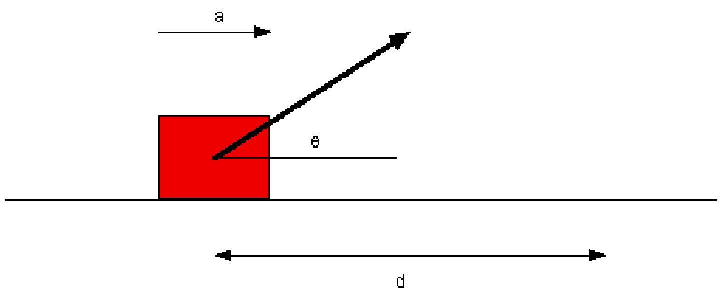
Lecture Note
Chapter 7
Work and Energy

1 Kinetic energy and work I



We consider the simplest case where the acceleration *a* is constant.

Newton's second law

$$ma = F\cos\theta \qquad \text{or} \qquad a = \frac{F}{m}\cos\theta$$

Here we use

$$v_f^{\ 2} - v_i^{\ 2} = 2ad = 2\frac{1}{m}Fd\cos\theta$$

where $\boldsymbol{v}_f$ is the final velocity and $\boldsymbol{v}_i$ is the initial velocity. Then we get

$$\frac{m}{2}(v_f^{\ 2} - v_i^{\ 2}) = Fd\cos\theta = \boldsymbol{F}\cdot\boldsymbol{d}$$

(1) The kinetic energy is defined by

$$K = \frac{1}{2}mv^2$$

((**Note**)) Units
In SI units, the units of work is J (Joule)

1 J (Joule) = kg m^2/s^2 = $N\,m$
1 N = kg m/s^2

In cgs units, the unit of work is erg.

1 erg = g cm^2/s^2 = dyne cm
1 dyne = g cm/s^2

(2) The work is defined by

$$W = \boldsymbol{F} \cdot \boldsymbol{d} \qquad \text{(work done by a constant force)}$$

Then Eq.(1) can be rewritten as

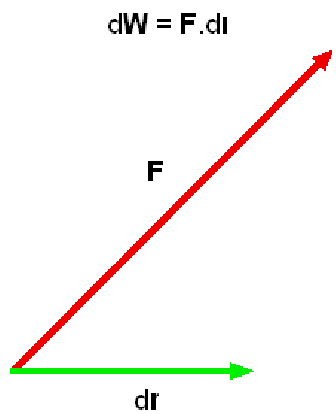
$$\Delta K = K_f - K_i = W_{net}$$

This is called as work-kinetic energy theorem, simply, work-energy theorem:

2 Work: general case

2.1 Definition

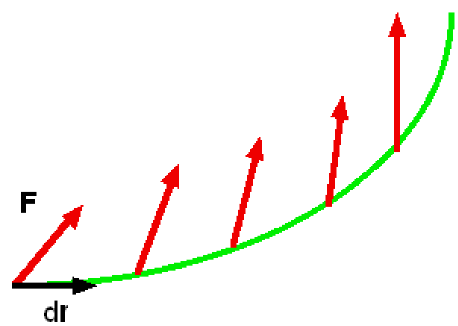
We now consider the work in the general case,



The work along a path is given by

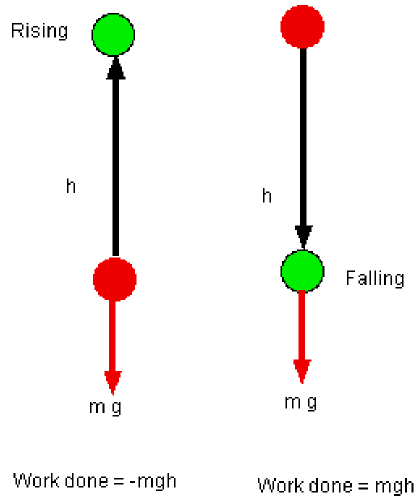
$$W = \int \boldsymbol{F} \cdot d\boldsymbol{r}$$

where the integral is made along a path (path integral).



2.2 Work done by the gravitational force (an example of W_c)

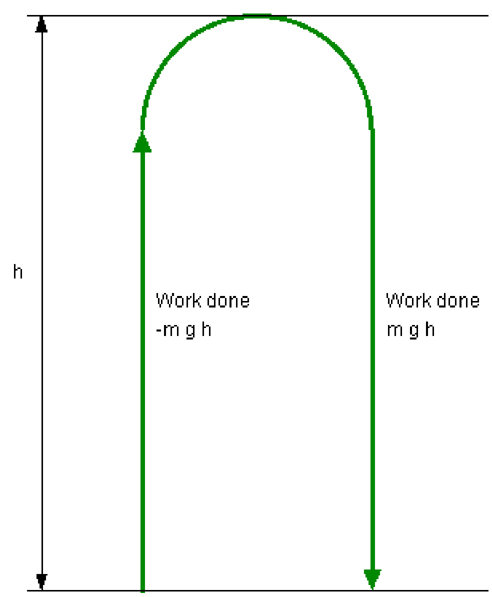
Conservative force



For the rising case, the work done by the gravitational force is $-mgh$ (<0).
For the falling case, the work done by the gravitational force is mgh (>0).

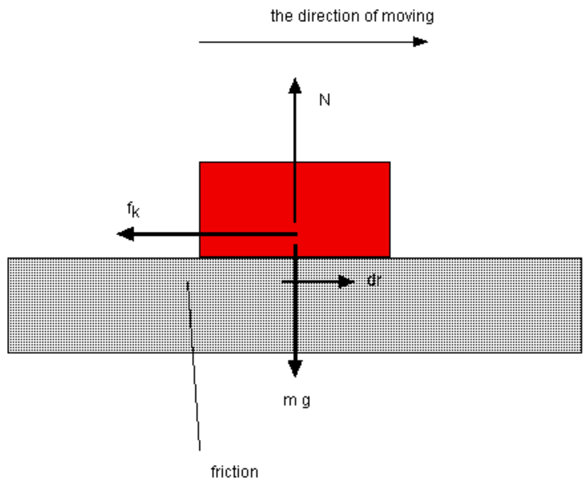
When the particles goes up and then falls down, the work done by the gravitational force is

$$-mgh + mgh = 0.$$



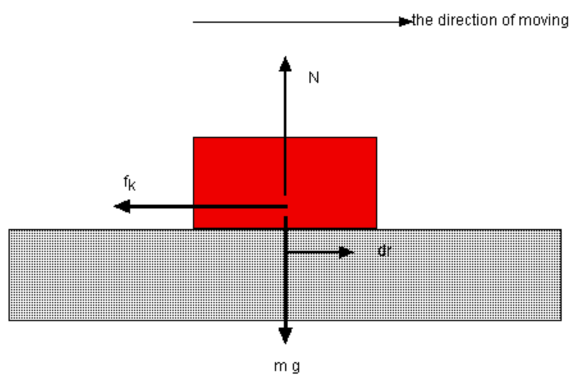
2.3 Work done by the frictional force (an example of W_{nc})

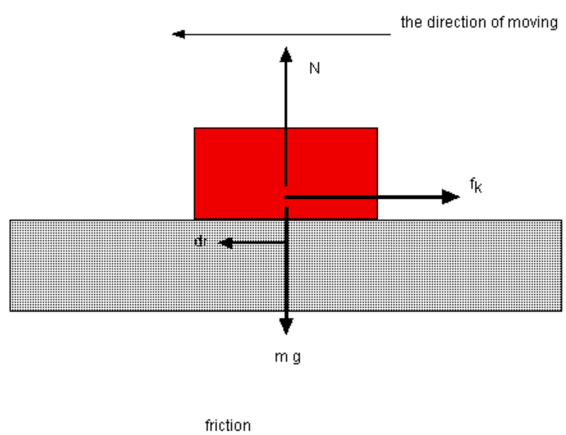
Nonconservative force



$dW_{nc} = -\mathbf{f}_k \cdot d\mathbf{r}$, work done by the frictional force. This work is always negative. That is not dependent on the direction of the movement.

