Chapter 14
Fluid

In this chapter we will explore the behavior of fluids. In particular we will study the following:

Static fluids
Pressure exerted by a static fluid
Methods of measuring pressure
Pascal's principle Archimedes' principle, buoyancy
Real versus ideal **Fluids in motion:** fluids
Equation of continuity
Bernoulli's equation

1 Pressure

The pressure is a scalar quantity, not a vector. In a fluid at rest the pressure is the same in all directions at a given point.

1.1 Units

Pressure is transmitted to solid boundaries or across arbitrary sections of fluid normal to these boundaries or sections at every point. It is a fundamental parameter in thermodynamics and it is conjugate to volume.

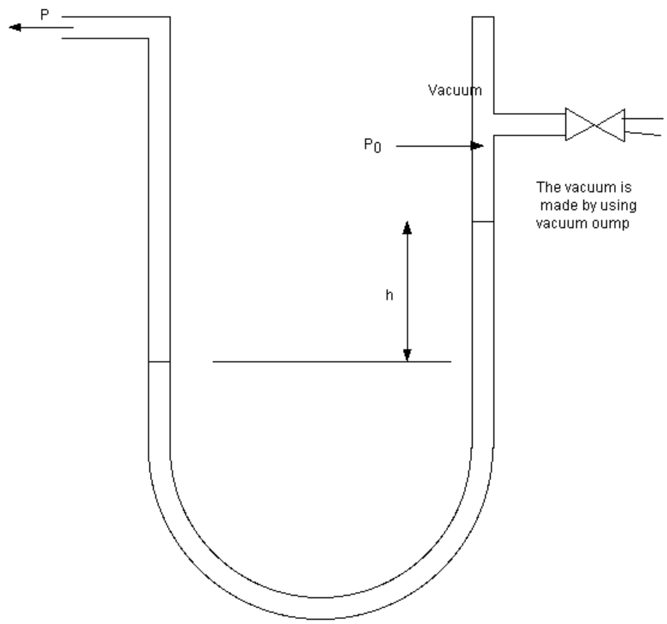
Pressure is a scalar, and has SI units of pascals

$1\text{Pa} = 1\ \text{N/m}^2 = 10\ \text{dyn/cm}^2$
$1\ \text{mbar} = 100\ \text{Pa} = 10^3\ \text{dyn/cm}^2$
$1\ \text{bar} = 10^5\ \text{Pa}$

Atmospheric pressure

$P_0 = 1\ \text{atm} = 1.01325 \times 10^5\ \text{Pa}$
$P_0 = 760\ \text{mmHg} = 760\ \text{Torr}$

1.2 Mercury manometer



Mercury manometer

$$P = P_0 + \rho gh$$

1 Torr = 1.0 mmHg

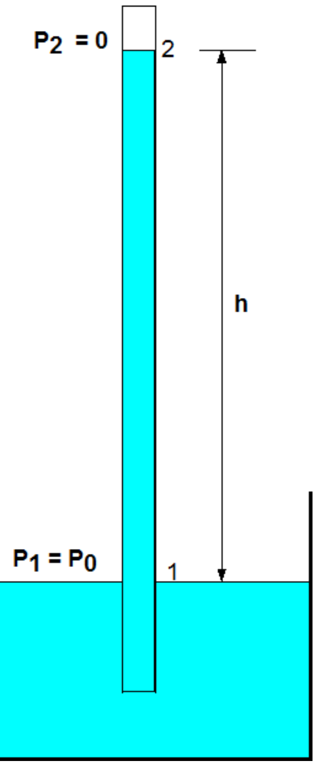
1 atm = 760 mmHg = 760 Torr

which means

$$\rho(Hg)gh = 0.76(m) \times 13.6 \times 10^3 (\frac{kg}{m^3}) \times 9.80(\frac{m}{s^2}) = 1.013 \times 10^5 \frac{N}{m^2} = 1.013 \times 10^5 Pa$$

1.3 Water suction

Suppose that one attempts to drink water through a very long straw. With his great strength he achieves maximum possible suction. The walls of the tubular straw do not collapse. What is the maximum height through which he can lift the water?



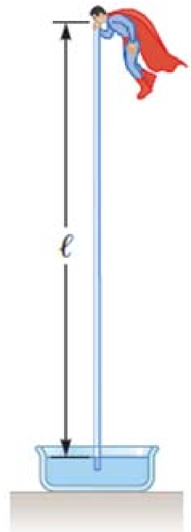
We imagine one to produce a perfect vacuum in the straw. Take point 1 at the water surface in the basin and point 2 at the water surface in the straw.

$$P_0 = P_1 + \rho g h = 0 + \rho g h$$

or

$$h = \frac{P_0}{\rho g} = \frac{1.013\times 10^5 Pa}{(1000 kg/m^3)(9.80 m/s^2)} = 10.3 m.$$

((Note)) **Walter Levin (Sucking cranberry)**



There is a very interesting experimental demonstration.
https://www.youtube.com/watch?v=Hlmpoo32QPE&index=29&list=PLUdYlQf0_sSsb2tNcA3gtgOt8LGH6tJbr

1.4 Pressure under water

The pressure *P* in the water increases with increasing the depth of water. In fact, the change of pressure ΔP is given by

$$\Delta P = \rho g \Delta h .$$

When ΔP = 1 atm = 1.013 x 10^5 Pa, Δh is evaluated as

$$\Delta h = \frac{\Delta P}{\rho g} = \frac{1.013 \times 10^5}{9.8 \times 10^3} = 10.3m$$

This means that the pressure at 100 m under the sea level is (100/10.3) +1 = 10.7 atm.

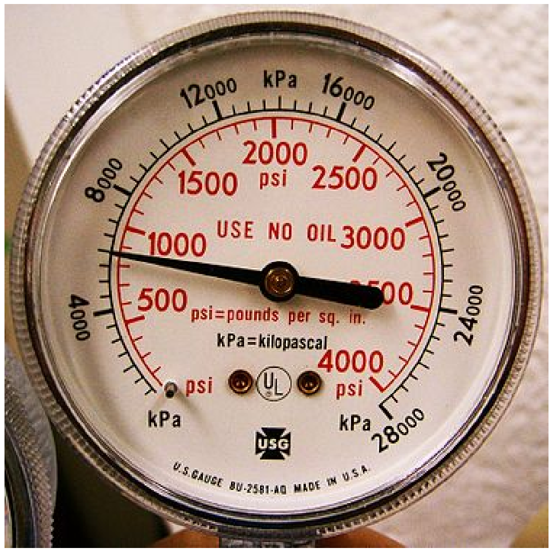
1.5 Units of psi (conventional units in the experiment in the U.S.A.)

psi: Pounds per square inch (redirected from Pound-force per square inch).

The pound per square inch or, more accurately, pound-force per square inch (symbol: psi or lbf/in^2) is a unit of pressure or of stress based on avoirdupois units. It is the pressure resulting from a force of one pound-force applied to an area of one square inch:

1 psi = 6,894.75729 Pa
1 psi = (6894.75729/101325) x 760 = = 51.715 Torr
1 mbar = 10^{-3} bar = 1.4503774 x 10^{-2} psi
1 bar = 10^6 dyn/cm^2 = 10^5 Pa = 14.503774 psi

1 atm = 14.69594554 psi



https://en.wikipedia.org/wiki/Pounds_per_square_inch

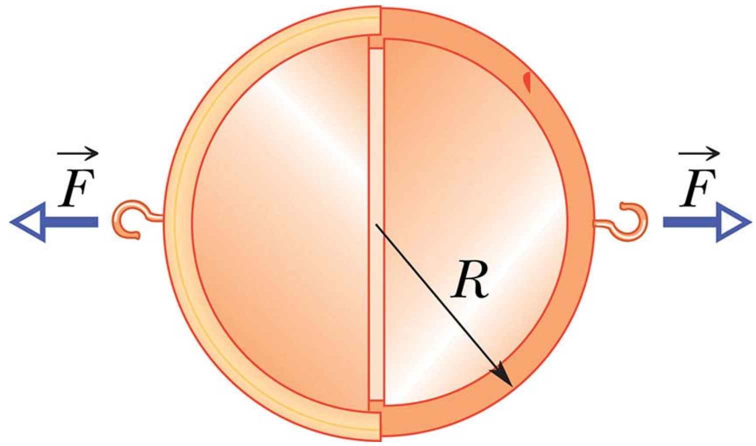
1.6 Hemisphere of Magdeburg

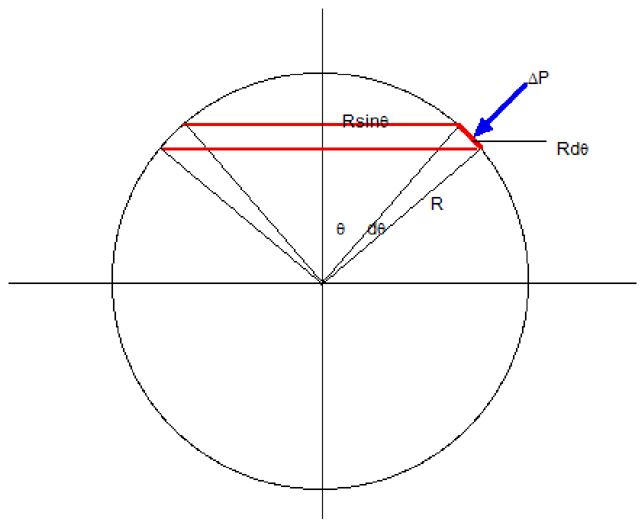
Problem 14-7 (10-th edition)**

In 1654, Otto von Guericke, inventor of the air pump, gave a demonstration before the noblemen of the Holy Roman Empire in which two teams of eight horses could not put apart two evacuated brass hemispheres. (a) Assuming the hemispheres have (strong) thin walls, so that *R* in Fig. may be considered both the inside and outside radius, show that the force *F* required to pull apart the hemisphere has magnitude

$$F = \pi R^2 \Delta p$$

where Δp is the difference between the pressures outside and inside the sphere. (b) Taking *R* as 30 cm, the inside pressure as 0.10 atm, and the outside pressure as 1.00 atm, find the force magnitude the team of horses would have to exert to pull apart the hemispheres. (c) Explain why one team of horses could have proved the point just as well if the hemispheres were attached to a sturdy wall.





