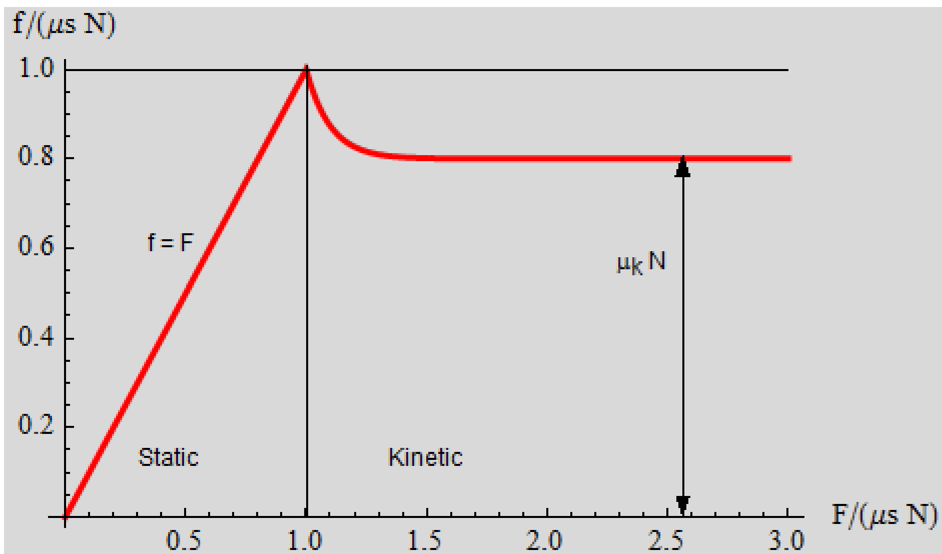
Lecture Note
Chapter 6

1. Overview: friction force

Friction forces are categorized as either static or kinetic. The coefficient of static friction μ_s characterizes friction when no movement exists between the two surfaces in question, and the kinetic coefficient μ_k characterizes friction where motion occurs.

μ_s	coefficient of static friction
μ_k	coefficient of kinetic friction

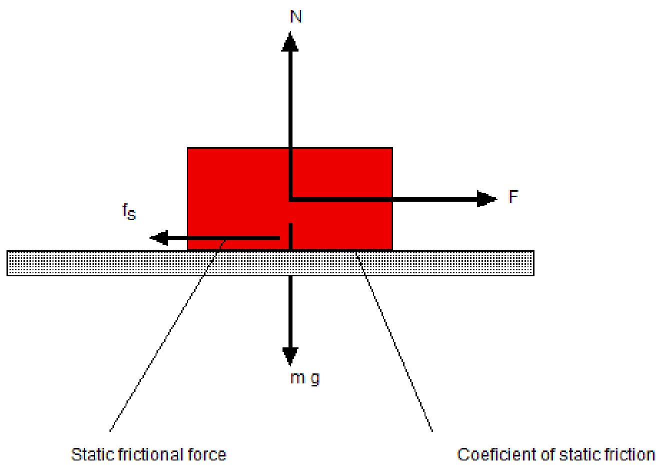


2. Friction force

2.1 Static friction

The static friction force must be overcome by an applied force before an object can move. The maximum possible friction force between two surfaces before sliding begins is the product of the coefficient of static friction and the normal force: $f_{max} = \mu_s N$. When there is no sliding occurring, the friction force can have any value from zero up to f_{max}. Any force smaller than f_{max} attempting to slide one surface over the other is opposed by a frictional force of equal magnitude and opposite direction.

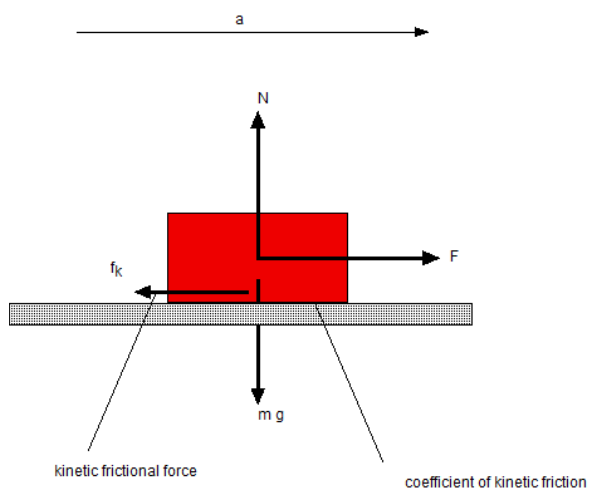
$$f_s \leq (f_s)_{\max} = \mu_s N$$



2.2 Kinetic friction

Any force larger than f_{max} overcomes the force of static friction and causes sliding to occur. The instant that sliding occurs, kinetic friction is applicable and static friction is no longer relevant. When one surface is sliding over the other, the friction force between them is always the same and is given by the product of the coefficient of kinetic friction and the normal force: $f_k = \mu_k N$.

$$f_k = \mu_k N$$



$$\begin{aligned} N &= mg \\ F - f_k &= ma, \\ f_k &= \mu_k N \end{aligned}$$

or

$$\begin{aligned} F &= f_k + ma \\ f = f_k &= \mu_k N = const \end{aligned}$$

2.3 Property of friction

The coefficient of static friction is larger than the coefficient of kinetic friction, since it takes more force to make surfaces start sliding over each other than it does to keep them sliding once started.

$$\mu_s > \mu_k$$

The friction force is directed in the opposite direction of the resultant force acting on a body.

3 Drag force and terminal speed

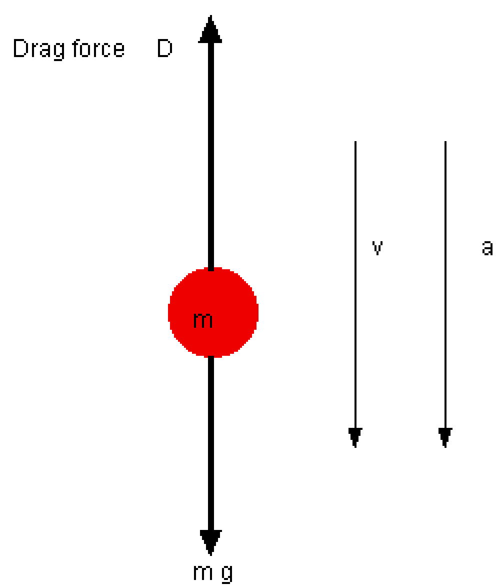
In fluid dynamics, drag (sometimes called fluid resistance) is the force that resists the movement of a solid object through a fluid (a liquid or gas). The most familiar form of drag is made up of friction forces, which act parallel to the object's surface, plus pressure forces, which act in a direction perpendicular to the object's surface.

Drag coefficient *D*

$$D = \frac{1}{2} C \rho A v^2$$

where

ρ is the air density,
A is the effective cross-sectional area,
C is the drag coefficient (typically, $C = 0.4 - 1.0$).



Newton's second law

$$\sum F = mg - kv^2 = m\frac{d}{dt}v$$

where

$$D = kv^2$$

with

$$k = \frac{1}{2}C\rho A$$

What is the terminal velocity?