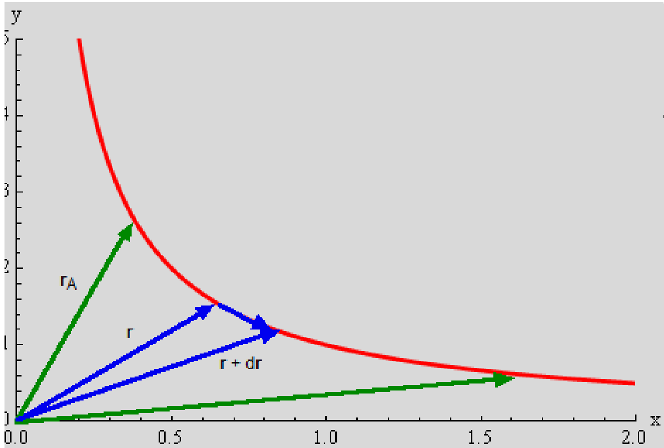
The frictional force is one of the non-conservative forces.





$W_{nc} = -\boldsymbol{F} \cdot d\boldsymbol{r} < 0$, which does not depend on the direction of motion.

3. Work-energy theorem



The work done in the displacement by the force is defined as

$$W(A \to B) = \int_A^B \boldsymbol{F} \cdot d\boldsymbol{r} \qquad (1)$$

where the limits A and B stand for the positions $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$. The substitution of the force $\boldsymbol{F}$ defined by

$$\boldsymbol{F} = m\frac{d\mathbf{v}}{dt}$$

into Eq.(1) leads to

$$W(A \to B) = m\int_A^B \frac{d\mathbf{v}}{dt} \cdot d\boldsymbol{r} \, .$$

Now

$$d\boldsymbol{r} = \frac{d\boldsymbol{r}}{dt} dt = \mathbf{v}dt$$

so that

$$W(A \to B) = m\int_A^B \frac{d\mathbf{v}}{dt} \cdot \mathbf{v}dt$$

where the limits A and B now stand for the times t_A and t_B when the particle is at the positions $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$. Here we can rearrange the integrand

$$\frac{d}{dt}v^2 = \frac{d}{dt}(\boldsymbol{v}\cdot\boldsymbol{v}) = \frac{d\boldsymbol{v}}{dt}\cdot\boldsymbol{v} + \boldsymbol{v}\cdot\frac{d\boldsymbol{v}}{dt} = 2\frac{d\boldsymbol{v}}{dt}\cdot\boldsymbol{v}$$

So that

$$W(A\to B) = \frac{m}{2}\int_A^B \left(\frac{d}{dt}v^2\right)dt = \frac{m}{2}\int_A^B d(v^2) = \frac{1}{2}m{v_B}^2 - \frac{1}{2}m{v_A}^2$$

where the limits A and B now stand for the velocities $\boldsymbol{v}_A$ and $\boldsymbol{v}_B$ when the particle is at the positions $\boldsymbol{r}_A$ and $\boldsymbol{r}_B$. This expression is rewritten as

$$W(A\to B) = K_B - K_A$$

where K is the kinetic energy of the particle.

In summary, the work-energy theorem is given by

$$\Delta K = W$$

where

$$\Delta K = \frac{1}{2}m({v_B}^2 - {v_A}^2)$$
$$W = \int_{\boldsymbol{r}_A}^{\boldsymbol{r}_B} \boldsymbol{F}\cdot d\boldsymbol{r}$$

4 Net work

The force $\boldsymbol{F}$ is the sum of $\boldsymbol{F}_c$ and $\boldsymbol{F}_{nc}$,

$$\boldsymbol{F} = \boldsymbol{F}_c + \boldsymbol{F}_{nc}$$

where $\boldsymbol{F}_c$ is the conserved force (such as a gravitational force) and $\boldsymbol{F}_{nc}$ is the non-conserved force (such as friction).

The net work W_{net} is the sum of W_c and W_{nc}

$$W_{net} = W_c + W_{nc}$$

where

$$W_c = \int_{\boldsymbol{r}_A}^{\boldsymbol{r}_B} \boldsymbol{F}_c \cdot d\boldsymbol{r}$$

$$W_{nc} = \int_{\boldsymbol{r}_A}^{\boldsymbol{r}_B} \boldsymbol{F}_{nc} \cdot d\boldsymbol{r}$$

Then we have the work-energy theorem:

$$\Delta K = W_{net} = W_{nc} + W_c$$

5 Potential energy *U*

We define a potential energy. This is a scalar function associated with a conservative force.

$$W_c = -\Delta U = -(U_f - U_i)$$

where W_c is the work done by the conservative force, U_i is the potential energy at the initial state and U_f is the potential energy at the final state.

(a) For the one dimensional system, we have

$$W_c = \int_{x_i}^{x} F_c(x')dx' = -U(x) + U(x_i)$$

Here we take a derivative of both sides with respect to *x*, we get

$$F_c(x) = -\frac{dU(x)}{dx}$$

((**Example**))
For the gravitational force (conservative force), the potential energy is given by

$U(z) = mgz,$

since

$$F_c(z) = -\frac{dU(z)}{dz} = -mg$$

(b) For the three dimensional system, we have

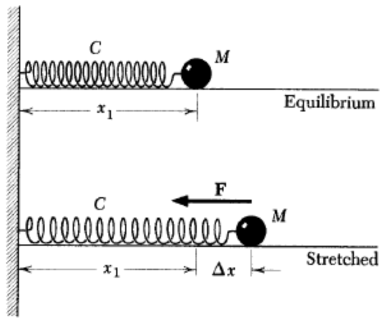
$$W_c = \int_i^f \boldsymbol{F}_c \cdot d\boldsymbol{r}$$

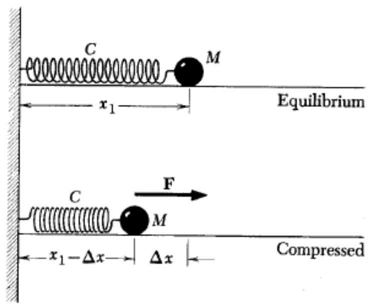
The conservative force $\boldsymbol{F}_c$ is expressed by the potential energy as

$$\boldsymbol{F}_c = -\nabla U = -gradU = (-\frac{\partial U}{\partial x}, -\frac{\partial U}{\partial y}, -\frac{\partial U}{\partial y})$$

((Note))
In Chapter 8 we will give a proof of this expression using Stokes theorem.

6 Spring Force
6.1 Hooke's law

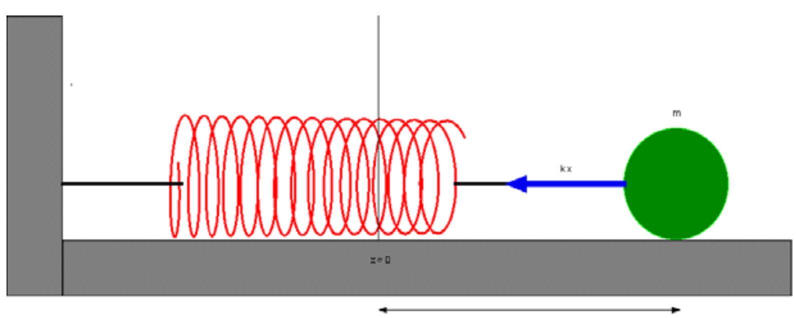




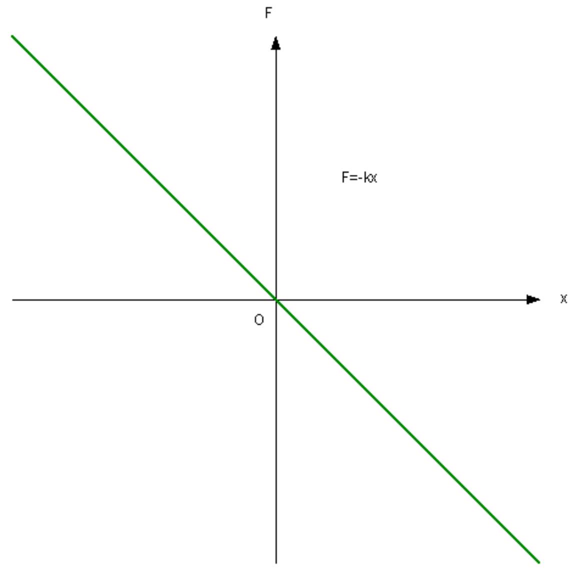
A particle is subject to a linear restoring force in the x direction. A linear restoring force (spring force) is one that is directly proportional to the displacement (x) measured from the equilibrium point. The spring force is defined by

$F=-kx$, or $\boldsymbol{F} = -kx\boldsymbol{i}$

where k is a positive constant. The unit of k is N/m. This is called a Hooke's law. For sufficiently small displacements, the spring force may be produced by a stretched or compressed spring. The sign of the force is such that the particle is always attracted toward the origin $x = 0$.