$$F_h = \int_0^{\pi/2} \Delta P(2\pi R\sin\theta)\cos\theta R d\theta = 2\pi R^2 \Delta P \int_0^{\pi/2} \sin\theta\cos\theta d\theta$$

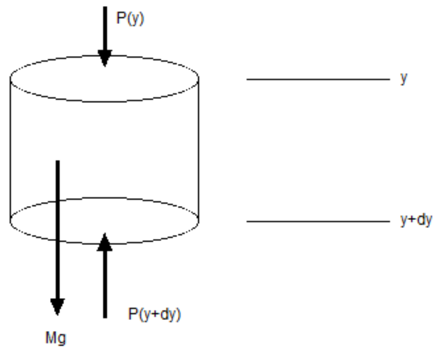
$$= \pi R^2 \Delta P \int_0^{\pi/2} \sin(2\theta) d\theta = \frac{\pi R^2 \Delta P}{2}[-\cos(2\theta)]_0^{\pi/2} = \pi R^2 \Delta P$$

We use 1 atm = 1.01 × 10^5 Pa to show that Δp = 0.90 atm = 9.09 × 10^4 Pa. The sphere radius is R = 0.30 m, so

$F_h = \pi(0.30\text{ m})^2(9.09 \times 10^4\text{ Pa}) = 2.6 \times 10^4$ N.

2. Variation of pressure with depth

Fluids have pressure that varies with depth. If a fluid is at rest in a container, all portions of the fluid must be in static equilibrium. All points at the same depth must be at the same pressure. Otherwise, the fluid would not be in equilibrium



A is the area of the cylinder and *P*(*y*) is the pressure at the depth *y* from the surface of liquid.

$$\sum F_y = AP(y) + Mg - AP(y + dy) = 0$$

$$Mg = A\rho(y)gdy$$

or

$$\frac{P(y + dy) - P(y)}{dy} = \rho(y)g$$

In the limit of dy→0, we have

$$\frac{dP}{dy} = \rho(y)g \,,$$

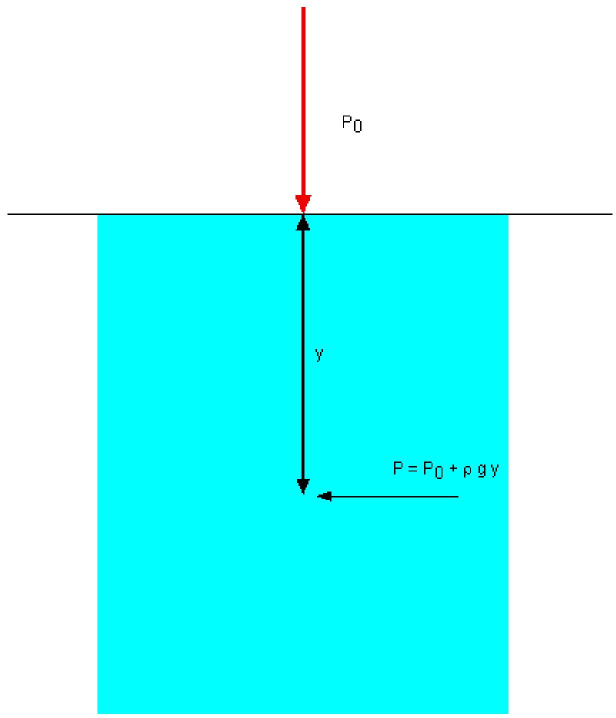
or

$$P(y) = g\int \rho(y)dy$$

When $\rho(y)$ is independent of *y*, one can get

$$P(y) = P_0 + \rho gy$$

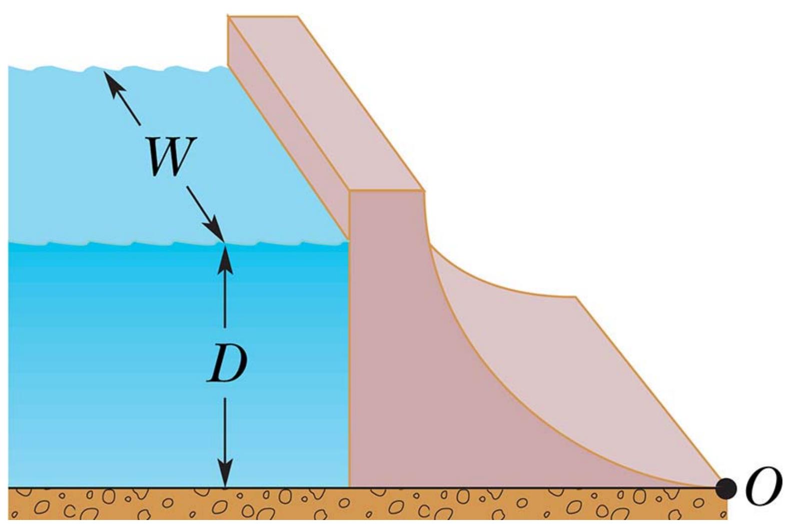
where P_0 is the atmospheric pressure at $y = 0$.

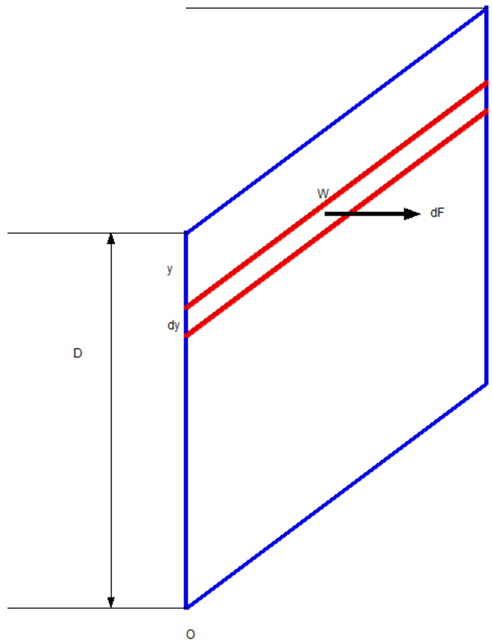


2.2 Example

Problem 14-24 (SP-14) (10-th edition)**

In Fig., water stands at depth D = 35.0 m behind the vertical upstream of a dam of width W = 314 m. Find (a) the net horizontal force on the dam from the gauge pressure of the water and (b) the net torque due to that force about a line through O parallel to the width of the dam. (c) Find the moment arm of this torque.





((**Solution**))
$D = 35$ m, $W = 314$ m
$g = 9.8$ m/s^2
$\rho = 10^3$ kg/m^3

The total horizontal force and the net torque are evaluated as follows;

The pressure:

$$P(y) = \rho g y$$

The force:

$$dF = WdyP(y) = Wdy(\rho g y)$$

or

$$F = \int_0^D Wdy(\rho g y) = \rho g W \frac{1}{2} D^2 = 1.885 \text{ x } 10^9 \text{ N}$$

The torque:

$$d\tau = (D - y)dF = (D - y)(Wdy\rho g y)$$

or

$$\tau = \rho g W \int_0^D (D-y) y dy = \rho g W \frac{D^3}{6} = 2.19892 \text{ x } 10^{10} \text{ Nm}$$

The average arm's length of torque

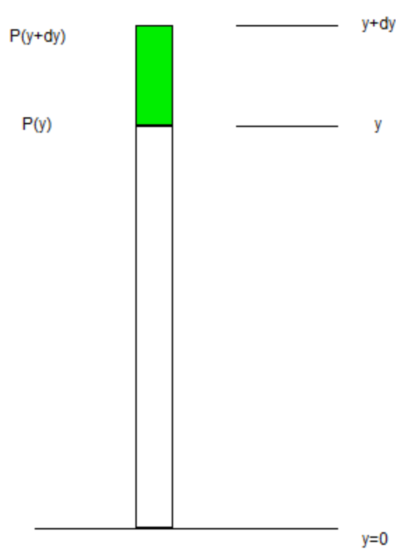
$$r = \frac{\tau}{F} = \frac{\rho g W \dfrac{D^3}{6}}{\rho g W \dfrac{1}{2} D^2} = \frac{D}{3} = 11.7 \text{ m.}$$

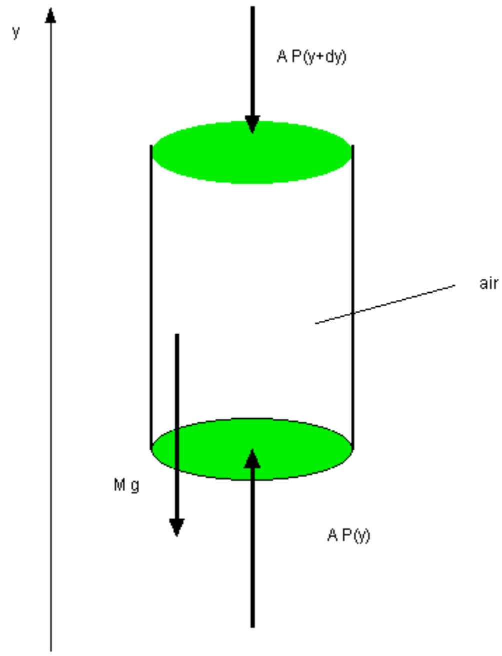
3 Variation of pressure with height

Problem 14-27 (10-th edition)

What would be the height of the atmosphere if the air density (a) were uniform and (b) decreased linearly to zero with height? Assume that at sea level the air pressure is 1.0 atm and the air density is 1.3 kg/m^3.