When $\frac{dv}{dt} = 0$,

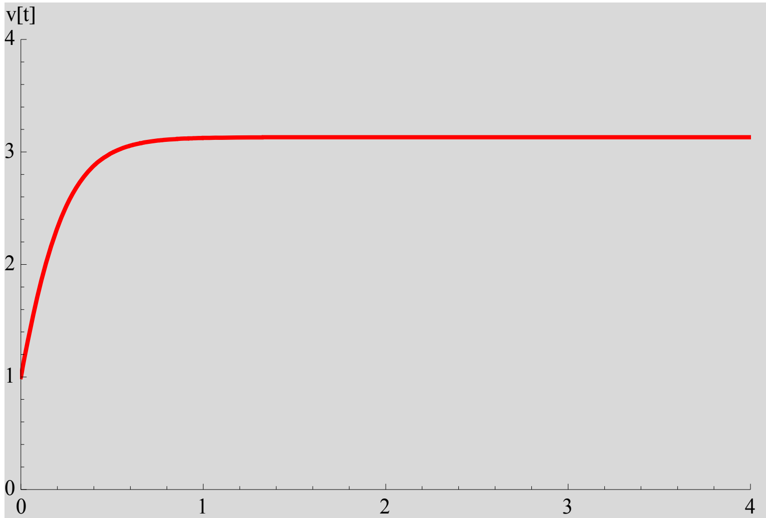
$$v = \sqrt{\frac{mg}{k}} \qquad \text{(terminal velocity)}$$

This is the first order differential equation

$$v(t) = \sqrt{\frac{mg}{k}}\tanh[\sqrt{\frac{kg}{m}}t + Arc\tanh\left(\sqrt{\frac{k}{mg}}v_0\right)]$$

For $m = 1$, $k = 1$, $g = 9.8$, and $v_0 = 1$.

We have the time dependence of $v(t)$.



((**Mathematica**))
Using Mathematica, you can easily solve the first-order differential equation and make a plot of the velocity as a function of *t* for typical numerical values of the initial velocity.

```
eq1 = {m g - k v[t]^2 == m v'[t], v[0] == v0}
```

$\{g\,m - k\,v[t]^2 == m\,v'[t],\ v[0] == v0\}$

```
eq2 = DSolve[eq1, v[t], t] // Simplify
```

$$\left\{\left\{v[t] \rightarrow \frac{\sqrt{g}\,\sqrt{m}\,\mathrm{Tanh}\left[\frac{\sqrt{g}\,\sqrt{k}\,t}{\sqrt{m}} + \mathrm{ArcTanh}\left[\frac{\sqrt{k}\,v0}{\sqrt{g}\,\sqrt{m}}\right]\right]}{\sqrt{k}}\right\}\right\}$$

```
v1 = v[t] /. eq2[[1]]
```

$$\frac{\sqrt{g}\,\sqrt{m}\,\mathrm{Tanh}\left[\frac{\sqrt{g}\,\sqrt{k}\,t}{\sqrt{m}} + \mathrm{ArcTanh}\left[\frac{\sqrt{k}\,v0}{\sqrt{g}\,\sqrt{m}}\right]\right]}{\sqrt{k}}$$

```
rule1 = {m → 1, k → 1, g → 9.8, v0 → 1}
```

{m → 1, k → 1, g → 9.8, v0 → 1}

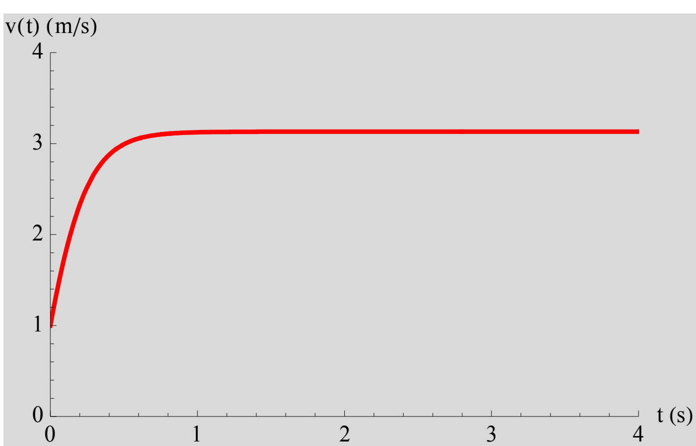
```
v11 = v1 /. rule1
```

3.1305 Tanh[0.331021 + 3.1305 t]

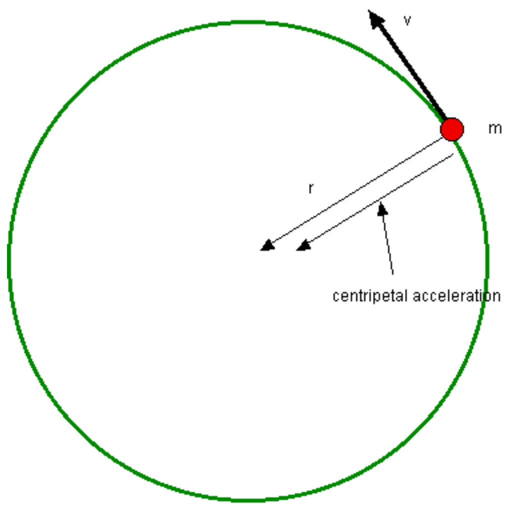
```
Limit[v11, t → ∞]
```

3.1305

```
Plot[v11, {t, 0, 4},
 PlotRange → {{0, 4}, {0, 4}},
 AxesLabel → {"t (s)", "v(t) (m/s)"},
 PlotStyle → {Thick, Red},
 Background → LightGray]
```



4 Uniform circular motion



a_r: centripetal acceleration

Newton's second law

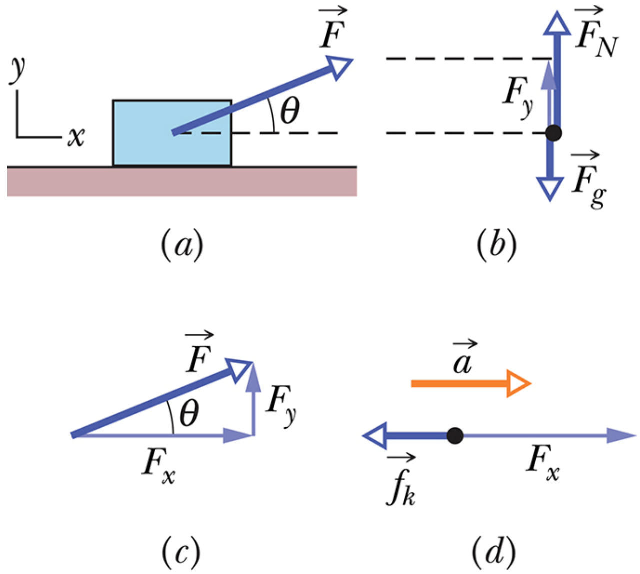
$$F_r = ma_r = m\frac{v^2}{r}$$

5 Sample problems

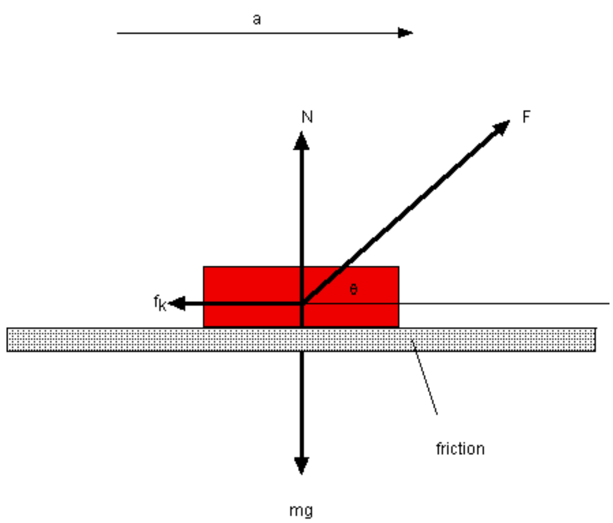
5.1 Friction

5.1.1 Sample Problem 6-2

In Fig.a, a block of mass m = 3.0 kg slides along a floor while a force $\boldsymbol{F}$ of magnitude 12.0 N is applied to it at an upward angle θ. The coefficient of kinetic friction between the block and the floor is μ_k = 0.40.. We can vary θ from 0 to 90° (the block remains on the floor). What θ gives the maximum value of the block's acceleration magnitude a?