A plot of the spring force is shown as a function of *x*.



The spring force is a conservative force.

$$\boldsymbol{F}\cdot d\boldsymbol{r} = (-kx\boldsymbol{i})\cdot dx\boldsymbol{i} = -kxdx$$

$$W(i\to f) = \int_{x_i}^{x_f} \boldsymbol{F}\cdot d\boldsymbol{r} = \int_{x_i}^{x_f} (-kxdx) = -\frac{1}{2}k(x_f{}^2 - x_i{}^2)$$

Note that

$$W(i\to f) + W(f\to i) = -\frac{1}{2}k(x_f{}^2 - x_i{}^2) - \frac{1}{2}k(x_i{}^2 - x_f{}^2) = 0,$$

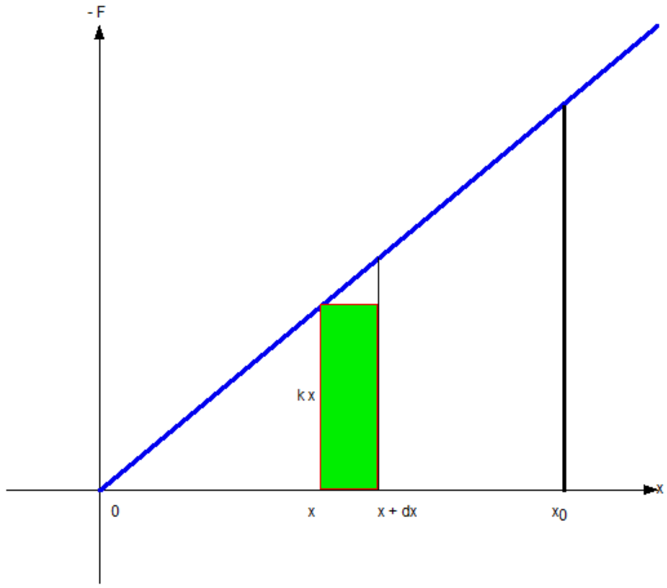
which means the spring force is a conservative force.

Then the potential energy *U* can be defined as

$$F_c(x) = -\frac{dU(x)}{dx} = -kx$$

or

$$\boxed{U = \int_0^x kxdx = \frac{1}{2}kx^2}$$



The energy conservation:

From the work-energy theorem

$$\Delta K = W = -\Delta U$$

we have

$$\Delta(K+U) = 0 \qquad \text{or} \qquad \Delta E = 0$$

where E is the total energy and is defined by

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2 \qquad \text{(Energy conservation law)}$$

and

$$E_f = \frac{1}{2}m{v_f}^2 + \frac{1}{2}k{x_f}^2 = \frac{1}{2}m{v_i}^2 + \frac{1}{2}k{x_i}^2 = E_i$$

Where

$K\ (=\frac{1}{2}mv^2)$ is the kinetic energy

$U\ (=\frac{1}{2}kx^2)$ is a potential energy.

((**Note**)) **Derivation of the energy conservation law for the simple harmonics. Approach from the Newton's second law**

We start from the Newton's second law for the simple harmonics

$$m\dot{v} = F = -kx \tag{1}$$

Multiplying both sides of Eq.(1) by $v = \dot{x}$, we get

$$mv\dot{v} = F = -kx\dot{x}$$

or

$$\frac{d}{dt}(\frac{m}{2}v^2 + \frac{1}{2}kx^2) = 0$$

Then we have the energy conservation law for the simple harmonics

$$\frac{1}{2}mv^2 + \frac{1}{2}kx^2 = E = \text{const} \tag{2}$$

6.2 Energy conservation in the spring system

Here we consider the energy conservation law for the spring,

$$E = K + U = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

At $x = 0$, $E = \frac{1}{2}mv_{\max}{}^2$

At $x = x_{max}$ (amplitude) $E = \frac{1}{2}kx_{\max}{}^2$

or

$$E = \frac{1}{2}mv_{\max}{}^2 = \frac{1}{2}kx_{\max}{}^2$$

or

$$v_{\max} = \omega x_{\max}$$

From the energy conservation law, we have

$$1 = \frac{1}{2E}mv^2 + \frac{1}{2E}kx^2$$

or

$$\left(\frac{v}{v_{\max}}\right)^2+\left(\frac{x}{x_{\max}}\right)^2=1$$

This is a circle in the x/x_{max} vs $v/v_{\max}$ plane. The center is at the origin. In general, the v-x plane is called a *phase space*.

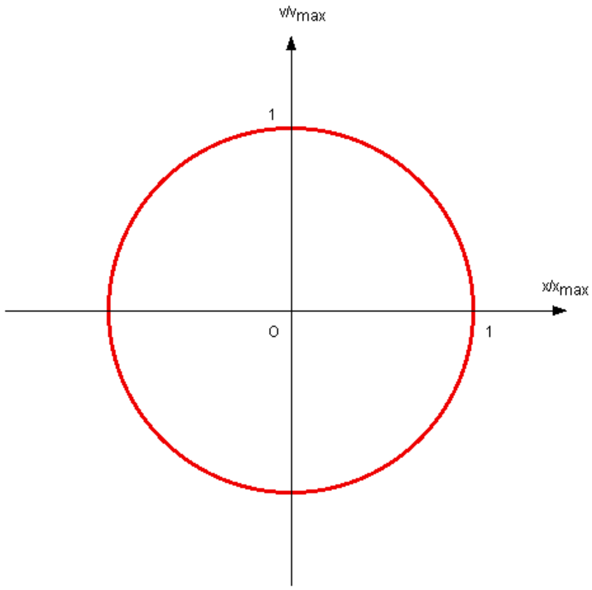
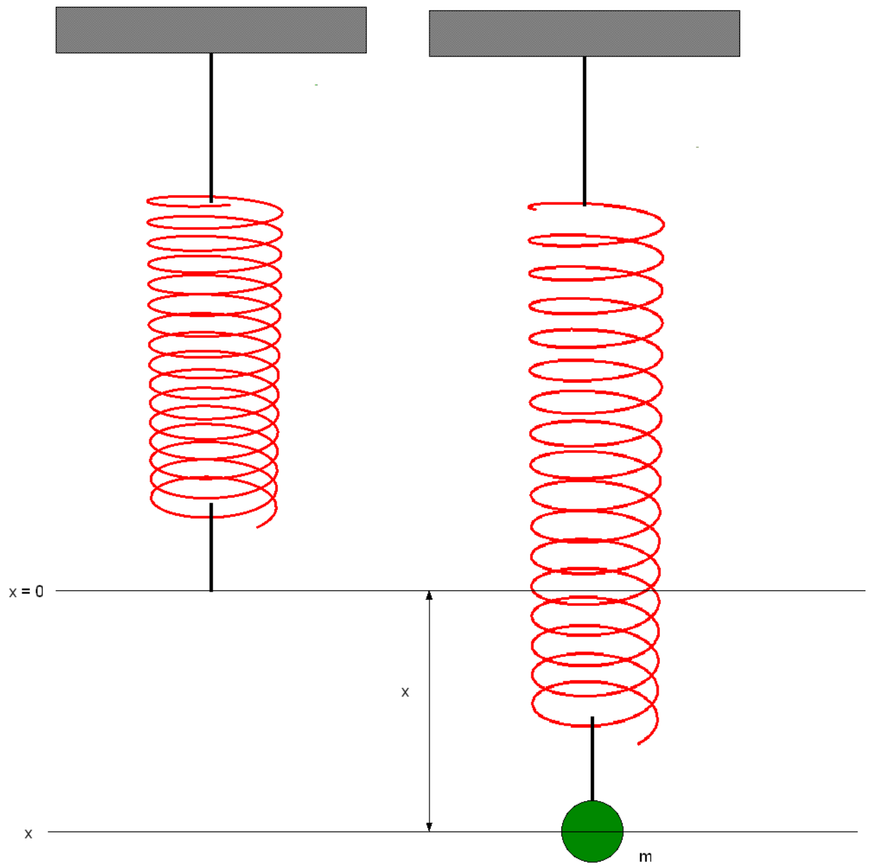
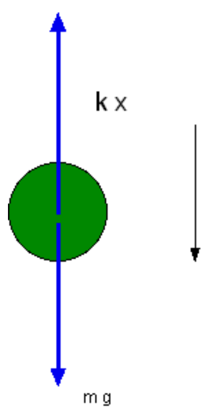