P_0 = 1 atm = 1.01325 x 10^5 Pa, Pa = 1 N m^2.
$\rho_{air} = 1.3 kg/m^3$





where Mg is the weight of air;

In equilibrium

$$\sum F_y = AP(y) - AP(y+dy) - Mg = 0$$

$$Mg = A\rho_{air}(y)gdy$$

or

$$P(y+dy) - P(y) = -\rho_{air}(y)gdy$$

$$\frac{dP(y)}{dy} = -\rho_{air}(y)g$$

Then we have

$$P(y) = \int dP = P(y=0) - \int_0^y \rho_{air}(y)gdy$$

(a) In the case of $\rho_{air}(y) = \rho_0$

$$P(y) = -\int_0^y \rho_0 gdy + P(y=0) = P_0 - \rho_0 gy$$

When $P(y) = 0$, we have

$$P(y=0) - \rho_0 g y = 0$$

$$y = \frac{P_0}{\rho_0 g} = 7.9km$$

(b) In the case of $\rho_{air}(y) = \rho_0(1-\frac{y}{h})$: h is a height to be determined.

$$P(y) = -\int_0^y \rho_0 g(1-\frac{y}{h})dy + P(y=0) = -g\rho_0(y - \frac{1}{2h}y^2) + P_0$$

When $P(h) = 0$, we have

$$P(h) = -g\rho_0(h - \frac{1}{2h}h^2) + P(y=0) = -g\rho_0\frac{h}{2} + P_0 = 0$$

or

$$h = \frac{2P_0}{g\rho_0} = 16km$$

(c) We assume that at any given height the density of air proportional to pressure;

$$\frac{P(y)}{P_0} = \frac{\rho(y)}{\rho_0}$$

((Note)) Here we assume that air is an ideal gas.

$$P_0 V_0 = P(y)V(y)$$

$$\rho_0 = \frac{M}{V_0}$$

$$\rho(y) = \frac{M}{V(y)}$$

or

$$\frac{P_0}{\rho_0}M = \frac{P(y)}{\rho(y)}M$$

$$\frac{P(y)}{P_0} = \frac{\rho(y)}{\rho_0}$$

Then we have

$$\frac{dP}{dy} = -\rho(y)g = -\rho_0 g\frac{P}{P_0}$$

or

$$\int \frac{1}{P}dP = -\frac{\rho_0 g}{P_0}\int dy$$

or

$$P = P_0 \exp(-\frac{\rho_0 gy}{P_0})$$

where ρ_0 (= 1.1929 kg/m^3) is the density of air at room temperature ($P = P_0$ = 1 atm)

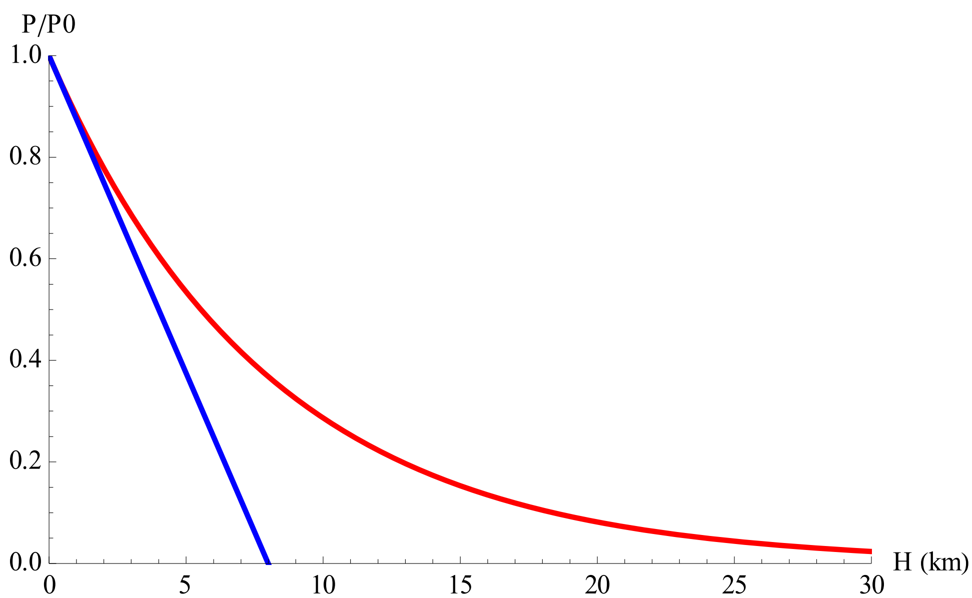


Fig. Red curve for the case (c) and the blue curve for the case (a). The straight line (blue) is a tangential line of the red curve at $H = 0$.

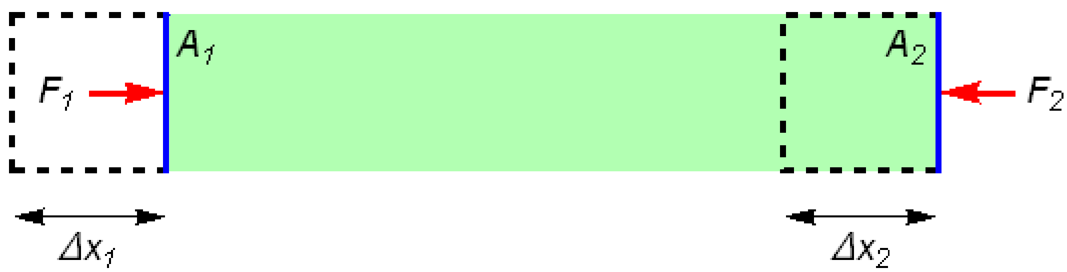
4 Pascal's principle

((**Pascals' Principle**))

Pascal's law or the principle of transmission of fluid-pressure (also *Pascal's Principle*) is a principle in fluid mechanics that states that pressure exerted anywhere in a confined incompressible fluid is transmitted equally in all directions throughout the fluid such that the pressure variations (initial differences) remain the same.

Here we derive the Pascal's principle by using three approaches.

(a) Approach I: Approach from the work-energy theorem



Work-energy theorem:

$$\Delta K = W_c + W_{nc} = -\Delta U + W_{nc}$$

Suppose that

$$\Delta K = 0, \qquad \Delta U = 0$$

Then we have

$$W_{nc} = 0$$

or

$$W_{nc} = F_1 \Delta x_1 - F_2 \Delta x_2 = 0$$

Since

$$A_1 \Delta x_1 = A_2 \Delta x_2 .$$

we get the relation

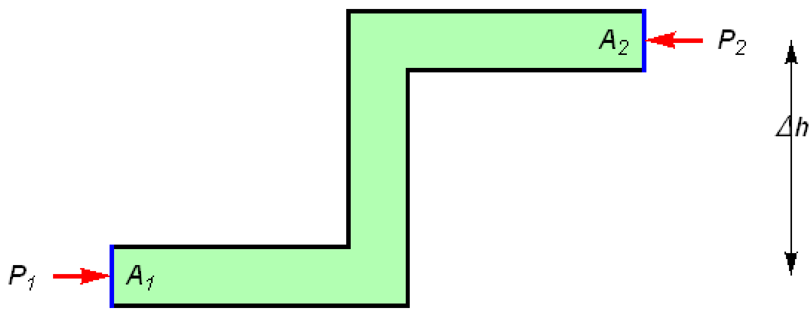
$$\frac{F_1 \Delta x_1}{A_1 \Delta x_1} = \frac{F_2 \Delta x_2}{A_2 \Delta x_2}$$

or

$$\frac{F_1}{A_1} = P_1 = \frac{F_2}{A_2} = P_2$$

This equation is valid for the case when the effect of fornce due to the potential