Free-body diagram



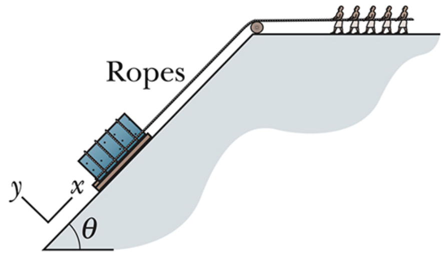
Newton's second law

$$
\begin{aligned}
&N + F\sin\theta - mg = 0\\
&F\cos\theta - f_k = ma\\
&f_k = \mu_k N
\end{aligned}
$$

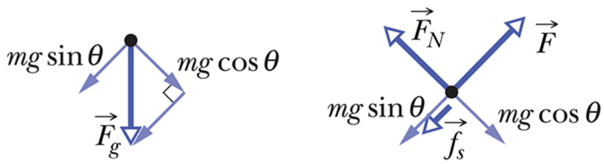
$$a = \frac{F\cos\theta + \mu_k F\sin\theta - \mu_k mg}{m}$$

5.1.2 Sample Problem 6-3

Although many ingenious schemes have been attributed to the building of the Great Pyramid, the stone blocks were probably hauled up the side of the pyramid by men pulling on ropes. Figure a represents a 2000 kg stone block in the process of being pulled up the finished (smooth) side of the Great Pyramid, which forms a plane inclined at angle . The block is secured to a wood sled and is pulled by multiple ropes (only one is shown). The sled's track is lubricated with water to decrease the coefficient of static friction to 0.40. Assume negligible friction at the (lubricated) point where the ropes pass over the edge at the top of the side. If each man on top of the pyramid pulls with a (reasonable) force of 686 N, how many men are needed to put the block on the verge of moving?

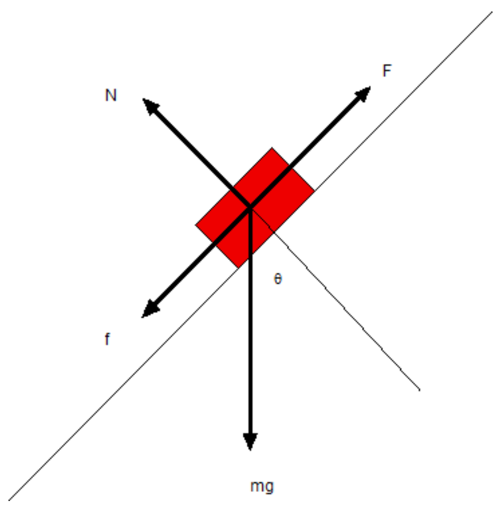


(a)



(b) (c)

Free-body diagram



Newton's second law

$$N = mg\cos\theta$$
$$F - f_s - mg\sin\theta = 0$$
$$f_s \le \mu_s N$$

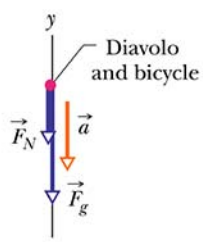
$$F \le mg(\mu_s \cos\theta + \sin\theta)$$

5.2 Circular motion

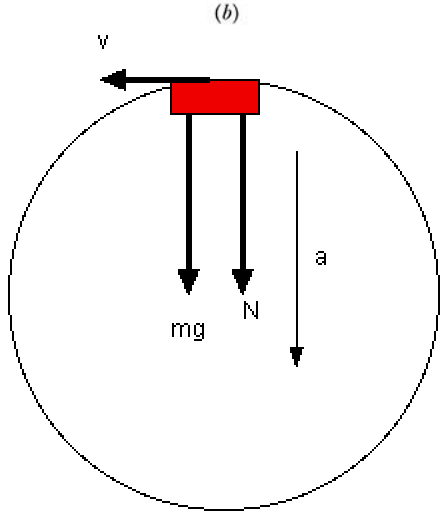
5.2.1 Sample Problem 6-7

In a 1901 circus performance, Allo "Dare Devil" Diavolo introduced the stunt of riding a bicycle in a loop-the-loop (Fig. a). Assuming that the loop is a circle with radius $R = 2.7$ m, what is the least speed v Diavolo could have at the top of the loop to remain in contact with it there?

(*a*)



(*b*)



$R = 2.7$ m

$$\sum F_y = N + mg = m\frac{v^2}{R}$$

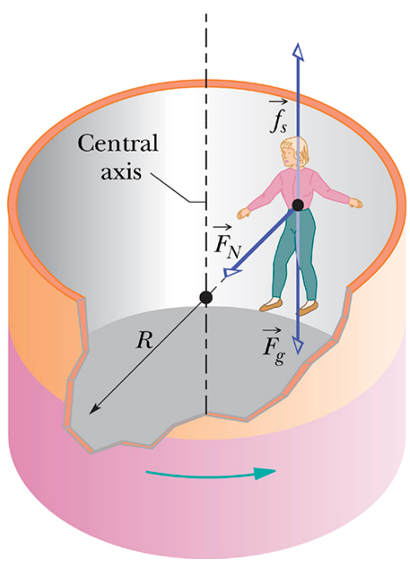
$$N = m(\frac{v^2}{R} - g) \geq 0$$

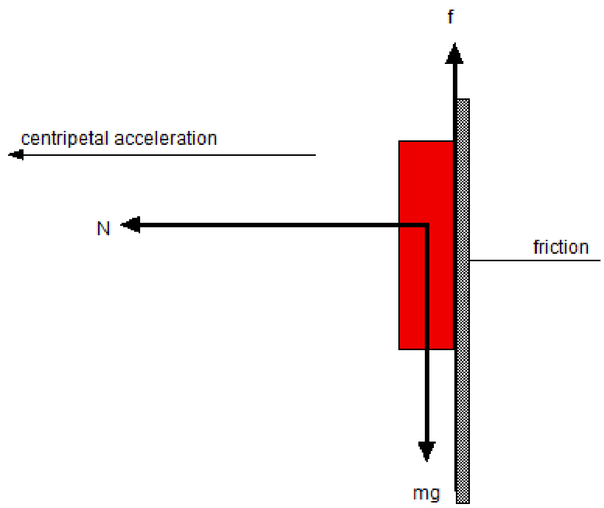
or

$$v \geq \sqrt{gR} = 5.1 m/s$$

5.2.2 Sample Problem 6-8

Even some seasoned roller-coaster riders blanch at the thought of riding the Rotor, which is essentially a large, hollow cylinder that is rotated rapidly around its central axis (Fig. 6-11). Before the ride begins, a rider enters the cylinder through a door on the side and stands on a floor, up against a canvas-covered wall. The door is closed, and as the cylinder begins to turn, the rider, wall, and floor move in unison. When the rider's speed reaches some predetermined value, the floor abruptly and alarmingly falls away. The rider does not fall with it but instead is pinned to the wall while the cylinder rotates, as if an unseen (and somewhat unfriendly) agent is pressing the body to the wall. Later, the floor is eased back to the rider's feet, the cylinder slows, and the rider sinks a few centimeters to regain footing on the floor. (Some riders consider all this to be fun.)