Fig. Phase space (x, v) for the simple harmonics.

6.3 Determination of the spring constant *k*



Free-body diagram



In equilibrium, we have

$$kx_0 = mg$$

or

$$x_0 = \frac{mg}{k}$$

In SI units, the spring constant is in the units of N/m.

In dynamics, we set up an equation of motion,

$$m\frac{d^2x}{dt^2} = mg - kx = -k(x - \frac{mg}{k}) = -k(x - x_0)$$

(simple harmonics, see Chapter 15 for detail)

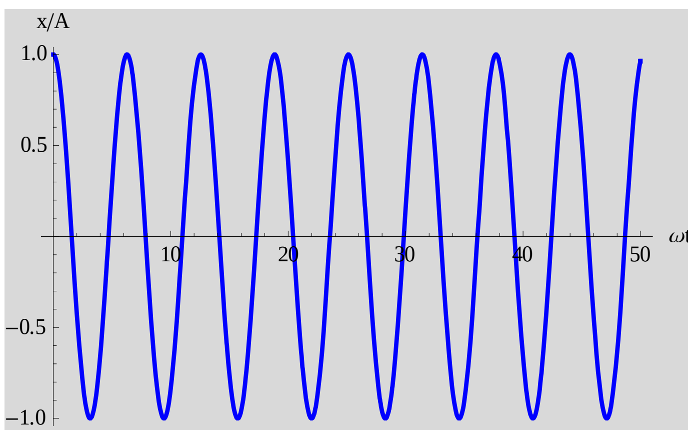
$$\frac{d^2x}{dt^2} = -\omega^2(x - x_0)$$

where ω is the angular frequency

$$\omega = \sqrt{\frac{k}{m}}$$

The solution of the second order differential equation is

$$x - x_0 = A\cos(\omega t + \phi)$$



The period T is given by

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}}$$

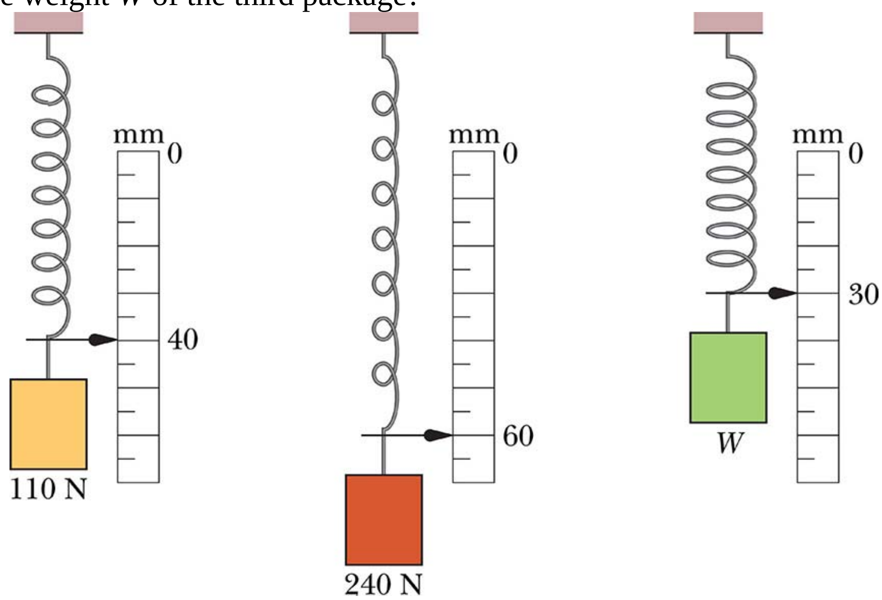
((**Note**))
The dimension of m/k

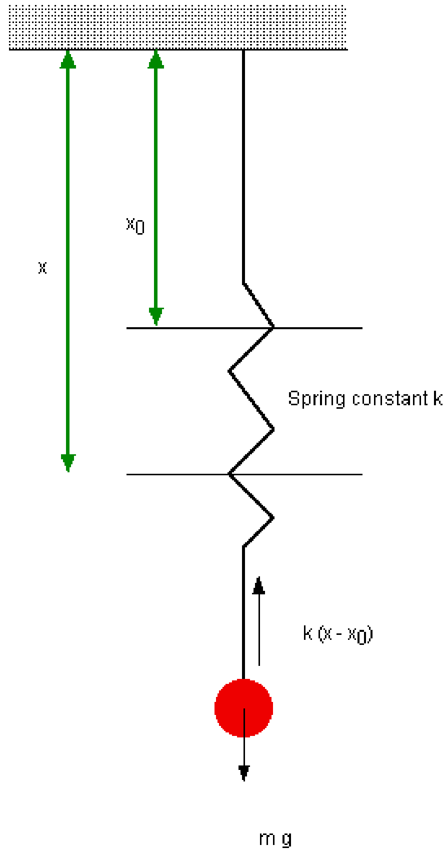
$$[\frac{m}{k}] = \frac{kg}{\frac{N}{m}} = \frac{kg}{kg\frac{m}{s^2}\frac{1}{m}} = s^2$$

6.4 Example

Problem 7-67 (SP-7) (10-th edition)

A spring with a pointer attached is hanging next to a scale marked in millimeters. Three different packages are hung from the spring, in turn, as shown in Fig. (a) Which mark on the scale will the pointer indicate when no package is hung from the spring? (b) What is the weight W of the third package?





m_1g = 110 N m_2g = 240 N
x_1 = 40 mm x_2 = 60 mm
x_3 = 30 mm

Determination of the spring constant

$$m_1 g = k(x_1 - x_0) \qquad m_2 g = k(x_2 - x_0) \qquad \text{or} \qquad (m_2 - m_1)g = k(x_2 - x_1)$$

The we have

$$k = \frac{(m_2 - m_1)g}{x_2 - x_1} = \frac{(240 - 110)N}{(60 - 40)\times 10^{-3} m} = 6.5\times 10^3 N/m$$

(a) $\frac{m_1 g}{k} = x_1 - x_0$ and $\frac{m_1 g}{k} = 16.9mm$

Then we have

$x_0 = x_1$-16.9 = 23.1 mm

since x_1 = 40.0 mm.

(b) $$W = k(x_3 - x_0) = 44.9 N$$

where x_3 = 30 mm and x_0 = 23.1 mm.

