eq3, eq4}, {T1, T2, a1, a2}]
```

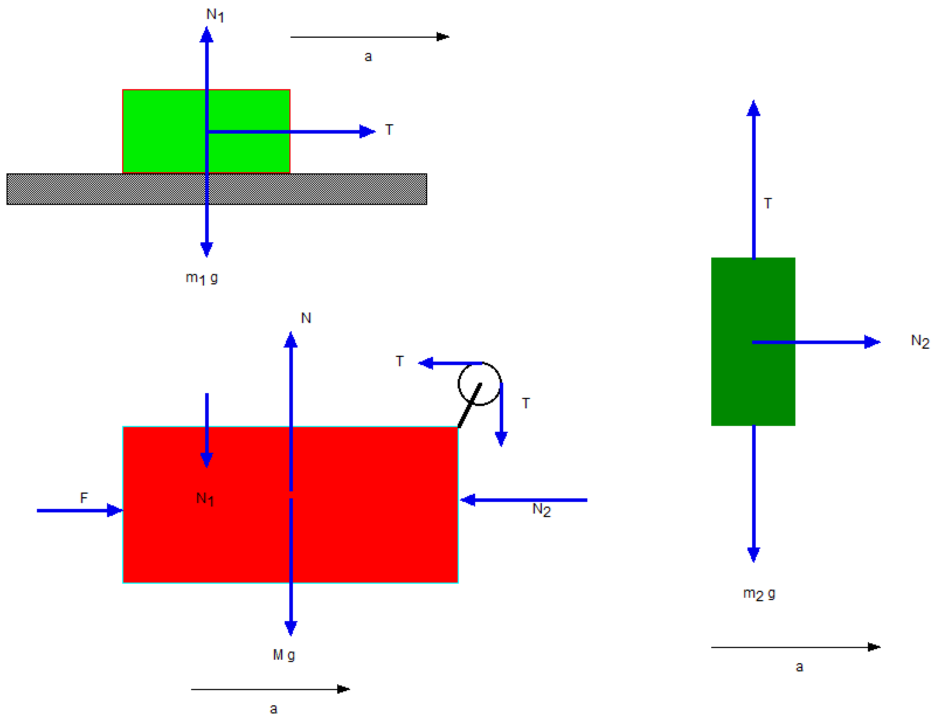
$$\left\{\left\{T1 \to \frac{2\, g\, m1\, m2}{4\, m1 + m2},\ T2 \to \frac{4\, g\, m1\, m2}{4\, m1 + m2},\ a1 \to \frac{2\, g\, m2}{4\, m1 + m2},\ a2 \to \frac{g\, m2}{4\, m1 + m2}\right\}\right\}$$

4.3 Serway Problem 4-47 (Advanced problem)

What horizontal force must be applied to the cart shown in Fig. so that the blocks remain stationary relative to the cart? Assume all surfaces, wheels, and pulley are frictionless. Notice that the force exerted by the string, accelerate m_1.



Free-body diagram



((**Note**))

The masses m_1, M, and m_2 move along the positive x direction at the same acceleration. Thus the length of string between m_1 and m_2 remains unchanged. In other words, there is no vertical motion of m_2. We only have to take into account of the horizontal motion for the mass m_2.

For m_1

$$\sum F_x = T = m_1 a$$
$$\sum F_y = N_1 - m_1 g = 0$$

For m_2

$$\sum F_x = N_2 = m_2 a$$
$$\sum F_y = m_2 g - T = 0$$

For M

$$\sum F_x = F - T - N_2 = Ma$$
$$\sum F_y = N - Mg - N_1 - T = 0$$

From these equations, we have

$$a = \frac{m_2}{m_1} g$$

$$F = (M + m_1 + m_2)a = (M + m_1 + m_2)\frac{m_2}{m_1} g$$

$$T = m_2 g$$

$$N_1 = m_1 g$$

$$N_2 = \frac{m_2^{\ 2}}{m_1} g$$

$$N = (M + m_1 + m_2)g$$

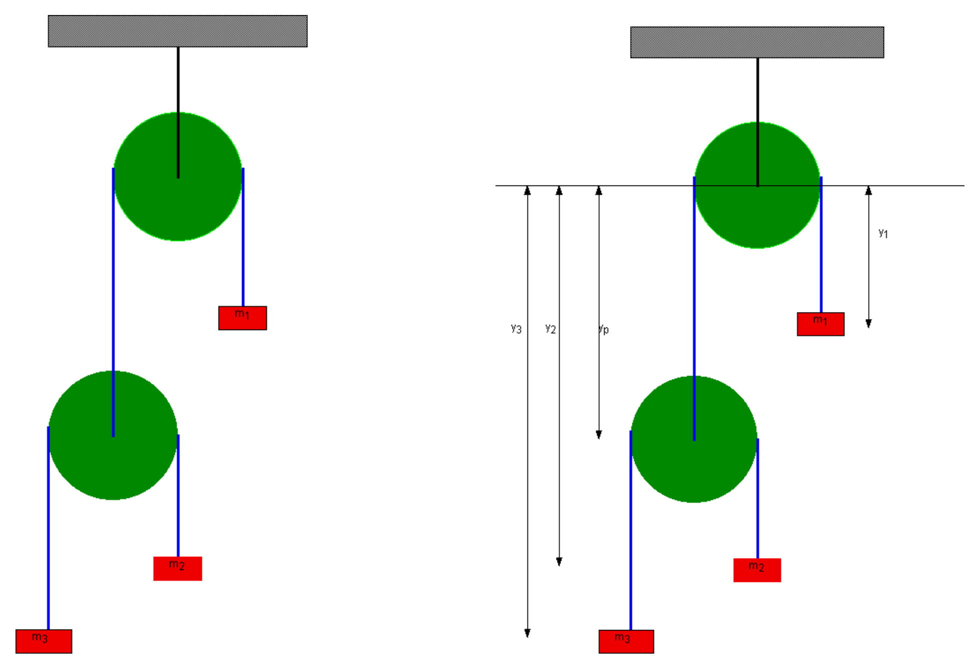
4.4 Constraint equations for the accelerations

In the system shown in this Fig., the cords are flexible and inextensible, and the pulleys have no friction and negligible mass.