(b) Approach II: Approach from the Bernoulli's equation



According to the Bernoulli's equation, we have

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho g h_2$$

When $v_1 = v_2 = 0$, we get

$$P_1 + \rho g h_1 = P_2 + \rho g h_2$$

or

$$P_2 = P_1 + \rho g (h_1 - h_2)$$

Suppose that

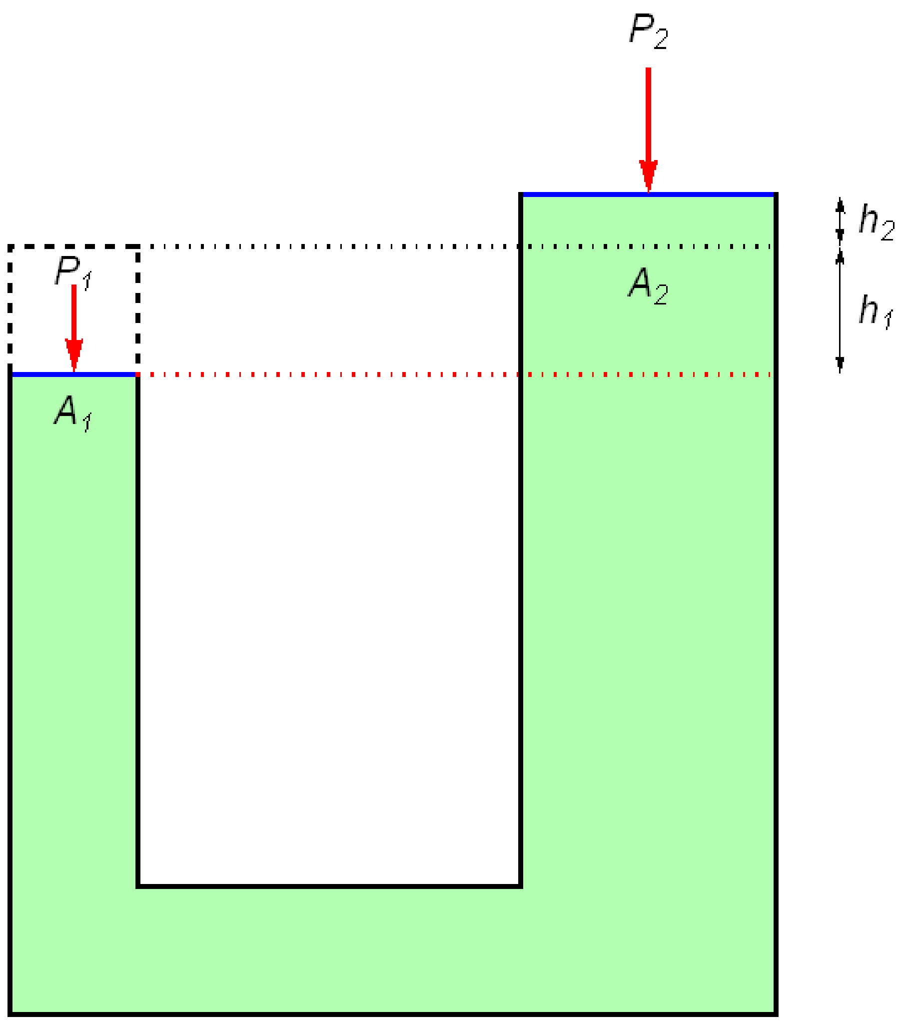
$$\left|\rho g (h_1 - h_2)\right| << P_1, P_2$$

Then we have

$$P_2 = P_1 . \qquad \text{(Pascal's principle)}$$

(c) Approach III: Standard approach

The approach is essentially the same as the above approaches.



The pressure at the height denoted by red line is

$$P_1 + P_0 = P_2 + P_0 + \rho g(h_1 + h_2)$$

or

$$P_1 = P_2 + \rho g(h_1 + h_2)$$

When $\rho g(h_1 + h_2)$ is much smaller than P_1 and P_2, the above equation is approximated by

$$P_1 = P_2 \qquad \text{(Pascal's principle)}$$

((**Example**))

The hydraulic lift at a car repair shop is filled with oil. The car rests on a 25-cm-diameter piston. To lift the car, compressed air is used to push down on a 6.0-cm-diameter piston What does the pressure gauge read when a 1300 kg car is 2.0 m above the compressed air piston?

((**Solution**))

$\rho = 900$ kg/m^3 for oil
$r_1 = 3$ cm, $r_2 = 12.5$ cm.
$m = 1300$ kg.
$F_2 = mg = 1.274 \times 10^4$ N.
$h = 2$ m

$$P_1 = P_2 + \rho g h$$

where

$$h = h_1 + h_2$$

$$A_1 h_1 = A_2 h_2$$

$$h_1 = \frac{A_2}{A_1} h_2 = \frac{r_2^{\ 2}}{r_1^{\ 2}} h_2$$

(b)

$\rho g h$ =900 x 9.8 x 2=1.764 x 10^4 Pa

F_2 is the weight of the car pressing down on the piston:

$$F_2 = mg = 1.274 \times 10^4 \text{ N}$$

The piston areas are

$$A_1 = \pi r_1^{\ 2} = 0.00283 \text{ m}^2. \qquad A_2 = \pi r_2^{\ 2} = 0.0491 \text{ m}^2.$$

The pressure P_2 is

$$P_2 = \frac{F_2}{A_2} = \frac{1.274 \times 10^4}{0.0491} = 2.595 \times 10^5 \text{ Pa.}$$

The pressure P_1 is

$$P_1 = P_2 + \rho g h = (25.95 + 1.27) \times 10^4 = 2.72 \times 10^5 \text{ Pa}$$

The force required to hold the car at height 2.0 m.

$$h_1 = \frac{A_2}{A_1} h_2 = 17.35 h_2, \quad h = h_1 + h_2 = 2.0 \text{ m}.$$

$$h_2 = 0.11 \text{ m}, \qquad h_1 = 1.89 \text{ m}$$

Note that P_1 is not always equal to P_2 because of the gravitational contribution.

(d) A proper understanding of the Pascal's principle

Here we discussed the derivation of the Pascal's principle using the three approaches. The approach I is independent of the gravitational potential (horizontal configuration). Thus the Pascal's principle is correctly derived. However, the approaches II and III are dependent on the gravitational potential because of the vertical configuration. The Pascal's principle can be derived when the gravitational pressure is negligibly small compared with the magnitudes of P_1 and P_2.

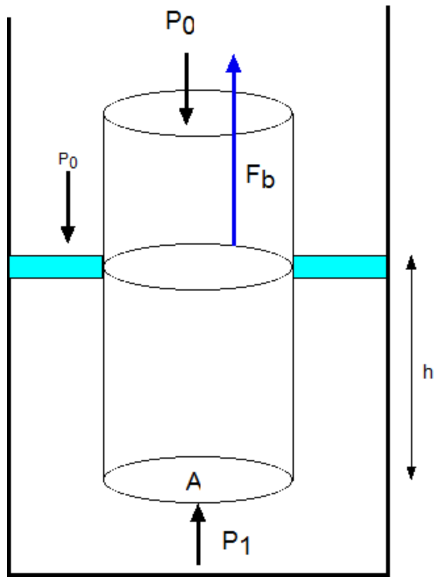
I checked several standard textbooks of general physics. It seems to me that there is some misunderstanding of the Pascal's Principle. In almost textbooks, the gravitational contribution for the vertical configuration is neglected. There is one exception. The approach III is discussed in Knight's book. It is noted that the Pascal's principle is valid only when the gravitational contribution is negligibly small compared with the pressure P_1 and P_2. When we use the Pascal's principle for the vertical configuration, we must make sure that $\rho g h$ should be much smaller than P_1 and P_2.

In conclusion, the Bernoulli's equation is based on the work-energy theorem. So the approach based on this equation leads to the most appropriate result.

5 Buoyant force

((Archimedes's principle))

Any object completely or partially immersed in a fluid is buoyed up by a force equal to the weight of the displaced fluid.



$$P_1 = P_0 + \rho_W gh$$

The buoyant force F_b is defined by

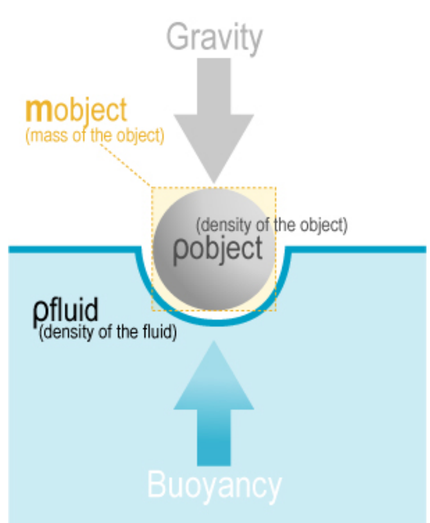
$$F_b = AP_1 - AP_0 = A\rho_W gh$$

where Ah is a part of volume under the surface.

Then the buoyant force is given by

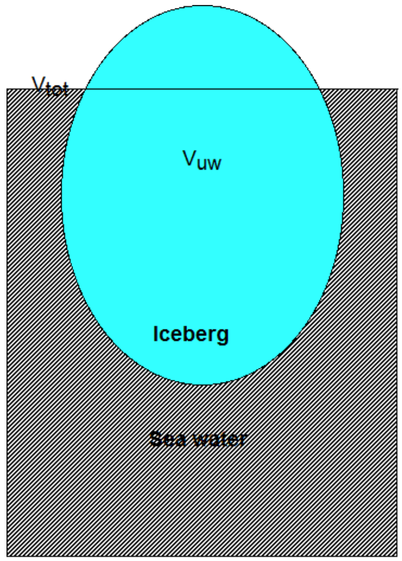
$$F_b = \rho_W gV$$

where $V = Ah$ is the volume of the cylinder under the water.



((Iceberg))

Because the density of pure ice is about ρ_{ice} = 920 kg/m³, and that of sea water about ρ_{sea} = 1025 kg/m³, typically only one-tenth of the volume of an iceberg is above water. The shape of the underwater portion can be difficult to judge by looking at the portion above the surface.