7 Power *P*

7.1 Average power

The power is the time rate of transfer energy. The average power is defined as

$$P_{avg} = \frac{\Delta W}{\Delta t} \qquad \text{(average power)}$$

where

$$W = \int_{rA}^{rB} \boldsymbol{F} \cdot d\boldsymbol{r}$$

7.2 Instantaneous power

If the force ***F*** causes a particle to undergo a displacement d***r***, the work done is

$$dW = \boldsymbol{F} \cdot d\boldsymbol{r}\,.$$

Since

$$d\boldsymbol{r} = \boldsymbol{v} dt$$

the instantaneous power (or simply power) provided by the force id

$$P = \frac{dW}{dt} = \boldsymbol{F} \cdot \frac{d\boldsymbol{r}}{dt} = \boldsymbol{F} \cdot \boldsymbol{v} \qquad \text{(instantaneous power)}$$

From this expression, *W* can be derived as

$$W(t_1 \to t_2) = \int_{t_1}^{t_2} P(t) dt$$

((**Units**))
In SI units, the units of power is W (Watt)

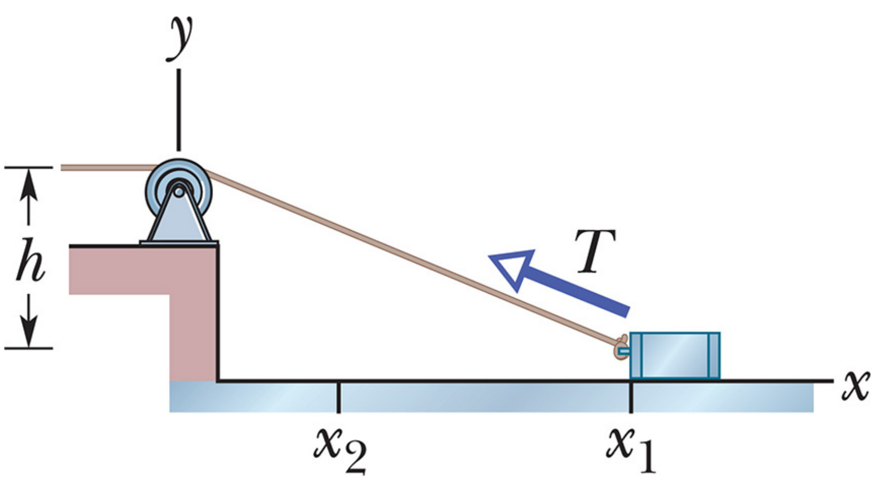
1W = 1 J/s
1 horsepower=1 hcp = 746 W
kWh =(10^3 J/s) 3600 s = 3.6 x 10^6 J

8. Example

8.1

Problem 7-42* (SP-7) (10-th edition)**

Figure shows a cord attached to a cart that can slide along a frictionless horizontal rail aligned along an x axis. The left end of the cord is pulled over a pulley, of negligible mass and friction and at cord height h = 1.20 m, so that the cart slides from x_1 = 3.00 m to x_2 = 1.00 m. During the move, the tension in the cord is a constant 25.0 N. What is the change in the kinetic energy of the cart during the move?



((**Solution**))

h_1 = 1.20 m. x_1 = 3.0 m
x_2 = 1.0 m. T = 25.0 N

$$\cos\theta = \frac{x}{\sqrt{x^2+h^2}}$$

$$W = \int \boldsymbol{T}\cdot d\boldsymbol{r} = -T\int \cos\theta dx = -T\int_{x1}^{x2} \frac{x}{\sqrt{x^2+h^2}}.dx$$
$$= T(\sqrt{x_1{}^2+h^2} - \sqrt{x_2{}^2+h^2})$$
$$= 41.7262J$$

8.2
Problem 7-52* (SP-7) (10-th edition)**

A funny car accelerates from rest through a measured track distance in time T with the engine operating at a constant power P. If the track crew can increase the engine power by a differential amount dP, what is the change in the time required for the run.

((**Solution**))

$$P = \frac{dW}{dt}$$

When P is constant,

$$W = \int_0^t Pdt = Pt = \Delta K = \frac{1}{2}m[v(t)]^2 - \frac{1}{2}m[v(t=0)]^2 = \frac{1}{2}m[v(t)]^2$$

using the work-energy theorem.

Then we have

$$v(t) = \frac{dx}{dt} = \sqrt{\frac{2Pt}{m}}$$

or

$$x = \int dx = \sqrt{\frac{2P}{m}} \int_0^T \sqrt{t} dt = \sqrt{\frac{2P}{m}} \frac{2}{3} T^{3/2} = L$$

or

$$PT^3 = \frac{9}{8} mL^2 = \text{const}$$

This can be rewritten as

$$\ln(PT^3) = \ln P + 3\ln T = \ln(\frac{9}{8} mL^2) = const$$

or

$$\frac{\Delta P}{P} + 3\frac{\Delta T}{T} = 0$$

or

$$\frac{\Delta P}{P} = -3\frac{\Delta T}{T}$$

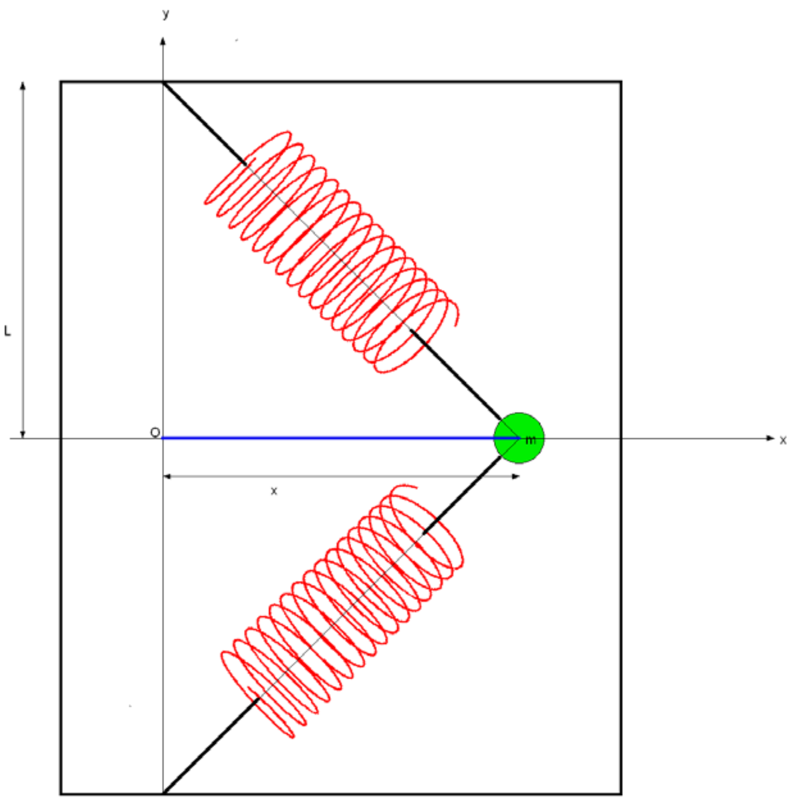
9. Advanced problem

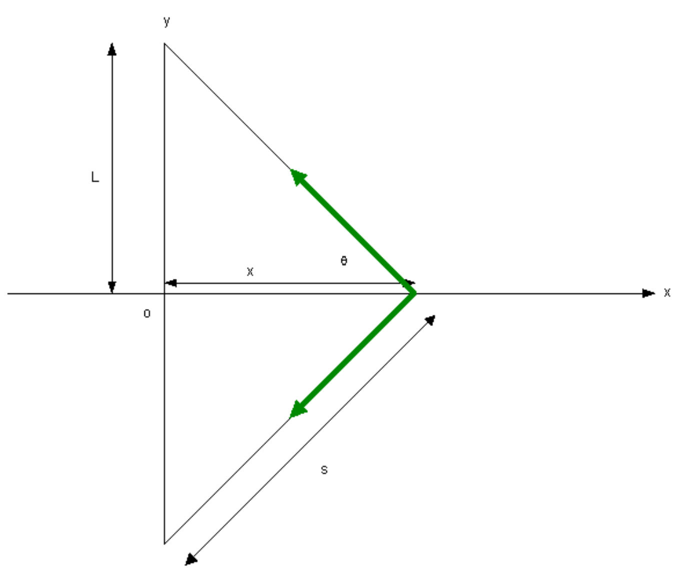
Serway Problem 6-52 and 7-14

A particle is attached between the identical springs on a horizontal frictionless table. Both springs have spring constant k and are initially unstressed.
(a) If the particle is pulled a distance x along a direction perpendicular to the initial configuration of the springs, as shown in Fig., show that the force exerted by the springs on the particle is

$$\mathbf{F} = -2kx(1 - \frac{L}{\sqrt{x^2 + L^2}})\hat{i}$$

(b) Determine the amount of work done by this force in moving the particle from $x = A$ to 0.