(a) After release from rest, what is the acceleration of m_1?

(b) What is the tension in the cord supporting the upper pulley?

(c) Is there any set of values for m_2 and m_3 such that m_1 will not move even when the system is released.



In this problem we note that the length of the cords is constant.

$$y_3 - y_p + y_2 - y_p = const$$

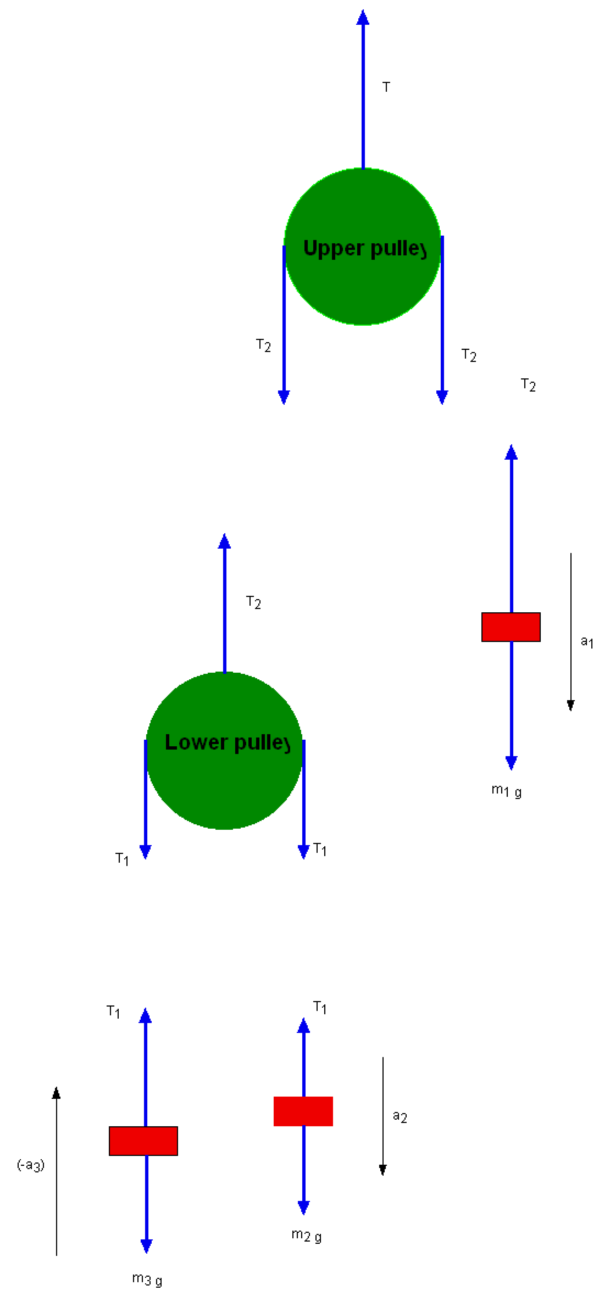
$$y_1 + y_p = const$$

In other words,

$$a_3 - 2a_p + a_2 = 0$$
$$a_1 + a_p = 0$$
, or $\boxed{a_3 + 2a_1 + a_2 = 0}$

This is the constraint equation needed in this problem.