$m = 49$ kg, $\mu_s = 0.40$

$$\sum F_r = N = m\frac{v^2}{R}$$
$$\sum F_y = f = mg$$
$$f \leq \mu_s N$$

From these equation we obtain

$$mg \leq \mu_s m\frac{v^2}{R}$$

or

$$v \geq \sqrt{\frac{gR}{\mu_s}} = 7.2 m/s$$

5.2.3 Sample Problem 6-6

Igor is a cosmonaut on the International Space Station, in a circular orbit around Earth, at an altitude *h* of 520 km and with a constant speed v of 7.6 km/s. Igor's mass *m* is 79 kg.

(a) What is his acceleration?
(b) What force does Earth exert on Igor?

((**Gravitational constant**))

The gravitational constant *G* is a key element in Newton's law of universal gravitation. The **gravitational constant**, denoted *G*, is a physical constant involved in the calculation of the gravitational attraction between objects with mass. It appears in Newton's law of universal gravitation and in Einstein's theory of general relativity. It is also known as the

universal gravitational constant, **Newton's constant**, and colloquially *G*. It should not be confused with "little g" (*g*), which is the local gravitational field (equivalent to the local acceleration due to gravity), especially that at the Earth's surface; see Earth's gravity and standard gravity.

According to the law of universal gravitation, the attractive force (F) between two bodies is proportional to the product of their masses (m_1 and m_2), and inversely proportional to the square of the distance (r) between them:

$$F = G\frac{m_1 m_2}{r^2}$$

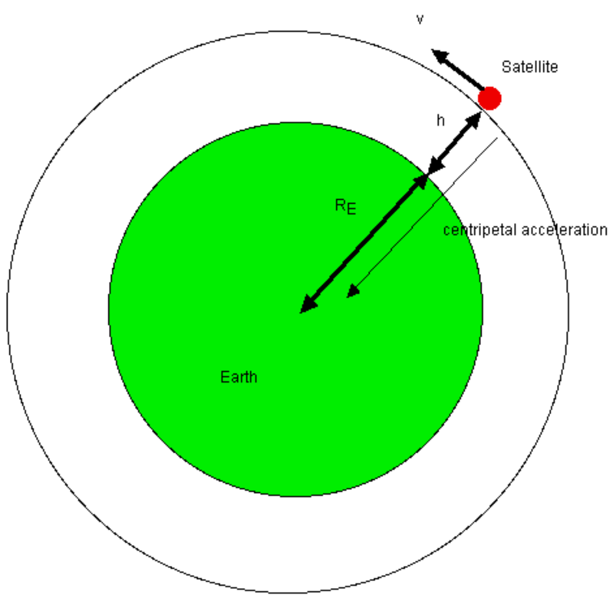
The constant of proportionality, G, is the gravitational constant.

$$G = 6.67384 \text{ x } 10^{-11} \text{ N m}^2/\text{kg}^2$$

((**Note**))

The **gravitational constant**, approximately 6.67×10^{-11} N·(m/kg)^2 and denoted by letter G, is an empirical physical constant involved in the calculation(s) of gravitational force between two bodies. It usually appears in Sir Isaac Newton's law of universal gravitation, and in Albert Einstein's general theory of relativity. It is also known as the **universal gravitational constant**, **Newton's constant**, and colloquially as **Big G**.
http://en.wikipedia.org/wiki/Gravitational_constant

Motion of satellite



$$r = R_E + h$$

$$\sum F_r = m\frac{M_E G}{r^2} = m\frac{v^2}{r}$$

We assume that $r = R_E$. In other words, h is much smaller than R_E.

$$v = \sqrt{\frac{M_E G}{R_E}} = 7.910 km/s = 4.9 miles/s = 17640 miles/h$$

$$T = \frac{2\pi R_E}{v} = 5061 s = 1 hour 24 \min 21 \sec$$

$$a = \frac{M_E G}{R_E{}^2} = 9.8 m/s^2$$

where

R_E is the radius of the Earth; $R_E = 6.372 \times 10^6$ m

M_E is the mass of the Earth; $M_E = 5.9736 \times 10^{24}$ kg

G is the gravitational constant; $G = 6.6742867 \times 10^{-11}$ N m^2/kg^2

((**Mathematica**))

G=gravitational constant (N m^2/kg^2),
Mea = 5.9736 x 10^{24} kg; Mass of the earth,
Rea=6372.797 km, radius of the earth,
Msun=mass of sun (kg) =Solar mass
Rsun=radius of Sun (m)=Solar radius
Mmoon=Mass of moon (m)
Rmoon=radius of moon (m)

```
Physconst = {G → 6.6742867 10^-11, Mea → 5.9736 10^24,
  Rea → 6.372  10^6 , Msun → 1.988435 10^30, Rsun → 6.9599 10^8,
  Mmoon → 7.3483 10^22, Rmoon → 1.738 10^6}
```

$\{G \to 6.67429\times10^{-11},\ Mea \to 5.9736\times10^{24},\ Rea \to 6.372\times10^{6},\ Msun \to 1.98844\times10^{30},\ Rsun \to 6.9599\times10^{8},\ Mmoon \to 7.3483\times10^{22},\ Rmoon \to 1.738\times10^{6}\}$

$$vea = \sqrt{\frac{Mea\ G}{Rea}}\ /.\ Physconst$$

7910.11

$$vsun = \sqrt{\frac{Msun\ G}{Rsun}}\ /.\ Physconst$$

436673.

$$vmoon = \sqrt{\frac{Mmoon\ G}{Rmoon}}\ /.\ Physconst$$

1679.85

$$\text{Tea} = \frac{2\,\pi\,\text{Rea}}{\text{vea}}\ /.\ \text{Physconst}$$

5061.43

$$\text{Tsun} = \frac{2\,\pi\,\text{Rsun}}{\text{vsun}}\ /.\ \text{Physconst}$$

10014.4

$$\text{Tmoon} = \frac{2\,\pi\,\text{Rmoon}}{\text{vmoon}}\ /.\ \text{Physconst}$$