(a)

$$s = \sqrt{x^2 + L^2}\,, \qquad \cos\theta = \frac{x}{s} = \frac{x}{\sqrt{x^2 + L^2}}$$

$$F_x = -2k(s-L)\cos\theta = -2k(\sqrt{x^2+L^2}-x)\frac{x}{\sqrt{x^2+L^2}} = -2kx(1-\frac{1}{\sqrt{x^2+L^2}})$$

(b)

$$W = \int_A^0 F_x dx = \int_A^0 -2kx(1-\frac{L}{\sqrt{x^2+L^2}})dx = \int_0^A 2kx(1-\frac{L}{\sqrt{x^2+L^2}})dx$$

or

$$W = k[A^2 + 2L(L-\sqrt{A^2+L^2})]$$

(c) The potential U

$$U = -\int_0^x F_x dx = k(2L^2 + x^2 - 2L\sqrt{x^2+L^2})$$

((**Mathematica**))

```
F = -2 k x (1 - L/Sqrt[x^2 + L^2])
```

$$-2\,k\,x\left(1-\frac{L}{\sqrt{L^2+x^2}}\right)$$

```
J1 = Simplify[Integrate[F, {x, A, 0}], {A > 0, L > 0}]
```

$$k\left(A^2+2\,L\left(L-\sqrt{A^2+L^2}\right)\right)$$

```
rule1 = {L → 1, k → 1}
```

$\{L \to 1,\ k \to 1\}$

```
F1 = F /. rule1
```

$$-2\,x\left(1-\frac{1}{\sqrt{1+x^2}}\right)$$

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Plot[F1, {x, -2, 2}, PlotStyle → {Red, Thick},
 Background → LightGray, AxesLabel → {"x", "F"}]
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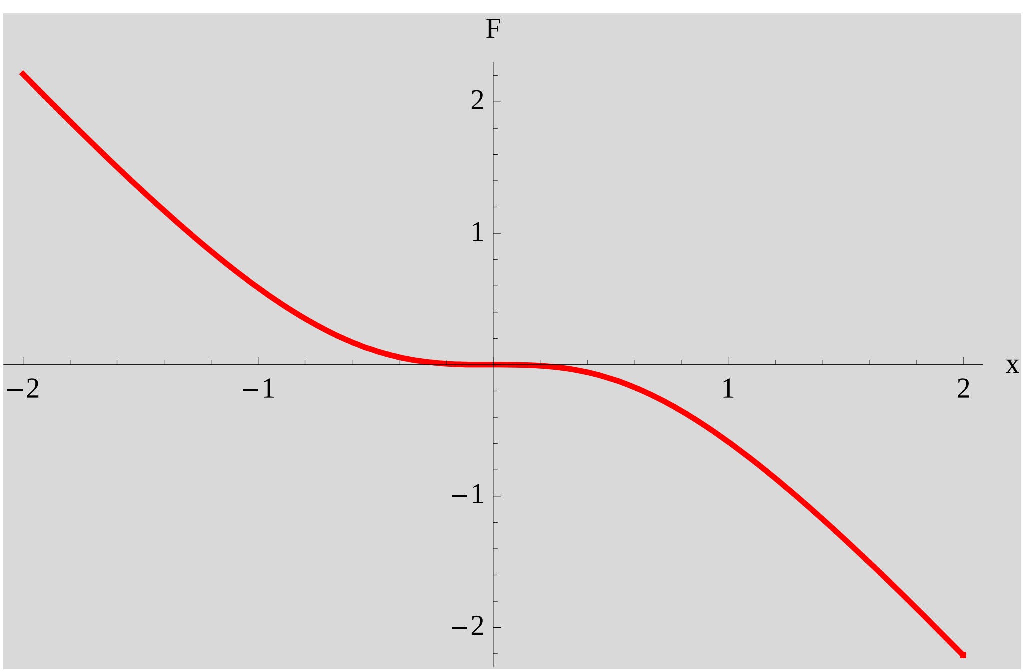

Potential energy

$$U1 = \text{Simplify}\left[-\int_0^x F\,dx,\ \{k>0,\ L>0,\ x>0\}\right]$$

$$k\left(2L^2 + x^2 - 2L\sqrt{L^2+x^2}\right)$$

$$U2 = U1\ /.\ \text{rule1}$$

$$2 + x^2 - 2\sqrt{1+x^2}$$

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Plot[U2, {x, -2, 2}, PlotStyle → {Red, Thick},
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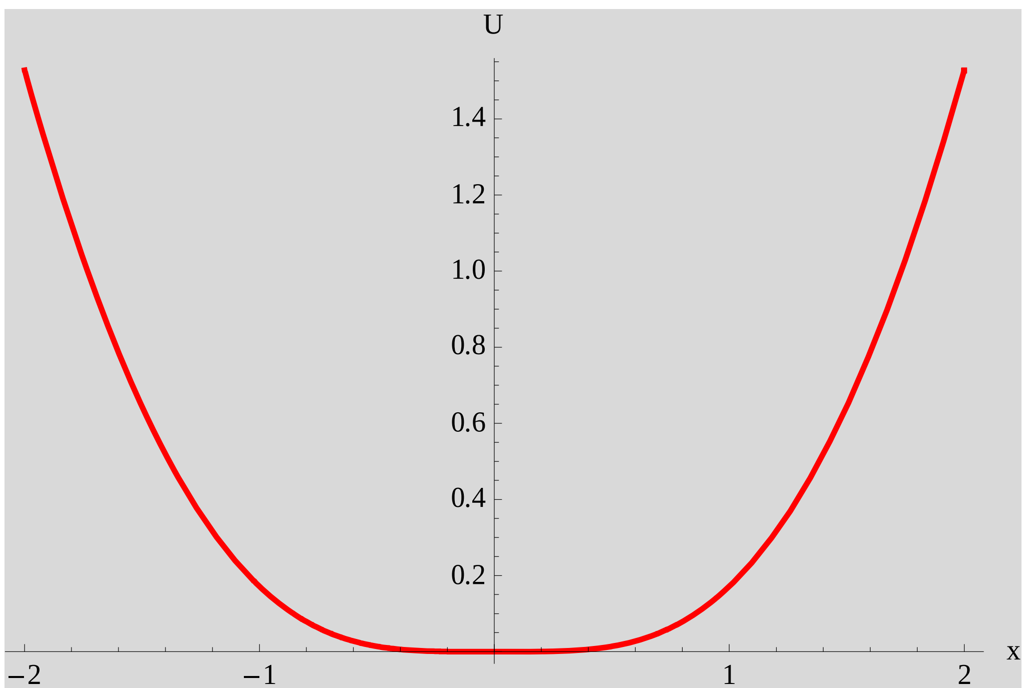
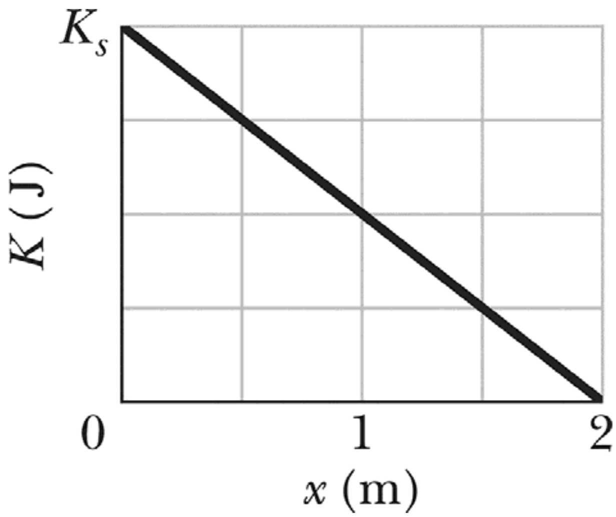