Free-body diagram



$$T = 2T_2$$

$$T_2 = 2T_1$$
$$T_1 - m_3 g = m_3(-a_3)$$
$$m_2 g - T_1 = m_2 a_2$$
$$m_1 g - T_2 = m_1 a_1$$
$$2a_1 + a_2 + a_3 = 0$$

((Mathematica))

```
F1 = {T2 == 2 T1, T1 - m3 g == m3 (-a3),
  m2 g - T1 == m2 a2, m1 g - T2 == m1 a1,
  2 a1 + a2 + a3 == 0}
```

```
{T2 == 2 T1, -g m3 + T1 == -a3 m3,
 g m2 - T1 == a2 m2, g m1 - T2 == a1 m1,
 2 a1 + a2 + a3 == 0}
```

```
eq1 = Solve[F1, {a1, a2, a3, T1, T2}]
```

$$\left\{\left\{a3 \to g - \frac{4\ g\ m1\ m2}{m1\ m2 + m1\ m3 + 4\ m2\ m3},\ a1 \to g - \frac{8\ g\ m2\ m3}{m1\ m2 + m1\ m3 + 4\ m2\ m3},\ a2 \to g - \frac{4\ g\ m1\ m3}{m1\ m2 + m1\ m3 + 4\ m2\ m3},\ T1 \to \frac{4\ g\ m1\ m2\ m3}{m1\ m2 + m1\ m3 + 4\ m2\ m3},\ T2 \to \frac{8\ g\ m1\ m2\ m3}{m1\ m2 + m1\ m3 + 4\ m2\ m3}\right\}\right\}$$

```
a11 = a1 /. eq1[[1]] // Factor // Simplify
```

$$\frac{g\ (-4\ m2\ m3 + m1\ (m2 + m3))}{4\ m2\ m3 + m1\ (m2 + m3)}$$

```
2 T2 /. eq1[[1]]
```

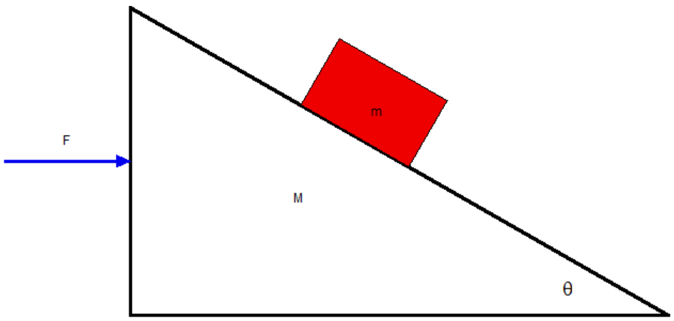
$$\frac{16\ g\ m1\ m2\ m3}{m1\ m2 + m1\ m3 + 4\ m2\ m3}$$

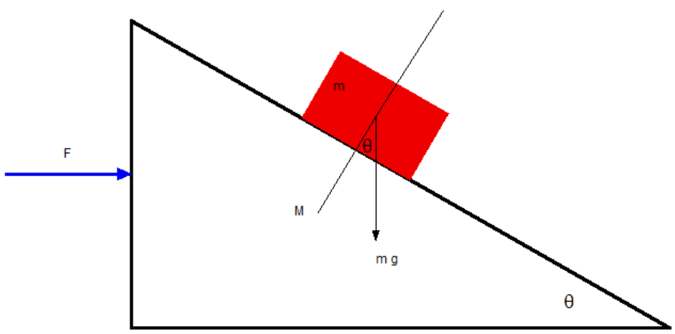
```
Solve[a11 == 0, m1]
```

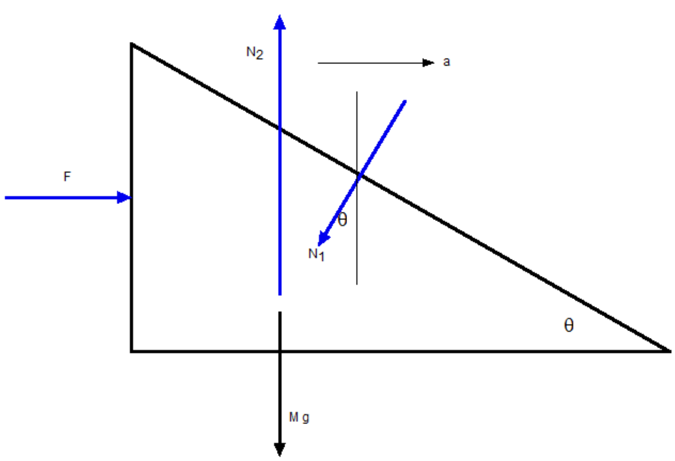
$$\left\{\left\{m1 \to \frac{4\ m2\ m3}{m2 + m3}\right\}\right\}$$

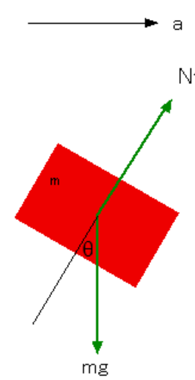
4.5 Moving wedge (Schaum's Outline M. Browne) p.70 Problem 5.18

A small block of mass m is placed on a wedge of angle θ and mass M. Friction is negligible. What horizontal force must be applied to the wedge so that the small block does not slide up or down the wedge surface.