6500.68

$$\text{aea} = \frac{\text{Mea G}}{\text{Rea}^2}\ /.\ \text{Physconst}$$

9.8195

$$\text{amoon} = \frac{\text{Mmoon G}}{\text{Rmoon}^2}\ /.\ \text{Physconst}$$

1.62365

$$\text{asun} = \frac{\text{Msun G}}{\text{Rsun}^2}\ /.\ \text{Physconst}$$

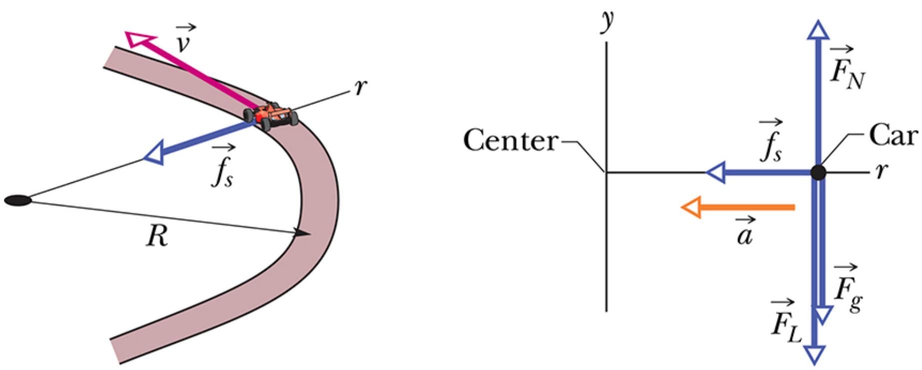
273.975

5.2.4 Sample Problem 6-9 Negative lift

Upside-down racing: A modern race car is designed so that the passing air pushes down on it, allowing the car to travel much faster through a flat turn in a Grand Prix without friction failing. This downward push is called negative lift. Can a race car have so much negative lift that it could be driven upside down on a long ceiling, as done fictionally by a sedan in the first Men in Black movie? Figure 6-12a represents a Grand Prix race car of mass $m = 600$ kg as it travels on a flat track in a circular arc of radius $R = 100$ m. Because of the shape of the car and the wings on it, the passing air exerts a negative lift $\boldsymbol{F}_L$ downward on the car. The coefficient of static friction between the tires and the track is 0.75. (Assume that the forces on the four tires are identical.)

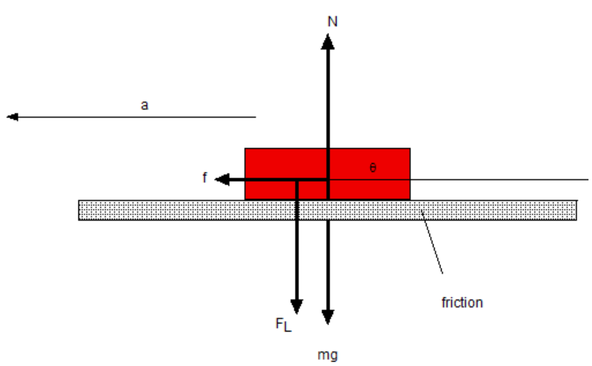
(a) If the car is on the verge of sliding out of the turn when its speed is 28.6 m/s, what is the magnitude of F_L?

(b) The magnitude FL of the negative lift on a car depends on the square of the car's speed v^2, just as the drag force does (Eq. 6-14). Thus, the negative lift on the car here is greater when the car travels faster, as it does on a straight section of track. What is the magnitude of the negative lift for a speed of 90 m/s?



(a) (b)

Free-body diagram



$v = 28.6$ m/s

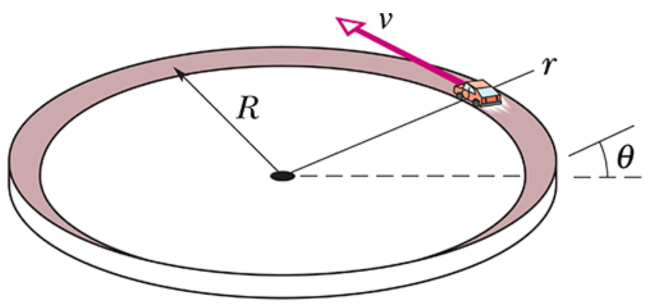
$$N = mg + F_L$$
$$f \le \mu_s N$$
$$\sum F_r = f = m\frac{v^2}{R}$$

When $f = \mu_s N$, we have

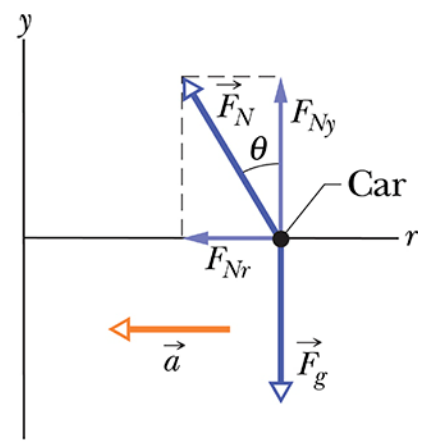
$$F_L = m(\frac{v^2}{\mu_s R} - g) = 660N$$

5.2.5 Sample Problem 6-10

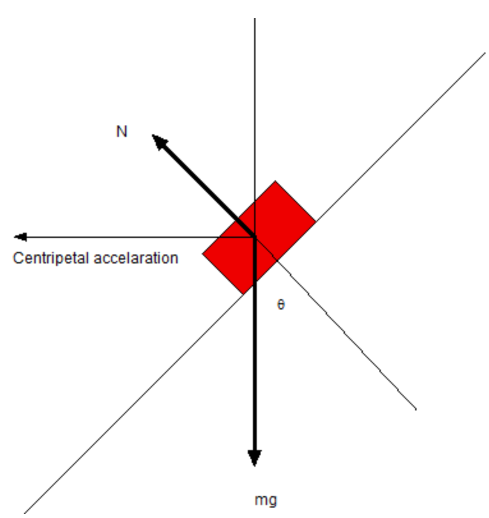
Curved portions of highways are always banked (tilted) to prevent cars from sliding off the highway. When a highway is dry, the frictional force between the tires and the road surface may be enough to prevent sliding. When the highway is wet, however, the frictional force may be negligible, and banking is then essential. Figure 6-13a represents a car of mass m as it moves at a constant speed v of 20 m/s around a banked circular track of radius R = 190 m. (It is a normal car, rather than a race car, which means any vertical force from the passing air is negligible.) If the frictional force from the track is negligible, what bank angle θ prevents sliding?



(a)



(b)



R = 190 m

$$\sum F_y = N\cos\theta - mg = 0$$
$$\sum F_x = N\sin\theta = m\frac{v^2}{r}$$

or

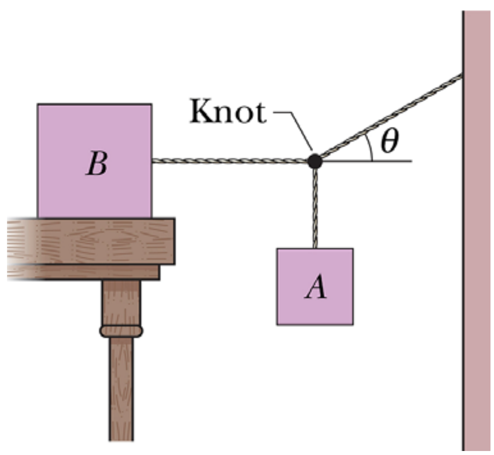
$$\tan\theta = \frac{v^2}{gR}$$

$$\theta = 12°$$

6 Selected Problems and Homeworks

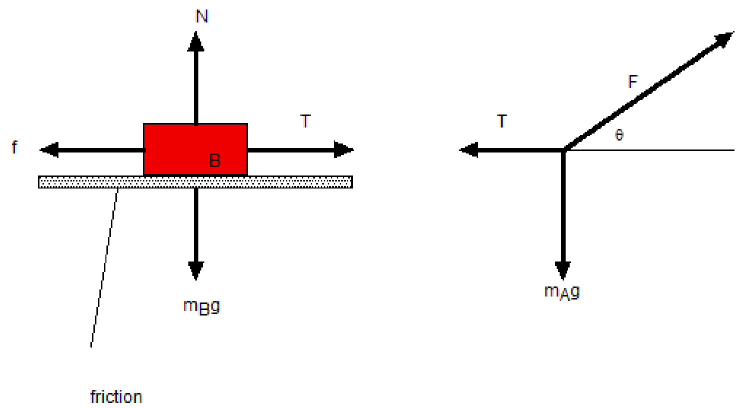
6.1
Problem 6-25 (SP-06) (10-th edition)