The mass of the iceberg, M, is given by

$$M = V_{tot}\rho_{ice},$$

where V_{tot} is the total volume of the iceberg. The condition that the iceberg floats in the seawater leads to

$$Mg = V_{uw}\rho_{sea}g = V_{tot}\rho_{ice}g$$

where V_{uw} is the volume of the iceberg under the sea water. Then we have

$$\frac{V_{uw}}{V_{tot}} = \frac{\rho_{ice}}{\rho_{sea}} = \frac{920}{1025} = 0.898$$

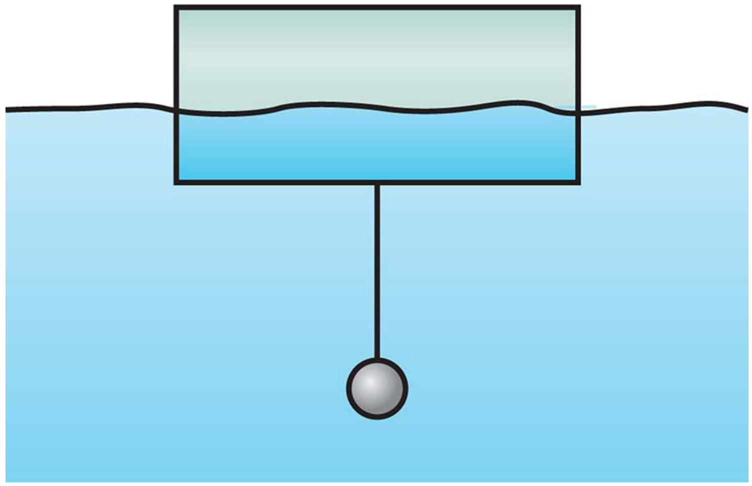
This means that 90 % of the volume of the iceberg is under the sea level.

6 Examples

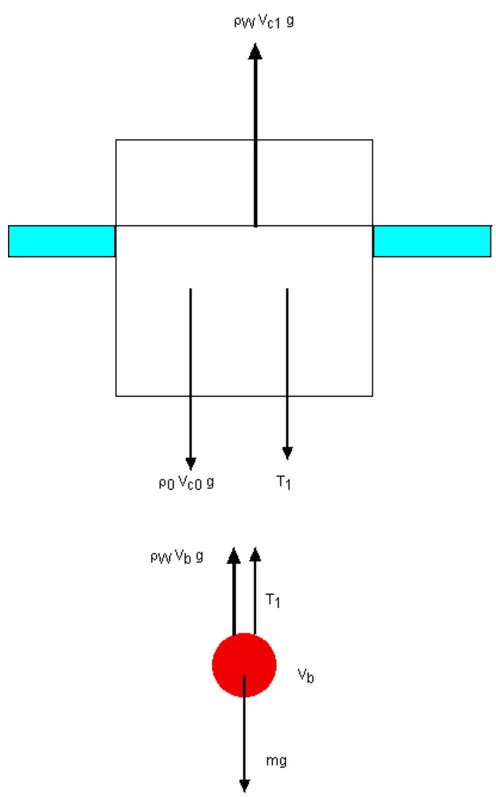
6.1

Problem 14-48*(SP-14) (10-th edition)**

Figure shows an iron ball suspended by thread of negligible mass from an upright cylinder that floats partially submerged in water. The cylinder has a height of 6.00 cm, a face area of 12.0 cm^2 on the top and bottom, and a density of 0.30 g/cm^3, and 2.00 cm of its height is above the water surface. What is the radius of the iron ball?



((**Solution**))



$$T_1 + \rho_W V_b g = \rho_b V_b g$$
$$\rho_W V_{c1} g = T_1 + \rho_0 V_{co} g$$

where

V_{c1} = 48 cm^3 = a part of volume under water
V_{c0} = 72 cm^3 = total volume of cylinder
$V_b = (4\pi/3)r^3$ = a volume of Fe ball
$\rho_b = \rho_{Fe}$ = 7.86 g/cm^3 = a density of Fe ball
ρ_0 = density of cylinder = 0.30 g/cm^3
ρ_W = density of water = 1/0 g/cm^3,
T_1 = tension

From the above equations, we have

$$T_1 = \rho_W V_{c1} g - \rho_0 V_{co} g = \rho_b V_b g - \rho_W V_b g$$

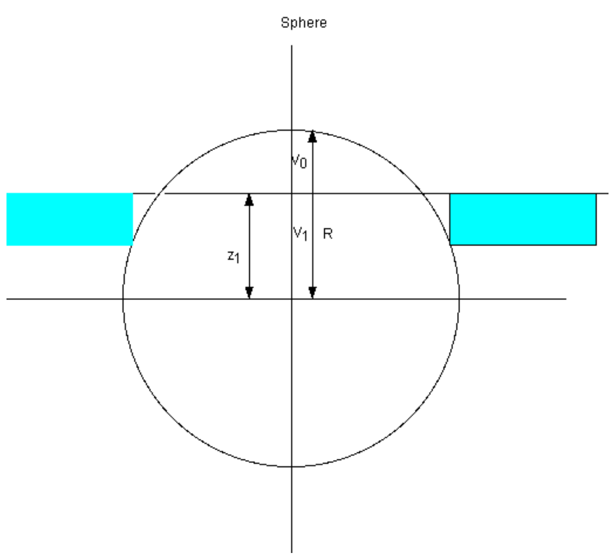
or

$$V_b = \frac{\rho_W V_{c1} - \rho_0 V_{co}}{\rho_b - \rho_W} = \frac{4\pi}{3} r^3 = 3.84 cm^3$$

r = 0.972 cm.

6.2 Density of the wooden sphere

A wooden sphere has a diameter of 1.20 cm. It floats in water with 0.40 cm of its diameter above water. Determine the density of the wooden sphere.

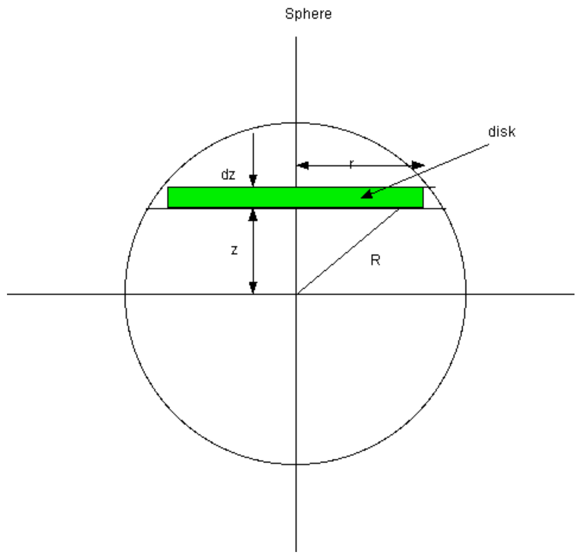


((Solution))

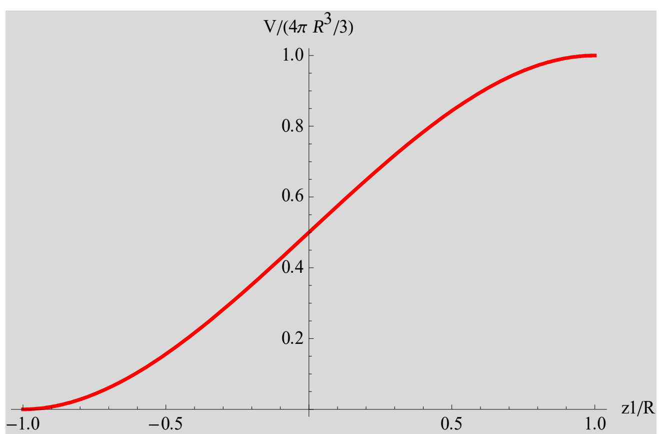
z_1 = 0.2 cm. 2 R = 1.20 cm

$$V_0 + V_1 = V = \frac{4\pi}{3} R^3$$

$$\sum F_y = \rho_{water} V_1 g - \rho V g = 0. \qquad \text{or} \qquad \rho_{water} V_1 = \rho V$$



$$V_1 = \int_{-R}^{z_1} \pi r^2 dz = \int_{-R}^{z_1} \pi (R^2 - z^2) dz = \frac{\pi}{3}(2R - z_1)(R + z_1)^2$$