$$N_1 \sin\theta = ma$$

$$N_1 \cos\theta = mg$$

$$F - N_1 \sin\theta = Ma$$

$$N_2 - N_1 \cos\theta = Mg$$

or

$$a = g \tan\theta$$
$$F = Ma + N_1 \sin\theta = (M + m)a = (M + m)g \tan\theta$$

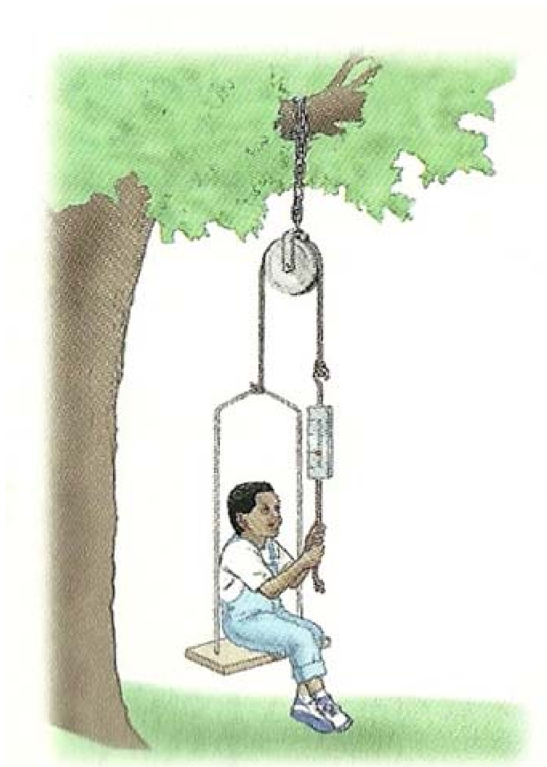
4.6 Problem 5-51 (Serway and Jewett) (Advanced problem)

An inventive child named Pat wants to reach an apple in a tree without climbing the tree. Sitting in a chair connected to a rope that passes over a frictionless pulley. Pat pulls on the loose end of the rope with such a force that the spring scale reads 250 N. Pats true weight is 320 N, and the chair weighs 160 N.

(a) Draw free-body diagrams for Pat and the chair considered as separate systems, and another diagram for Pat and the chair considered as one system.

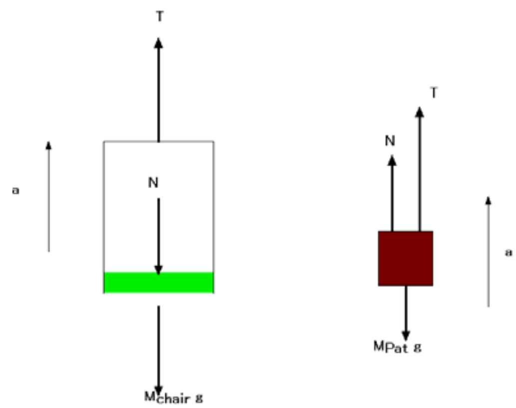
(b) Show that the acceleration of the system is upward and find its magnitude.

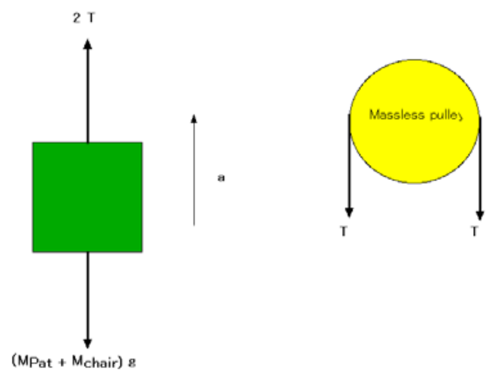
(c) Find the force Pat exerts on the chair.



((**Solution**))

Free-body diagrams:





g = 9.8 m/s^2, $M_{\text{Chair}}\, g$ = 160 N, M_{Patt} = 320 N, T = 250 N.
Newton's 2nd law:

$$T + N - M_{Pat}g = M_{Pat}a \qquad (1)$$

$$T - N - M_{Chair}g = M_{Chair}a \qquad (2)$$

Eq.(1) + Eq.(2):

$$2T - (M_{Patt} + M_{Chair})g = (M_{Patt} + M_{Chair})a$$

We have

N = 83.3 N
a = 0.408 m/s^2.

((**Mathematica**))

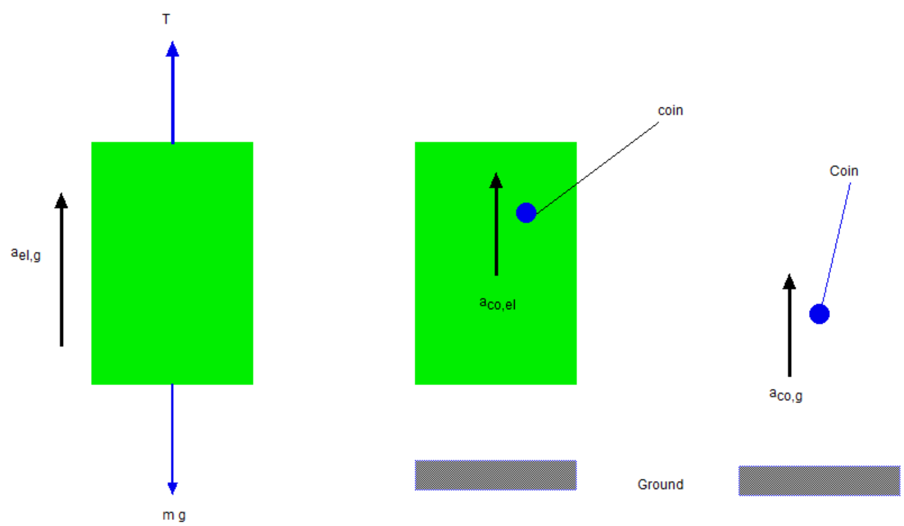
```
Clear["Global`*"]; eq1 = T1 + N1 - Mpat g == Mpat a;
eq2 = T1 - N1 - Mchair g == Mchair a;
rule1 = {g → 9.8, T1 → 250, Mchair → 160 / 9.8, Mpat → 320 / 9.8};
eq11 = eq1 /. rule1; eq22 = eq2 /. rule1;
Solve[{eq11, eq22}, {N1, a}]
```

```
{{N1 → 83.3333, a → 0.408333}}
```

5. Relative motion

Problem 5-46 (SP-5) (10-th edition)**

An elevator cab is pulled upward by a cable. The cab and its single occupant have a combined mass of 2000 kg. When that occupant drops a coin, its acceleration relative to the cab is 8.00 m/s^2 downward. What is the tension in the cable.