Block in Fig. weighs 711 N. The coefficient of static friction between block and table is 0.25; angle θ is 30°; assume that the cord between B and the knot is horizontal. Find the maximum weight of block A for which the system will be stationary.



$$\mu_s = 0.25$$
$$\theta = 30°$$
$$m_B g = 711N$$

Free-body diagram



Free body diagram

$$f \le \mu_s N$$
$$N = m_B g$$
$$T = f$$

$$F\cos\theta = T$$
$$F\sin\theta = m_A g$$

From these equations, we have

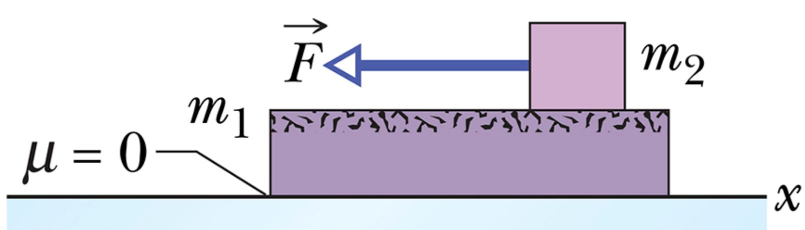
$$T = \frac{m_A g}{\tan\theta} = f \le \mu_s m_B g$$

or

$$m_A g \le \mu_s \tan\theta m_B g = 102.6N \, .$$

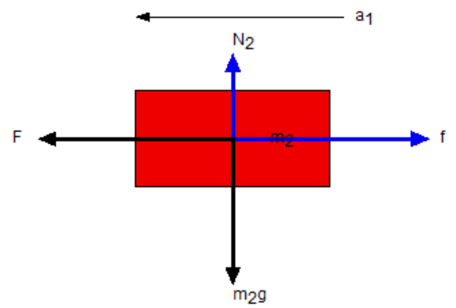
6.2
Problem 6-34 * (SP-06)** **(10-th edition)**

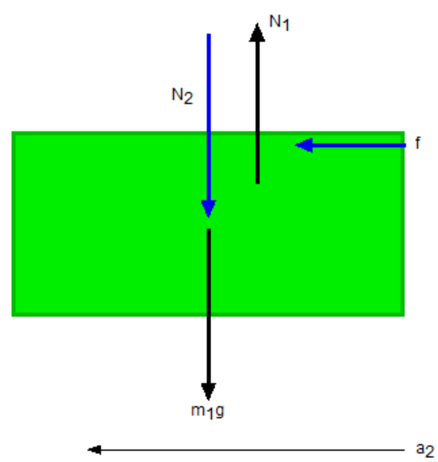
In Fig., a slab of mass m_1 = 40 kg rests on a frictionless floor, and a block of mass m_2 = 10 kg rests on top of the slab. Between block and slab, the coefficient is 0.60, and the coefficient of kinetic friction is 0.40. The block is pulled by a horizontal force ***F*** of magnitude 100 N. In unit-vector notation, what are the resulting accelerations of (a) the block and (b) the slab?



$m_1 = 40$ kg $\quad m_2 = 10$ kg $\quad F = 100$ N
$\mu_k = 0.4$, $\quad \mu_s = 0.6$

Free-body diagram





(a) Suppose that m_2 does not move on the mass m_1 $\qquad a_1 = a_2 = a$

$$F - f = m_2 a$$
$$N_2 = m_2 g$$
$$f = m_1 a$$
$$f \le \mu_s N_2$$
$$N_1 - N_2 - m_1 g = 0$$

We have

$$a = \frac{F}{m_1 + m_2} = \frac{100}{50} = 2 m/s^2$$

which does not satisfy the condition obtained from the above equations,

$$a \le \frac{\mu_s N_2}{m_1} = \frac{\mu_s m_2}{m_1} g = \frac{10}{40} 0.6 \times 9.8 = 1.47 m/s^2$$

So we can conclude that a_1 is not equal to a_2. In other words, the mass m_2 moves on the mass m_1.

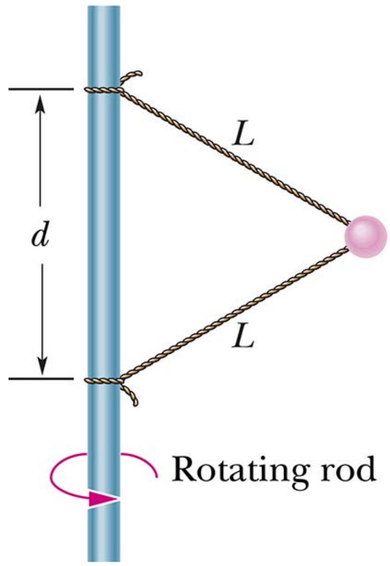
$$F - f = m_2 a_1$$
$$N_2 = m_2 g$$
$$f = \mu_k N_2$$
$$f = m_1 a_2$$
$$N_1 - N_2 - m_1 g = 0$$

or

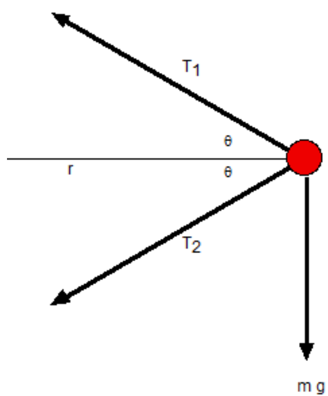
$$a_1 = \frac{F - f}{m_2} = \frac{F - \mu_k m_2 g}{m_2} = 6.1 m/s^2$$
$$N_2 = m_2 g$$
$$f = \mu_k m_2 g$$
$$a_2 = \mu_k \frac{m_2}{m_1} g = 0.98 m/s^2$$
$$N_1 = (m_1 + m_2) g$$

6.3
Problem 6-59* (SP-06) (10-th edition)**

In Fig., a 1.34 kg ball is connected by means of two massless strings, each of length L = 1.70 m, to a vertical rotating rod. The strings are tied to the rod with separation d = 1.70 m and are taut. The tension in the upper string is 35 N. What are the (a) tension in the lower string, (b) magnitude of the net force $\boldsymbol{F}_{net}$ on the ball, and (c) speed of the ball? (d) What is the direction of $\boldsymbol{F}_{net}$?