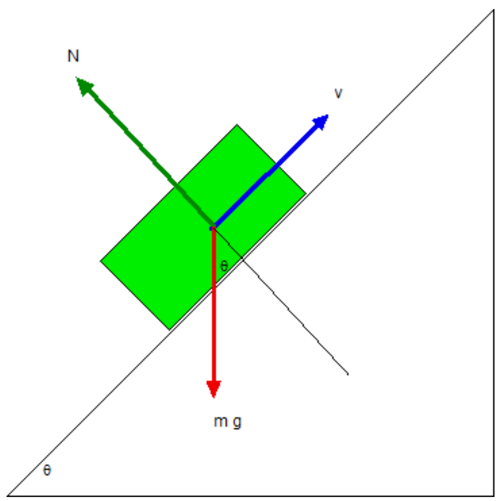
10. Problems of HW-7, SP-7, and so on

10.1
Problem 7-20 (SP-07) (10-th edition)

A block is set up a frictionless ramp along which an x axis extends upward. Figure gives the kinetic energy of the block as a function of position x; the scale of the figure's vertical axis is set by K_s = 40.0 J. If the block's initial speed is 4.00 m/s, what is the normal force on the block?



((**Solution**))
v_i = 4.00 m/s
K_s = 40.0 J.



The work-energy theorem

$$\Delta K = W_c = -\Delta U = -mg\sin\theta x$$

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mv_i^2 = K - K_s$$

$$K = K_s - mg\sin\theta x$$

((**Note**)) One can apply directly the energy conservation law to this problem.
The slope of K vs x is $-mg\sin\theta$.

The normal force N is given by

$$N = mg\cos\theta$$

10.2
Problem 7-38 (SP-07) (10-th edition)

A 1.5 kg block is initially at rest on a horizontal frictionless surface when a horizontal force along an x axis is applied to the block. The force is given by

$$\vec{F}(x) = (2.5 - x^2)\mathbf{i}N\,,$$

where x is in meters and the initial position of the block is x = 0. (a) What is the kinetic energy of the block as it passes through x = 2.0 m? (b) What is the maximum kinetic energy of the block between x = 0 and x = 2.0 m?

((**Solution**))
m = 1.50 kg.
v_i = 0.

The work energy theorem

$$\Delta K = K_f - K_i = K_f = W = \int_{x_i}^{x_f} F(x)dx = \int_{x_i}^{x_f}(2.5 - x^2)dx$$

Since

$$K_i = 0,\ \text{and} \qquad K_f = \frac{1}{2}m{v_f}^2$$

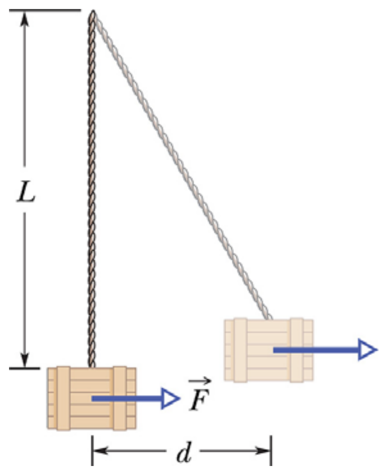
$$\frac{dW}{dx} = 2.5 - x^2 = 0$$
$$\frac{d^2W}{dx^2} = -2x$$

10.3

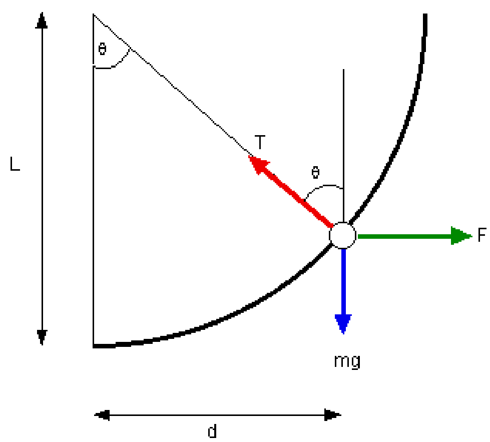
Problem 7-57 (SP-07) (10-th edition)

A 230 kg crate hangs from the end of a rope of length L = 12.0 m. You push horizontally on the crate with a varying force $\boldsymbol{F}$ to move its distance d = 4.00 m to the side (see Fig). (a) What is the magnitude of $\boldsymbol{F}$ when the crate is in this final position?

During the crate's displacement, what are (b) the total work done on it, (c) the work done by the gravitational force on the crate, and (d) the work done by the pull on the crate from the rope? (e) Knowing that the crate is motionless before and after its displacement, use the answers to (b), (c), and (d) to find the work your force $\boldsymbol{F}$ does on the crate. (f) Why is the work of your force not equal to the product of the horizontal displacement and the answer to (a)?



m = 230 kg, L = 12.0 m, d = 4.0 m, θ_0= arcsin(d/L) = 19.5°
F varies with the angle θ.



(a)

$$T\cos\theta - mg = 0$$
$$T\sin\theta = F$$

$$F = mg\tan\theta.$$

(b) The total work

$$\Delta K = W = 0 \qquad \text{(work-energy theorem)}$$

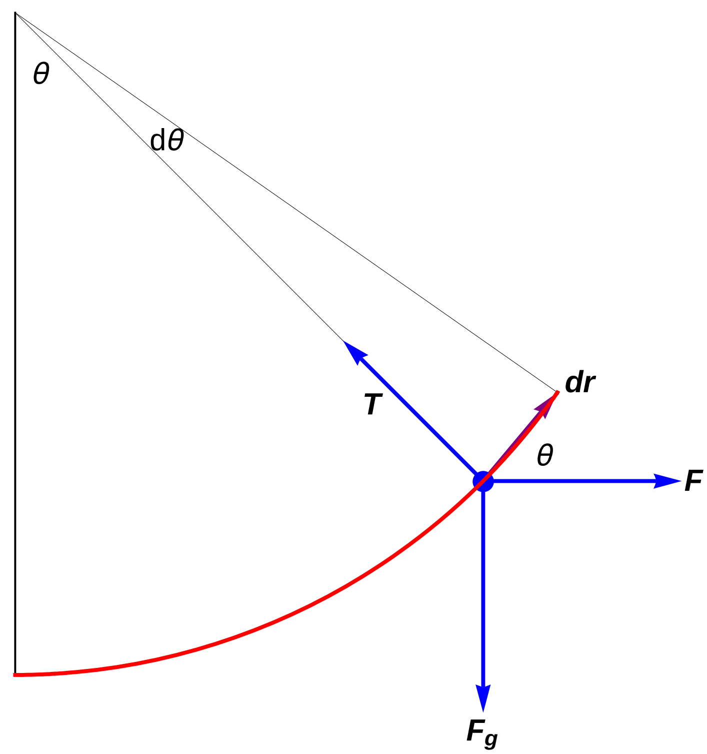
(c) $W_g = -\Delta U_g = -mgL(1-\cos\theta_0)$ work due to gravity.

(d) $\Delta K = W_g + W_F + W_T = 0$. $W_T = 0$

$$W_F = -W_g = mgL(1-\cos\theta_0)$$

((**Note-1**)

We calculate the work done by the force ***F***, tension ***T***, and gravitational force (mg) separately.