$$T - mg = ma_{el,g}$$

$$a_{co,e_l} = a_{co,g} + a_{g,el} = a_{co,g} - a_{el,g}$$

where co denotes coin, el, denotes elevator, and g denotes ground.

$$a_{co,el} = -8.0 m/s^2$$
$$a_{co,g} = -9.8 m/s^2$$

Then we have

$$a_{el,g} = a_{co,g} - a_{co,e_l} = -9.8 - (-8.0) = -1.8 m/s^2$$

The tension T is given by

$$T = m(g + a_{el}, g) = 2000 \times (9.8 - 1.8) = 16{,}000N = 16.0kN$$

6. Summary Problem-Solving Hints (Newton's law)

1. Conceptualize the problem – draw a diagram
2. Categorize the problem
3. Equilibrium ($\sum \boldsymbol{F} = 0$) or Newton's Second Law ($\sum \boldsymbol{F} = m\boldsymbol{a}$)
4. Analyze
   Draw free-body diagrams for each object
   Include only forces acting on the object
   Establish coordinate system
   Be sure units are consistent
   Apply the appropriate equation(s) in component form
   Solve for the unknown(s)
5. Finalize
   Check your results for consistency with your free- body diagram
   Check extreme values

REFERENCES

I. Newton, Principia Mathematica, New translation by I.B. Cohen and A. Whitman (University of California Press, Berkeley, 1999).

D.T. Whiteside, The Mathematical Principles underlying Newton's Principia Mathematica (University of Glasgow, 1970).

S. Chandrasekhar, Newton's Principia for the Common Reader (Oxford, 1995).

J.B. Brackenridge, The Key to Newton's Dynamics (University of California Press, 1995).

N. Guicciardini, Reading the Principia: The Debate on Newton's Mathematical Methods for Natural Philosophy from 1687 to 1736 (Cambridge, 1999).

C. Pask, Magnificient Principia: Exploring Isaac Newton's Materpiece (Prometheus Book, 2013).

A.Rupert Hall. Isac Newton, Adventure in Thought (Cambridge, 1992).

R.S. Westfall, The Life of Isaac Newton, 7-th edition (Cambridge, 2007)

J. Gleick, Isaac Newton (Vintage Books, 2004).

APPENDIX-I

Newton's law is clearly described in Principia Mathematica (1687) by Iasc Newton.

Isac Newton; Principia Mathematica (1687) p.13-15
Newton's first, second, and third laws

AXIOMS, OR LAWS OF MOTION[1]

LAW I

Every body continues in its state of rest, or of uniform motion in a right line, unless it is compelled to change that state by forces impressed upon it.

PROJECTILES continue in their motions, so far as they are not retarded by the resistance of the air, or impelled downwards by the force of gravity. A top, whose parts by their cohesion are continually drawn aside from rectilinear motions, does not cease its rotation, otherwise than as it is retarded by the air. The greater bodies of the planets and comets, meeting with less resistance in freer spaces, preserve their motions both progressive and circular for a much longer time.

LAW II[2]

The change of motion is proportional to the motive force impressed; and is made in the direction of the right line in which that force is impressed.

If any force generates a motion, a double force will generate double the motion, a triple force triple the motion, whether that force be impressed altogether and at once, or gradually and successively. And this motion (being always directed the same way with the generating force), if the body moved before, is added to or subtracted from the former motion, according as they directly conspire with or are directly contrary to each other; or obliquely joined, when they are oblique, so as to produce a new motion compounded from the determination of both.

LAW III

To every action there is always opposed an equal reaction: or, the mutual actions of two bodies upon each other are always equal, and directed to contrary parts.

Whatever draws or presses another is as much drawn or pressed by that other. If you press a stone with your finger, the finger is also pressed by the

[1 Appendix, Note 14.] [2 Appendix, Note 15.]

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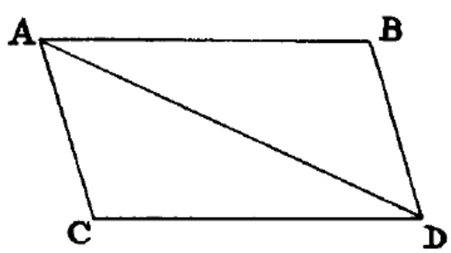
[[1] Appendix, Note 14.] [[2] Appendix, Note 15.]