Free-body diagram



$\theta = 30°$, $L = 1.70\ m$, $d = 1.70\ m$
$T_1 = 35$ N, $m = 1.34$ kg

$$T_1 \cos\theta + T_2 \cos\theta = m\frac{v^2}{r}$$
$$T_1 \sin\theta = T_2 \sin\theta + mg$$
$$r = L\cos\theta$$

From these equations we have

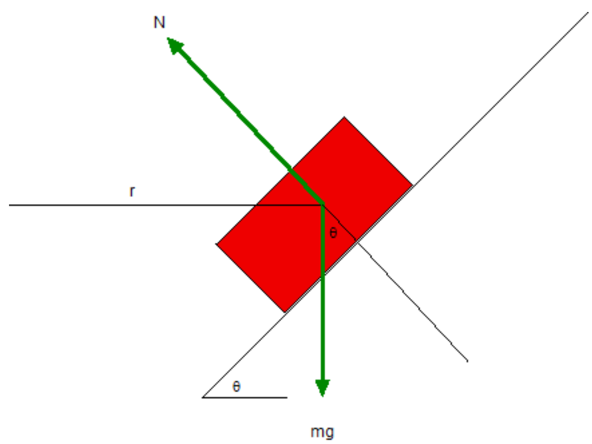
$$T_1 = 35N,$$
$$T_2 = 8.74N$$
$$v = 6.45m/s$$
$$r = 1.47m$$

6.4
Problem 6-92 (HW-06, Hint)   (10-th edition)

A circular curve of highway is designed for traffic moving at 60 km/h. Assume the traffic consists of cars without negative lift. (a) If the radius of the curve is 150 m, what is the correct angle of banking of the road? (b) If the curve were not banked, what would be the minimum coefficient of friction between tires and road that would keep traffic from skidding out of the turn when travelling at 60 km/h?

v = 60 km/h =16.7 m/s r = 150 m

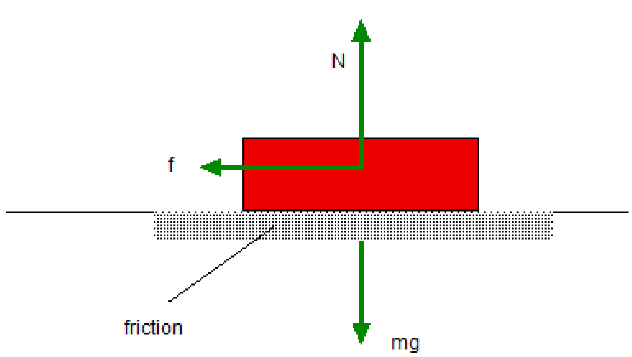
Free-body diagram



$$\sum F_y = N\sin\theta = m\frac{v^2}{r}$$
$$\sum F_x = N\cos\theta = mg$$

or

$$v = \sqrt{gr\tan\theta}$$



$$N = mg$$
$$f \le \mu_s N$$
$$\sum F_r = f = m\frac{v^2}{r}$$

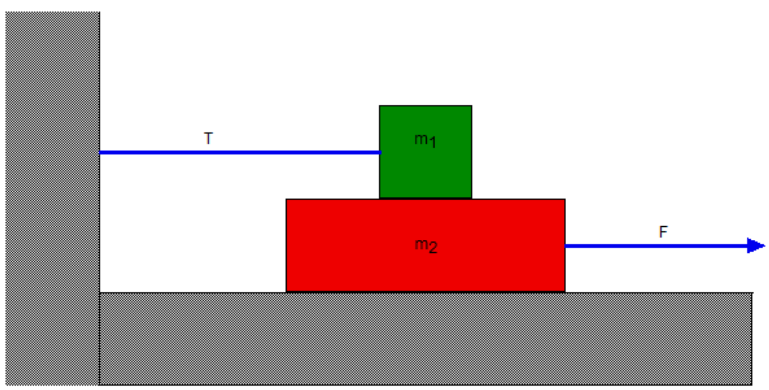
or

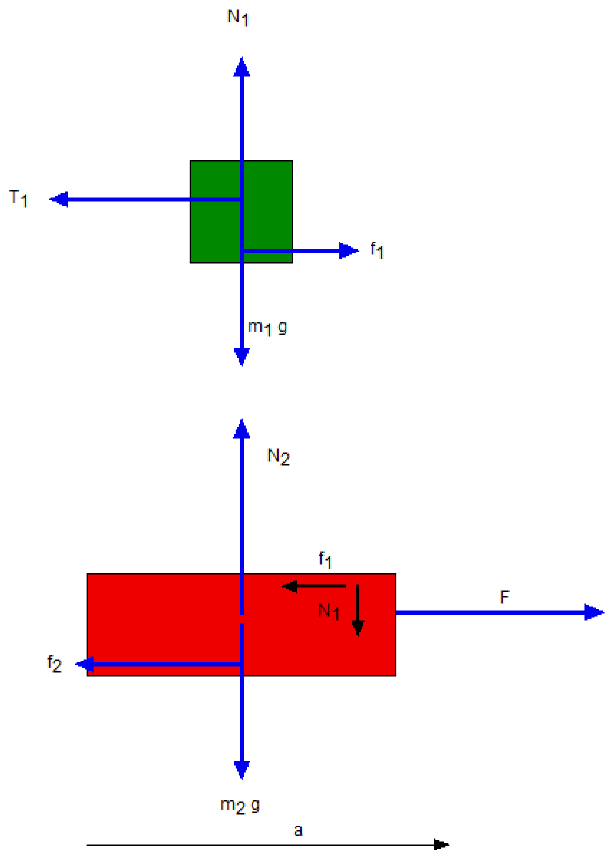
$$\mu_s \ge \frac{v^2}{gr}$$

7. Advanced Problems from other sources

7.1 Serway Problem 5-44

A 5.00 kg (m_1) is placed on top of a 10.0-kg (m_2) block. A horizontal force of 45.0 N (F) is applied to the 10-kg block, and the 5-kg block is tied to the wall. The coefficient of kinetic friction (μ_k) between all moving surfaces is 0.2.
(a) Draw a free-body diagram for each block and identify the action-reaction force between the bocks.
(b) Determine the tension (T) in the string and the magnitude of the acceleration (a) of the 10-kg block.





((**Solution**))

$m_1 = 5.00$ kg, $m_2 = 10.00$ kg

$\mu_k = 0.20$ $F = 45$ N.

For the block-1 which does not move,

$$f_1 = T_1$$
$$N_1 = m_1 g \qquad (1)$$
$$f_1 = \mu_k N_1 = \mu_k m_1 g$$

For the block-2 which moves along the positive x direction with the acceleration a,

$$N_2 - N_1 - m_2 g = 0$$
$$F - f_1 - f_2 = m_2 a \qquad (2)$$
$$f_2 = \mu_k N_2$$

From Eqs.(1) and (2), we have

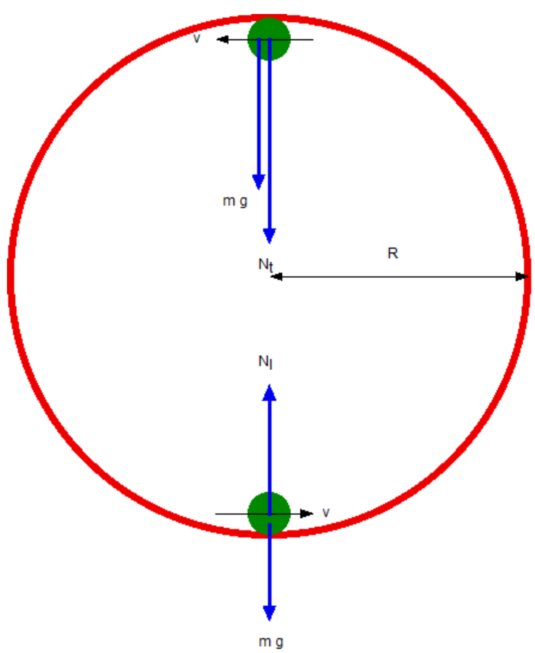
$$T_1 = f_1 = \mu_k N_1 = \mu_k m_1 g$$
$$N_2 = (m_1 + m_2)g$$
$$f_2 = \mu_k (m_1 + m_2)g$$
$$F - \mu_k (2m_1 + m_2)g = m_2 a$$

$a = 0.58$ m/s^2
$f_1 = T_1 = 9.8$ N, $f_2 = 29.4$ N.