(i) Work done by the force *F*

F varies with the angle θ,

$$F = mg\tan\theta.$$

Since $dr = Ld\theta$, we have

$$dW_F = \boldsymbol{F}\cdot d\boldsymbol{r} = Fdr\cos\theta = FL\cos\theta d\theta = mgL\tan\theta\cos\theta d\theta = mgL\sin\theta d\theta.$$

Then we get

$$W_F = mgL\int_0^{\theta_0} \sin\theta d\theta = mgL(1-\cos\theta_0)\,.$$

(ii) Work done by the force ***T***

Since the tension *T* is always perpendicular to the displacement d***r***,

$$W_T = 0$$

(iii) Work done by the gravitational force ($\boldsymbol{F}_g = m\boldsymbol{g}$)

$$dW_g = m\boldsymbol{g}\cdot d\boldsymbol{r} = mgdr\cos(\theta + \frac{\pi}{2}) = -mgL\sin\theta d\theta$$

or

$$W_g = -mgL\int_0^{\theta_0} \sin\theta d\theta = -mgL(1-\cos\theta_0)$$

The total work done is

$$W_{total} = W_F + W_T + W_g = mgL(1-\cos\theta_0) - mgL(1-\cos\theta_0) = 0$$

((**Note-2**))

The result $W_{total} = 0$ is evident since

$$W_{total} = \int d\boldsymbol{r}\cdot(\boldsymbol{F} + \boldsymbol{T} + \boldsymbol{F}_g) = 0$$

and

$$\boldsymbol{F} + \boldsymbol{T} + \boldsymbol{F}_g = 0$$

along any point on the path.

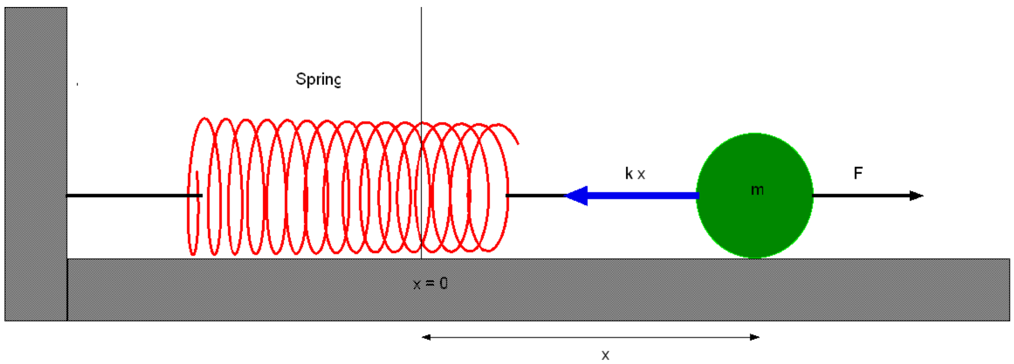
10.4

Problem 7-33* (from SP-7) (10-th edition)**

The block in Fig. lies on a horizontal frictionless surface, and the spring constant is 50 N/m. Initially, the spring is at its relaxed length and the block is stationary at position $x = 0$. Then an applied force with a constant magnitude of 3.0 N pulls the block in the positive direction of the x axis, stretching the spring until the block stops. When that stopping point is reached, what are

(a) the position of the block,
(b) the work that has been done in the block by the applied force, and
(c) the work that has been done on the block by the spring force?

During the block's displacement, what are
(d) the block's position when its kinetic energy is maximum and
(e) the value that maximum kinetic energy?



((**Solution**))
k = 50 N/m
F = 3.0 N
x = 0 and v = 0 initially.

The work-energy theorem

$$\Delta K = W_c + W_{nc} = -\Delta U + W_{nc}$$
$$\Delta E = \Delta K + \Delta U = W_{nc}$$

In the present case, we have

$$W_{nc} = Fx$$
$$\Delta U = \frac{1}{2}kx^2 - \frac{1}{2}kx_0{}^2 = \frac{1}{2}kx^2$$

with x_0 = 0.

or

$$\Delta K = W_{nc} - \Delta U = Fx - \frac{1}{2}kx^2$$

(a) When the stopping point is reached, ΔK = 0

$$\Delta K = 0 = Fx_{\max} - \frac{1}{2}kx_{\max}{}^{2}$$

$$x = x_{\max} = \frac{2F}{k} = 0.12m$$

(b)

$$W_{nc} = Fx_{\max} = 0.12 \times 3 = 0.36J$$

(c)

$$W_c = -\Delta U = -\frac{1}{2}kx_{\max}{}^{2} = -Fx_{\max} = -0.36J$$

(d)

$$\Delta K = Fx - \frac{1}{2}kx^2$$

$$\frac{d(\Delta K)}{dx} = F - kx = 0$$

$$x_1 = \frac{F}{k} = 0.06m$$

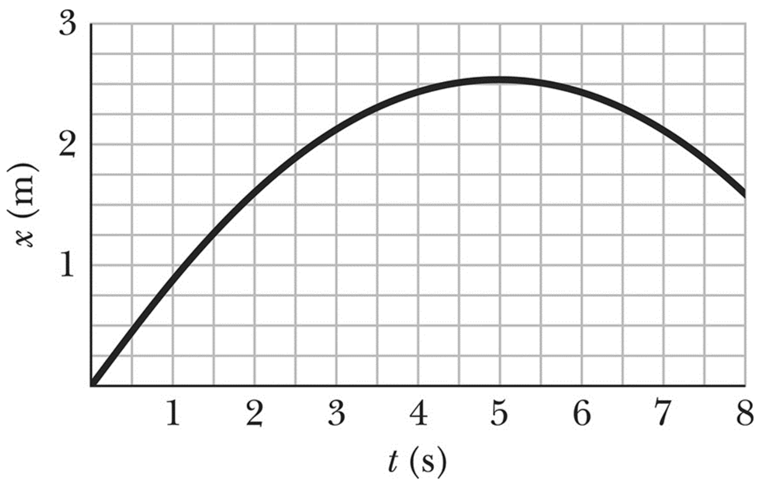
(e)

$$(\Delta K)_{\max} = Fx_1 - \frac{1}{2}kx_1{}^{2} = \frac{F^2}{2k} = 0.09J$$

10.5

Problem 7-79 (HW-7 Hint) (10-th edition)

A 2.0 kg lunchbox is sent sliding over a frictionless surface, in the positive direction of an *x* axis along the surface. Beginning at time *t* = 0, a steady wind pushes on the lunchbox in the negative direction of the *x* axis. Figure shows the position *x* of the lunchbox as a function of time *t* as the wind pushes on the lunchbox. From the graph, estimate the kinetic energy of the lunchbox at (a) *t* = 1.0 s and (b) *t* = 5.0 s. (b) How much work does the force from the wind do on the lunchbox from *t* = 1.0 s to *t* = 5.0 s.



((**Solution**))

m = 2.0 kg

$$\Delta K = W = \frac{1}{2}m(v^2 - v_0{}^2) = -Fx$$

or

$$K = \frac{1}{2}mv^2 = \frac{1}{2}mv_0{}^2 - Fx$$

((Note))

Equation of motion

$$ma = -F$$
$$a = -\frac{F}{m}$$
$$v = at + v_0$$
$$x = v_0 t + \frac{1}{2}at^2 = v_0 t - \frac{F}{2m}t^2$$
$$v^2 - v_0{}^2 = 2ax$$

11. Summary

Work-energy thorem ((from Wikipedia))

In physics, mechanical work is the amount of energy transferred by a force. Like energy, it is a scalar quantity, with SI units of joules. The term work was first coined in the 1830s by the French mathematician Gaspard-Gustave Coriolis.

According to the work-energy theorem if an external force acts upon an object, causing its kinetic energy to change from E_{k1} to E_{k2}, then the mechanical work (W) is given by

$$W = \Delta E_k = E_{k2} - E_{k1} = \Delta(\frac{1}{2}mv^2)$$

where m is the mass of the object and v is the object's velocity.