stone. If a horse draws a stone tied to a rope, the horse (if I may so say) will be equally drawn back towards the stone; for the distended rope, by the same endeavor to relax or unbend itself, will draw the horse as much towards the stone as it does the stone towards the horse, and will obstruct the progress of the one as much as it advances that of the other. If a body impinge upon another, and by its force change the motion of the other, that body also (because of the equality of the mutual pressure) will undergo an equal change, in its own motion, towards the contrary part. The changes made by these actions are equal, not in the velocities but in the motions of bodies; that is to say, if the bodies are not hindered by any other impediments. For, because the motions are equally changed, the changes of the velocities made towards contrary parts are inversely proportional to the bodies. This law takes place also in attractions, as will be proved in the next Scholium.

COROLLARY I

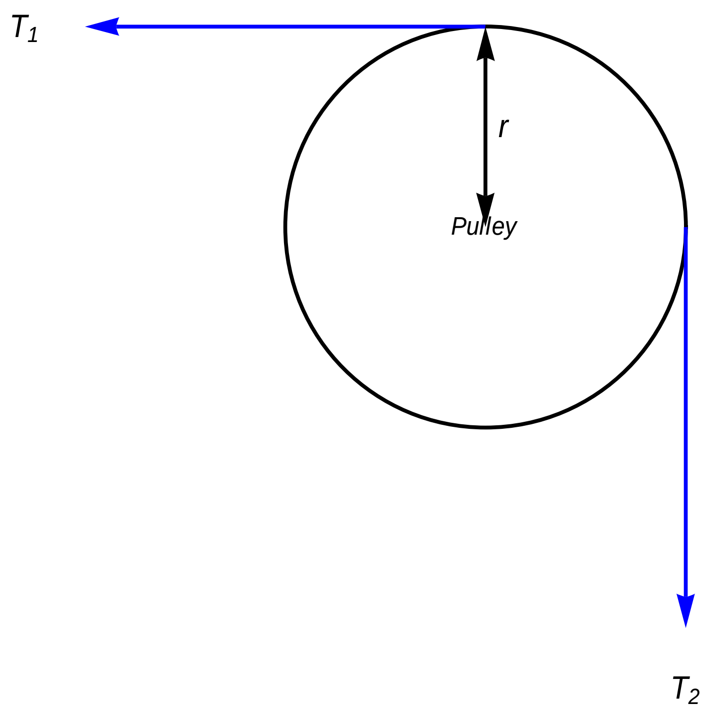
A body, acted on by two forces simultaneously, will describe the diagonal of a parallelogram in the same time as it would describe the sides by those forces separately.