7.2. Young and Freedman Problem 5-118

Example for the uniform circular motion

A small remote car with mass 1.60 kg moves at a constant velocity v = 12.0 m/s in a vertical circle inside a hollow metal cylinder that has a radius of 5.00 m. What is the magnitude of the normal force exerted on the car by the wall of the cylinder at (a) point A (at the bottom of the vertical circle) and (b) point B (at the top of the vertical circle).



For the normal force at the bottom of the vertical circle,

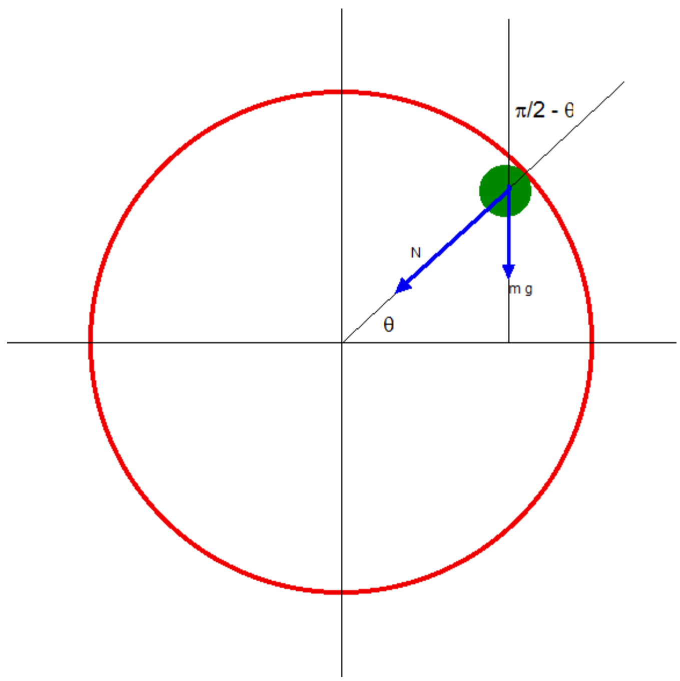
$$N_l - mg = m\frac{v^2}{R}$$
$$N_l = m(\frac{v^2}{R} + g)$$

For the normal force at the top of the vertical circle,

$$N_t + mg = m\frac{v^2}{R}$$

$$N_t = m(\frac{v^2}{R} - g)$$

((Note))
We consider the general case.



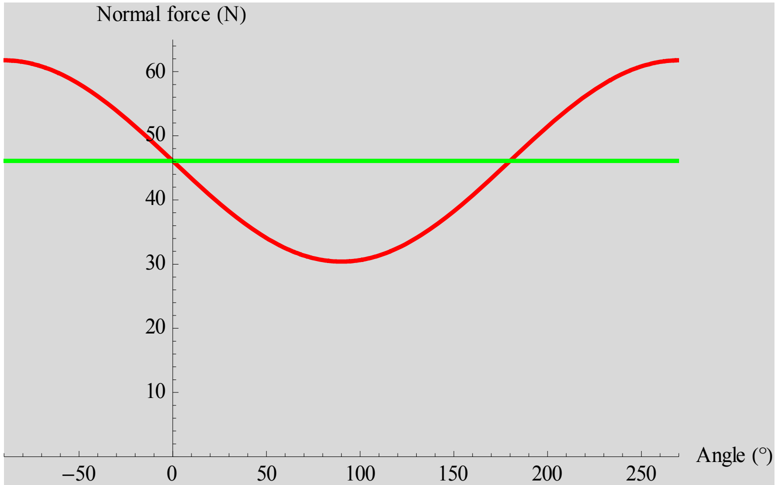
$$N + mg\cos(\frac{\pi}{2} - \theta) = m\frac{v^2}{R}$$

or

$$N = m(\frac{v^2}{R} - g\sin\theta)$$

When m = 1.6 kg, g = 9.80, v = 12.0 m/s, and R = 5.0 m

$$N = 46.08 - 15.68\sin\theta$$



((**Appendix**))

A.1 Terminal velocity

The terminal velocity can be determined from

$$mg = C_1 r v + C_2 r^2 v^2$$

where r is the radius of the system. The critical velocity v_{cr} is defined from the condition that

$$C_1 r v = C_2 r^2 v^2$$

or

$$v_{cr} = \frac{C_1}{C_2 r}.$$

(a) For $v \ll v_{cr}$, the terminal velocity is obtained as

$$v_t = \frac{mg}{C_1 r} \propto r^2$$

since m is proportional to r^3.

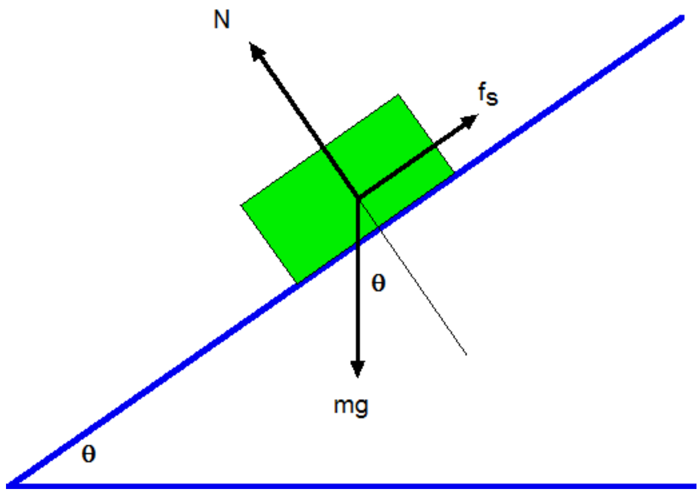
(b) For $v \gg v_{cr}$, the terminal velocity is obtained as

$$v_t = \sqrt{\frac{mg}{C_2 r^2}} \propto r^{1/2}$$

since *m* is proportional to r^3.

A.2 Experimental determination of μ_s

We put our system in the incline with an angle θ. We assume that the coefficient of static friction ms between our system and the incline. The coefficient of static friction ms is determined from the tangent of the maximum angle above which the system starts to slide on the incline.



We apply the Newton's second law;

$$N = mg\cos\theta$$
$$f_s - mg\sin\theta = 0$$
$$f_s \le \mu_s N$$

From these equations, we have an inequality,

$$\tan\theta \le \mu_s$$

or

$$\mu_s = \tan\theta_{\max}$$

where $\theta_{\max}$ is the maximum angle above which the system starts to slide on the incline. $\theta_{\max}$ can be determined experimentally.