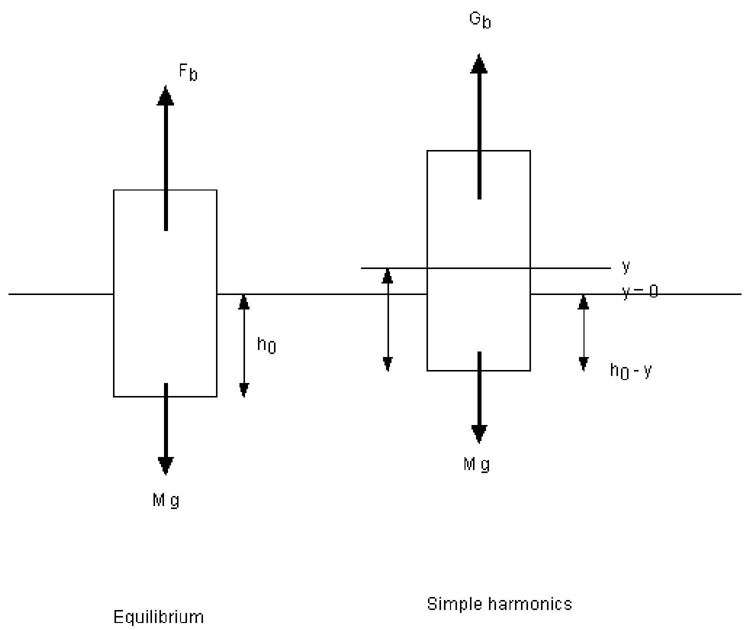


$$\rho = \rho_{water} \frac{V_1}{V} = \rho_{water} \frac{\frac{\pi}{3}(2R - z_1)(R + z_1)^2}{\frac{4\pi}{3} R^3} = \rho_{water} \frac{1}{4}(2 - \xi)(1 + \xi)^2$$

with $\xi = \frac{z_1}{R}$. When $\xi = \frac{0.2}{0.6} = \frac{1}{3}$, we have

$$\rho = 0.74 g / cm^3.$$

6.3 Simple harmonics in fluid mechanics



In equilibrium

$$F_b = A\rho g h_0 = Mg$$

or

$$h_0 = \frac{M}{A\rho}$$

Simple harmonics

$$M\ddot{y} = G_b - Mg = (h_0 - y)\rho A g - Mg$$

where G_b is the buoyant force.

$$M\ddot{y} = (h_0 - y)\rho A g - Mg = -y\rho A g$$

$$\ddot{y} = -\frac{\rho A g}{M} y = -\omega^2 y \qquad \text{(simple harmonics)}$$

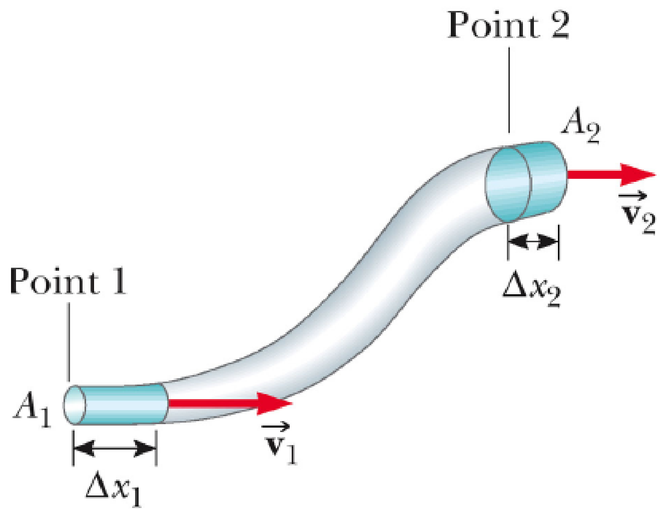
where

$$\omega = \sqrt{\frac{\rho A g}{M}}$$

$$T = 2\pi\sqrt{\frac{M}{\rho A g}}$$

From the measurement of the period T, one can determine the value of density ρ.

7 Equation of continuity



Consider a fluid moving through a pipe of nonuniform size (diameter). The particles move along streamlines in steady flow. The mass that crosses A_1 in some time interval is the same as the mass that crosses A_2 in the same time interval.

$$A_1 \Delta x_1 = A_2 \Delta x_2$$

$$A_1 v_1 \Delta t = A_2 v_2 \Delta t$$

or

$$A_1 v_1 = A_2 v_2$$

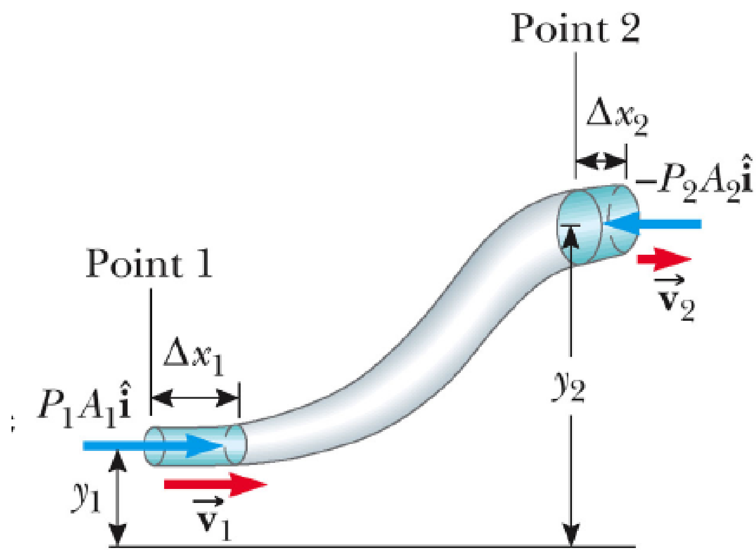
The equation of continuity for fluids.

The product of the area and the fluid speed at all points along a pipe is constant for an incompressible fluid.

8 Bernoulli equation

8.1 Derivation of the Bernoulli equation

As a fluid moves through a region where its speed and/ or elevation above the Earth's surface changes, the pressure in the fluid varies with these changes. The relation between fluid speed, pressure, and elevation was first derived by Daniel Bernoulli.



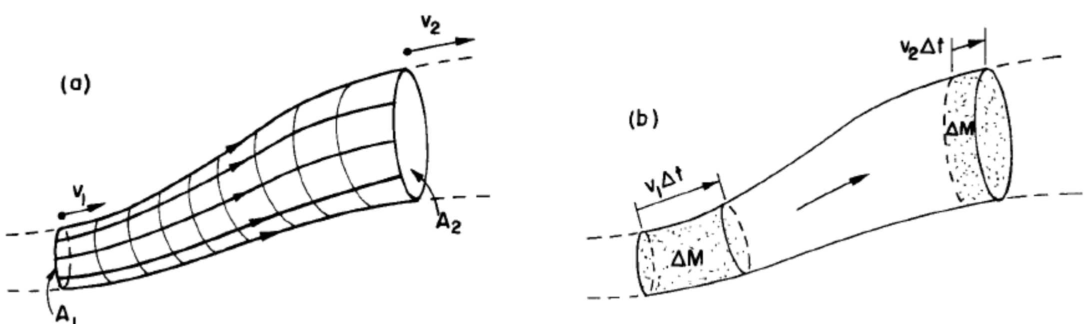
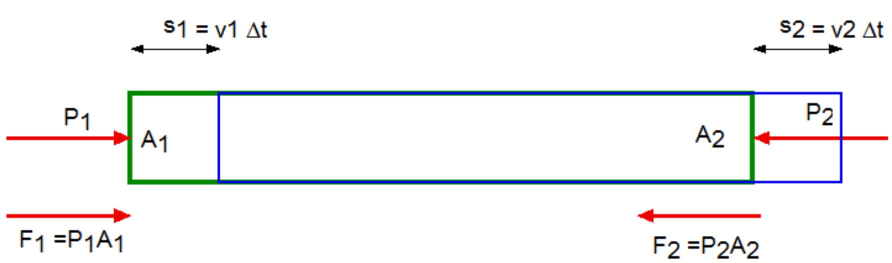


Fig. 40–6. Fluid motion in a flow tube.

This figure comes from Feynman Lecture on Physics.



((Note)) We use the Work-energy theorem to derive the Bernoulli's equation.

The equation of continuity:

$A_1 v_1 = A_2 v_2$.

Applying conservation of energy in form of the work-kinetic energy theorem we find that the change in KE of the system equals the net work done on the system;

$$\Delta K = W = W_c + W_F$$
$$\Delta E = \Delta(K + U) = W_F$$

where $W_c = -\Delta U$. The work done by the forces (= W_F) is

$$W_F = -F_2 s_2 + F_1 s_1 = -P_2 A_2 v_2 \Delta t + P_1 A_1 v_1 \Delta t$$

Note that the direction of P_2 is anti-parallel to that of $\mathbf{v}_2$. The change of potential energy is

$$\Delta U = mgy_2 - mgy_1 = \rho g A_2 v_2 y_2 \Delta t - \rho g A_1 v_1 y_1 \Delta t ,$$

where

$$m = \rho A_2 v_2 \Delta t = \rho A_1 v_1 \Delta t .$$

The increase in kinetic energy is

$$\Delta K = \frac{1}{2} m v_2^{\,2} - \frac{1}{2} m v_1^{\,2} = \frac{1}{2} \rho A_2 v_2 \Delta t v_2^{\,2} - \frac{1}{2} \rho A_1 v_1 \Delta t v_1^{\,2}$$