If a body in a given time, by the force M impressed apart in the place A, should with an uniform motion be carried from A to B, and by the force N impressed apart in the same place, should be carried from A to C, let the parallelogram ABCD be completed, and, by both forces acting together, it will in the same time be carried in the diagonal from A to D. For since the force N acts in the direction of the line AC, parallel to BD, this force (by the second Law) will not at all alter the velocity generated by the other force M, by which the body is carried towards the line BD. The body therefore will arrive at the line BD in the same time, whether the force N be impressed or not; and therefore at the end of that time it will be found somewhere in the line BD. By the same argument, at the end of the same time it will be found somewhere in the line CD. Therefore it will be found in the point D, where both lines meet. But it will move in a right line from A to D, by Law I.



APPENDIX-II Role of pulley

(a) Pulley-I



We consider the case when the pulley is at rest. We show that

$$T_1 = T_2$$

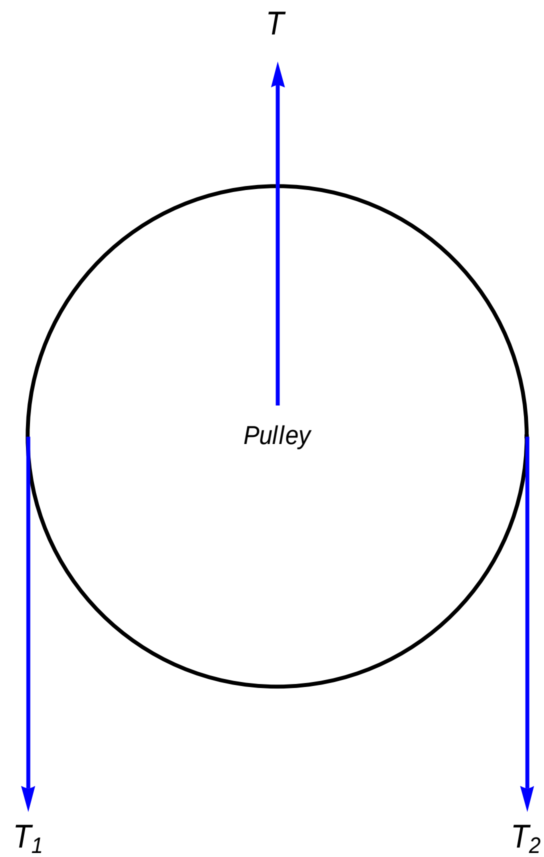
when the mass of pulley is zero. We consider the torque applied on the pulley. Suppose that the radius of the pulley is r. We apply the Newton's 2nd law for the rotation (we will discuss this later chapter) around the center of pulley. Then we have

$$\text{Torque} = r(T_2 - T_1) = I\alpha$$

where I is the moment of inertia for the pulley; $I = Mr^2$ (M is the mass of pulley). When $M = 0$, we get $I = 0$, leading to

$$T_2 = T_1 .$$

(b) Pulley-II



In the above figure, we assume that the mass of pulley is zero. The pulley is at rest. Then we have

$$T = T_1 + T_2$$

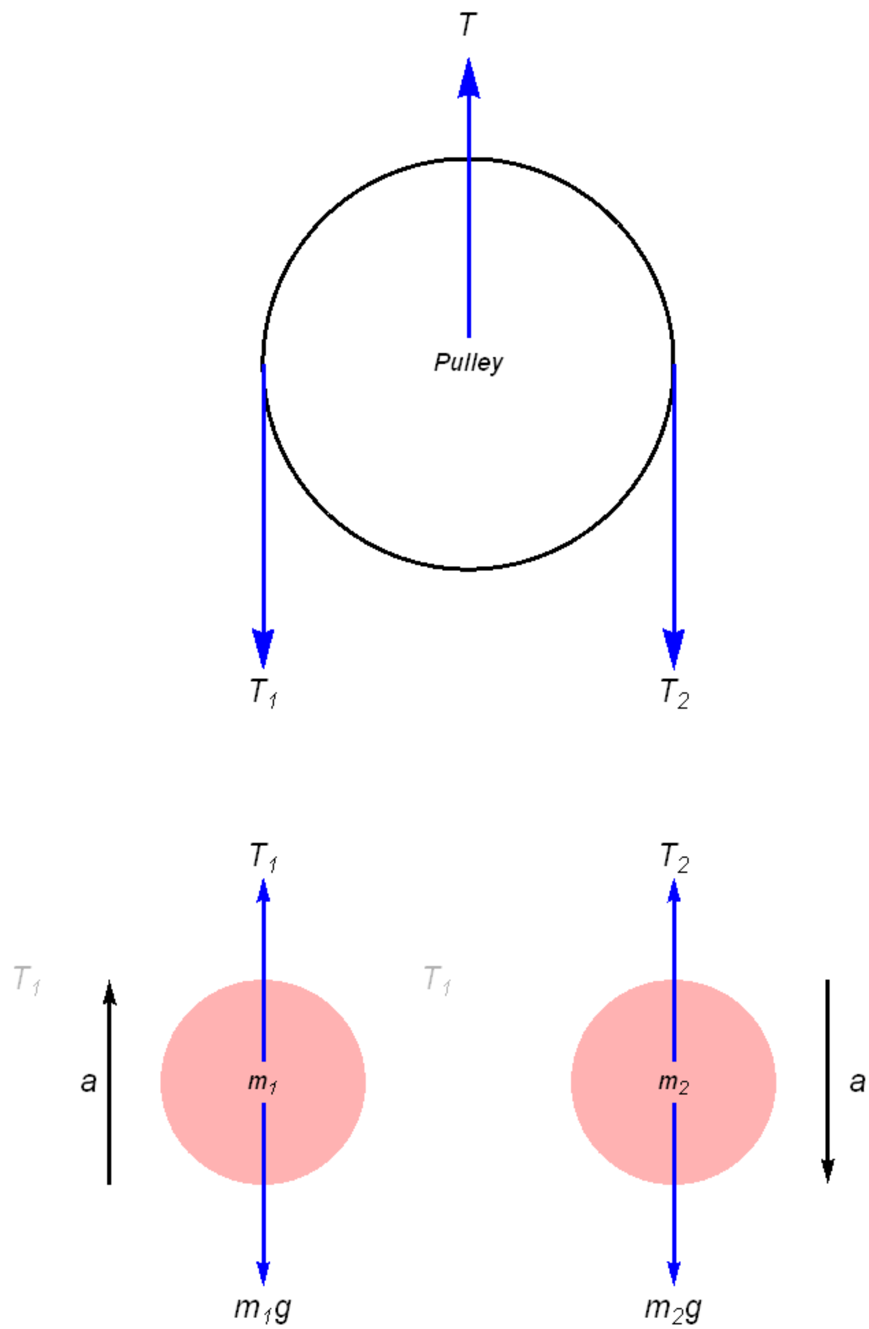
We consider the torque applied on the pulley. Suppose that the radius of the pulley is *r*. We apply the Newton's 2nd law for the rotation (we will discuss this later chapter) around the center of pulley. Then we have

$$\text{Torque} = r(T_2 - T_1) = I\alpha \,,$$

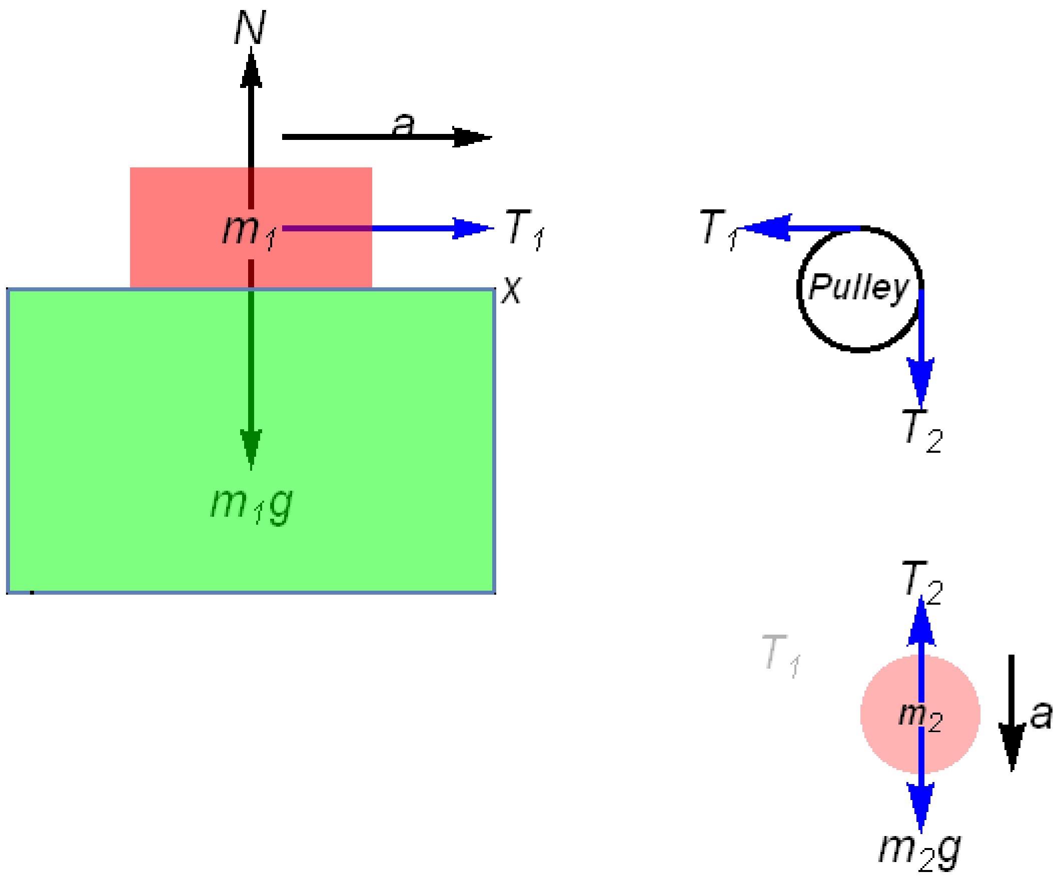
where α is I is the moment of inertia for the pulley; $I = Mr^2$ (*M* is the mass of pulley). When $M = 0$, we get $I = 0$, leading to

$$T_2 = T_1 . \qquad T = 2T_1 = 2T_2$$

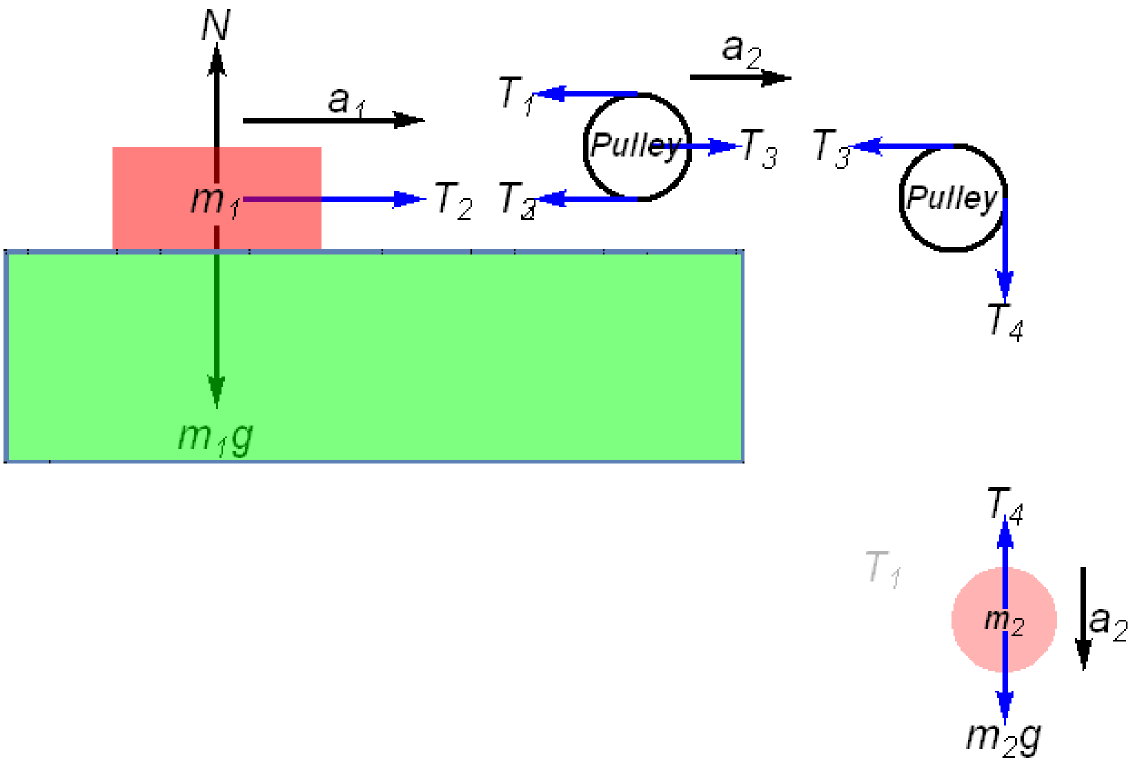
(c) Atwood's machine



(d)



(e)



$T_1 = T_2$