Putting these together,

$$\begin{aligned} \Delta E &= \Delta(K + U) \\ &= (\frac{1}{2} \rho A_2 v_2 \Delta t v_2^{\,2} - \frac{1}{2} \rho A_1 v_1 \Delta t v_1^{\,2}) + (\rho g A_2 v_2 y_2 \Delta t - \rho g A_1 v_1 y_1 \Delta t) \end{aligned}$$

and

$$W_F = P_1 A_1 v_1 \Delta t - P_2 A_2 v_2 \Delta t .$$

Then we get

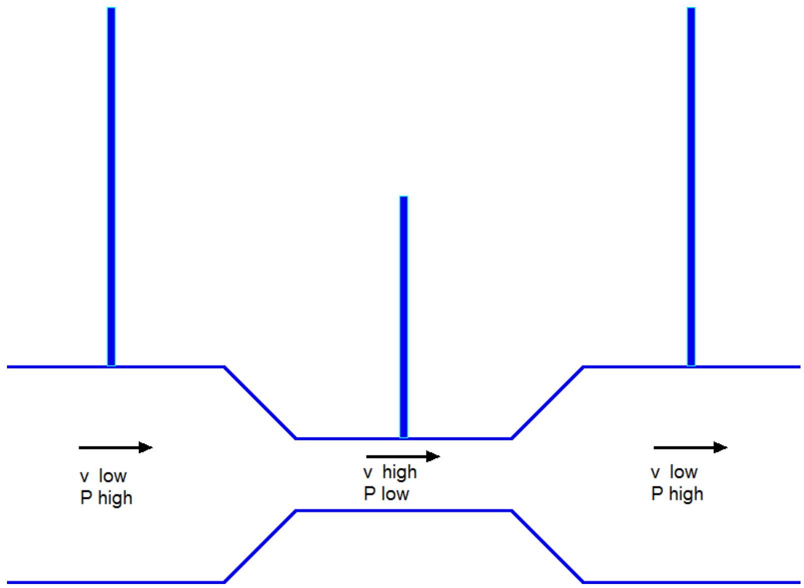
$$\frac{1}{2} \rho A_2 v_2 \Delta t v_2^{\,2} + P_2 A_2 v_2 \Delta t + \rho g A_2 v_2 h_2 \Delta t = \frac{1}{2} \rho A_1 v_1 \Delta t v_1^{\,2} + P_1 A_1 v_1 \Delta t + \rho g A_1 v_1 h_1 \Delta t .$$

After dividing by Δt, ρ and $A_1 v_1$ (=$A_2 v_2$) as the fluid is incompressible:

$$\frac{1}{2}\rho v_2{}^2 + P_2 + \rho g y_2 = \frac{1}{2}\rho v_1{}^2 + P_1 + \rho g y_1 = const$$

This equation is called a Bernoulli's equation. This equation is in fact nothing more than a statement of the conservation of energy.

((**Typical example resulting from Bernoulli's equation**))

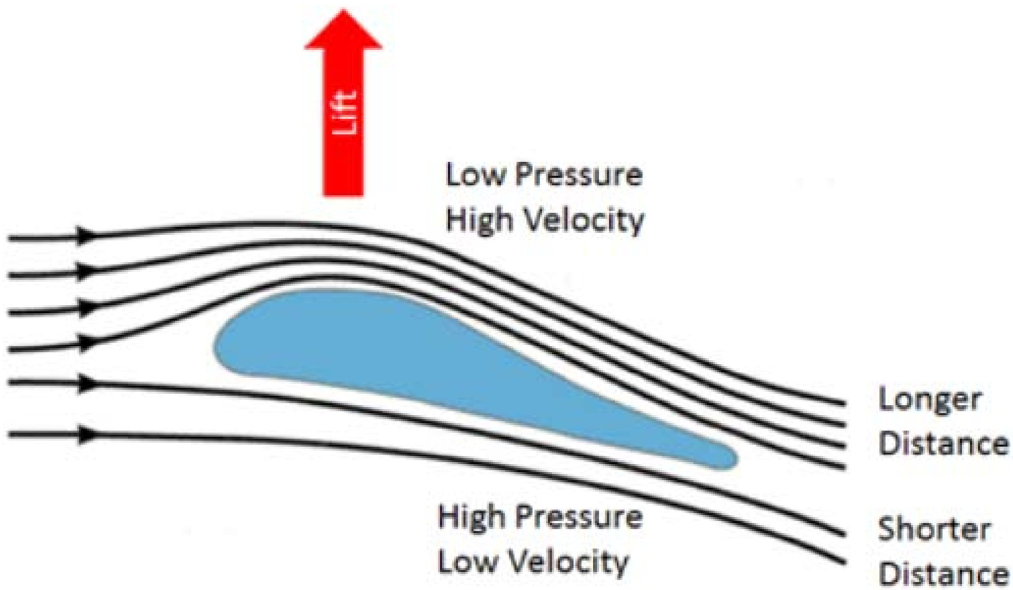


The pressure is the lowest where the velocity is the highest.

Note that if the velocity is equal to zero, the level is independent of the cross sectional area.

8.2 Example

(a) How airplane wings generate lift



1. Over the wing, the flow tube decreases in size due to the compression of steam lines. The higher speed lowers the pressure to $p<p_{atmos}$.
2. The pressure is p_{atmos} beneath the wing
3. The pressure difference above and below the wing causes *lift*.

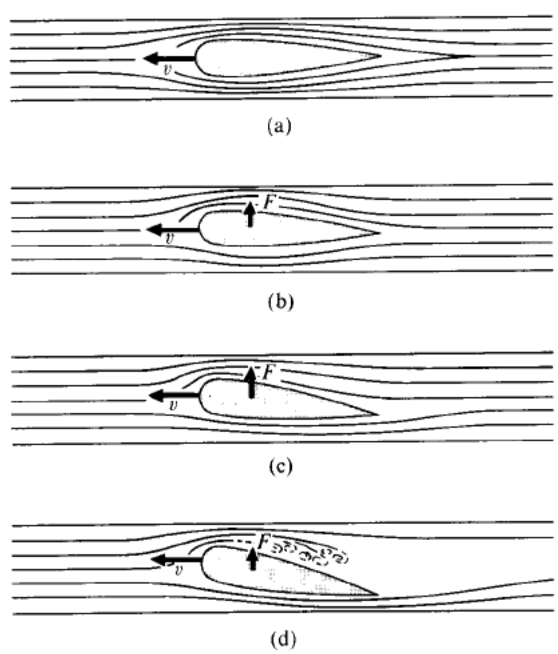


Fig. Air flow past a wing. At (a) the flow is the same on both surfaces, so that no lift results. The lift at (c) is greater than that at (b) because of the greater pressure difference between upper and lower surfaces (the pressure is least where the streamlines are closest together). At (d) turbulence reduces the available lift.

(b) Bernoulli ball

Bernoulli's principle tells us that air that is moving at high speed has lower pressure than still air. The air moves around the ball to create a pocket around the ball of low pressure air. When the ball moves to the side of the pocket, it will be pushed back in. And the upward force from the air stream keeps the ball aloft.

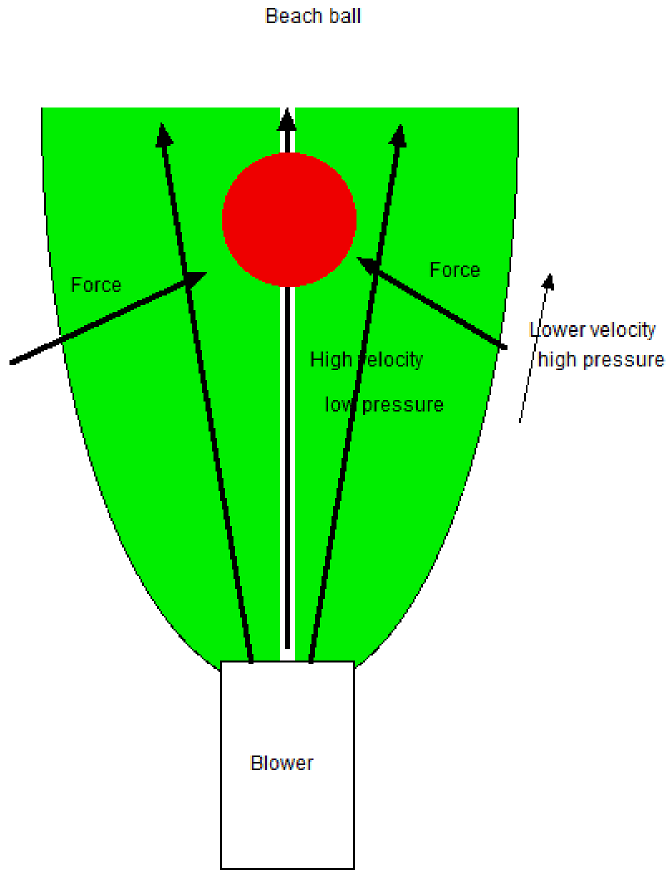
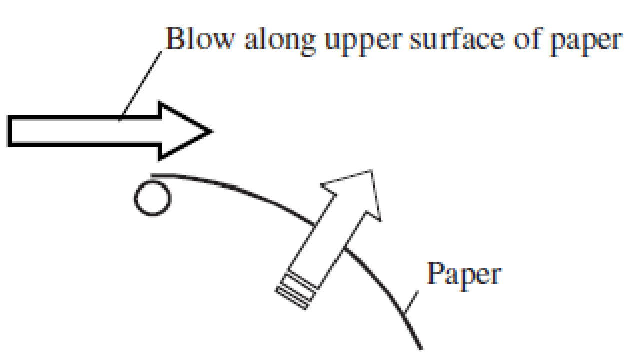


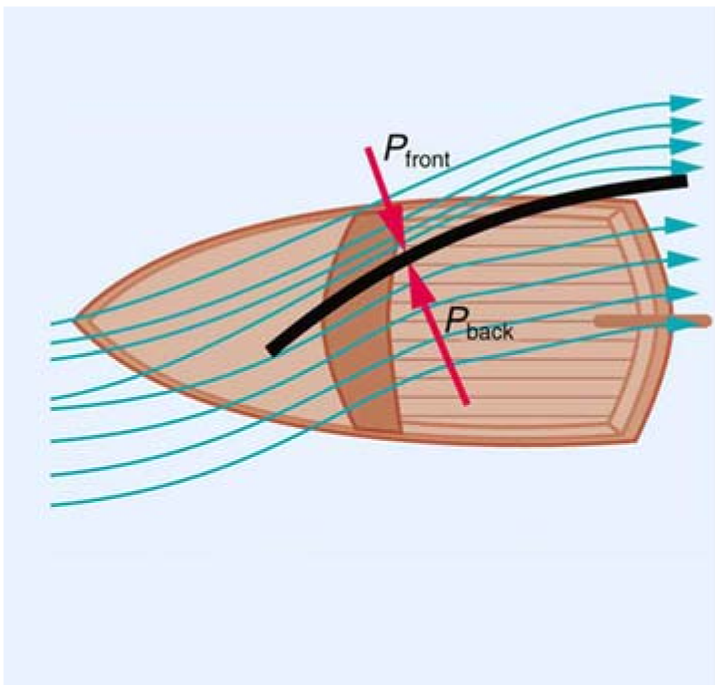
Fig. Bernoilli ball

(c) **Blowing along upper surface of paper**



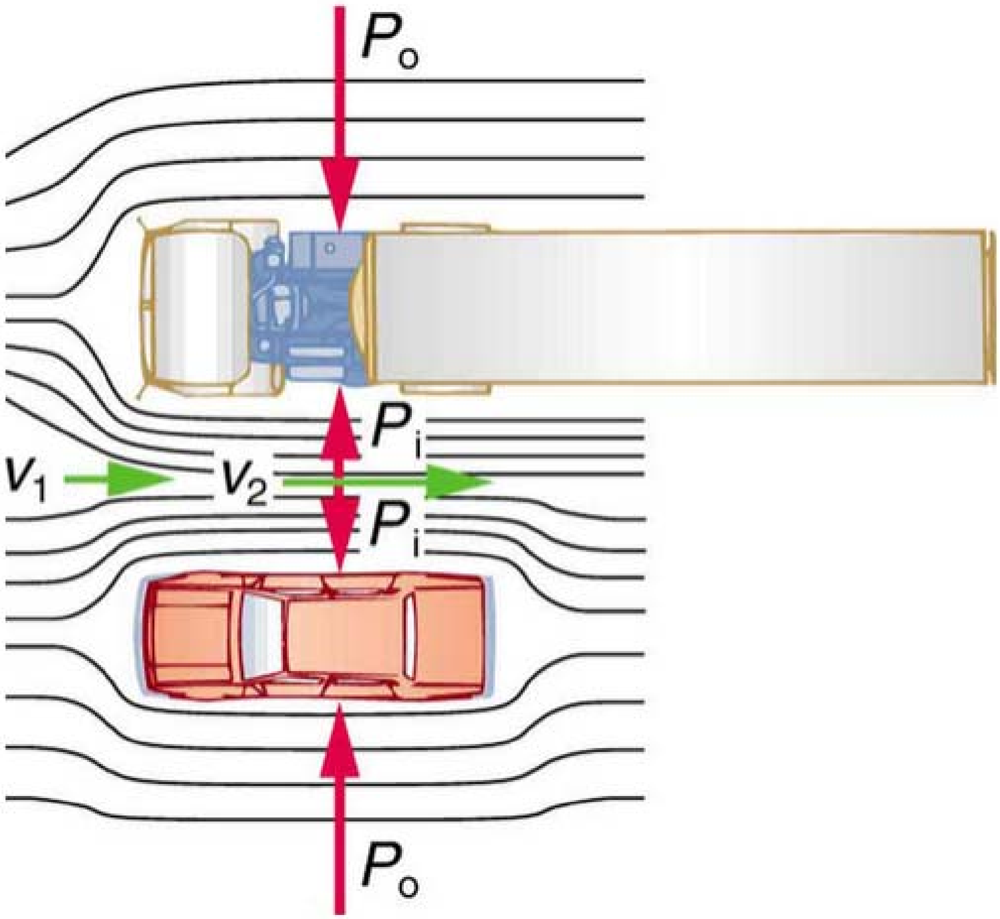
Bernoulli's equation is quoted, which states that larger velocities imply lower pressure and thus a net upwards pressure force is generated. Bernoulli's equation is often demonstrated by blowing over a piece of paper held between both hands as demonstrated in the above Fig. As air is blown along the upper surface of the sheet of paper it rises and, it is said, this is because the average velocity on the upper surface is greater (caused by blowing) than on the lower surface (where the air is more or less at rest). According to Bernoulli's equation, this should mean that the pressure must be lower above the paper, causing lift (H. Babinsky, How do wings work, w.w.w.iop.org.journals/physed)

(d) Flow along the cross section of a sail



The velocity of wind is different between the front side and back side of the sail. Because of the Bernoulli's equation, the pressure difference occurs, leading the moving of yacht.

(e)



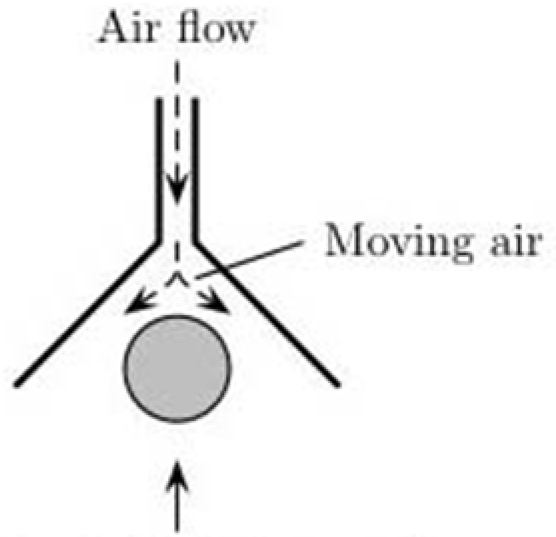
You may notice that when passing a truck on the highway, your car tends to veer toward it. The high velocity of the air between the car and the truck creates a region of lower pressure, and the vehicles are pushed together by greater pressure on the outside. (above Fig.).

(e) Shower curtain

Shower curtains have a disagreeable habit of bulging into the shower stall when the shower is on. The high-velocity stream of water and air creates a region of lower pressure inside the shower, and standard atmospheric pressure on the other side. The pressure difference results in a net force inward pushing the curtain in.

http://cnx.org/contents/0c8ac66c-a41b-4861-8fc6-e9633182091f@6/Bernoulli%E2%80%99s_Equation

(f) Funnel ball



The velocity of moving air from the upper part is faster than that around the lower part.

9 Application of the Bernoulli's equation

9.1 Example-1

(a) We consider the case when the area of the water tank (A) is relatively large. So that the level of water remains unchanged with time t.

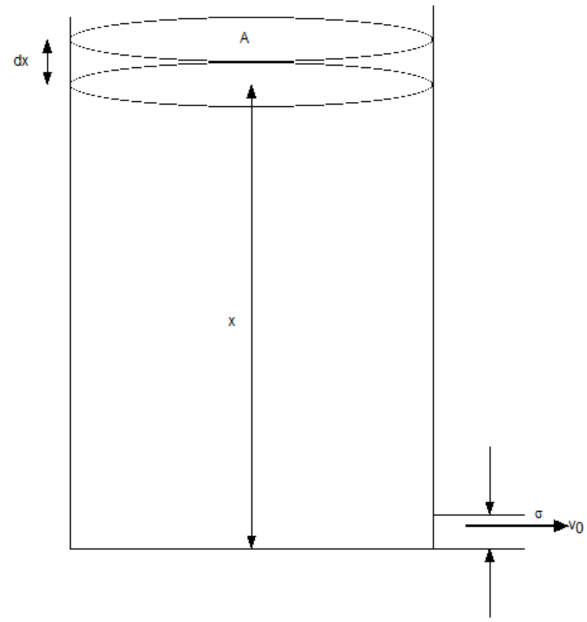
Bernoulli's equation

$$P_0 + \rho g H = P_0 + \frac{1}{2}\rho {v_0}^2$$

$$v_0 = \sqrt{2gH}$$

where H is the level of water. v_0 is the velocity of water at the hole of the bottom of the tank.

(b) Next we consider the case when the area of the water tank (A) is relatively small. So the height of water decreases with increasing time.



Bernoulli's equation

$$P_0 + \rho g x = P_0 + \frac{1}{2}\rho {v_0}^2$$
$$v_0 = \sqrt{2gx}$$

where x is the level of water, as a function of t. v_0 is the velocity of water at the hole of the bottom of the tank.

Equation of continuity

$$A(-\frac{dx}{dt}) = \sigma v_0$$

where σ is the area of the hole at the bottom. Then we have

$$\frac{dx}{dt} = -\frac{\sigma}{A}\sqrt{2gx} = -\frac{\sigma}{A}\sqrt{2g}\sqrt{x}$$

or

$$\int \frac{1}{\sqrt{x}}dx = -\int \frac{\sigma}{A}\sqrt{2g}dt$$

or

$$x = (\sqrt{x_0} - \frac{\sigma}{2A}\sqrt{2g}t)^2$$

or

$$\frac{x}{x_0} = (1 - \frac{\sigma}{2A}\sqrt{\frac{2g}{x_0}}\sqrt{t}) = (1 - \alpha\sqrt{t})^2$$

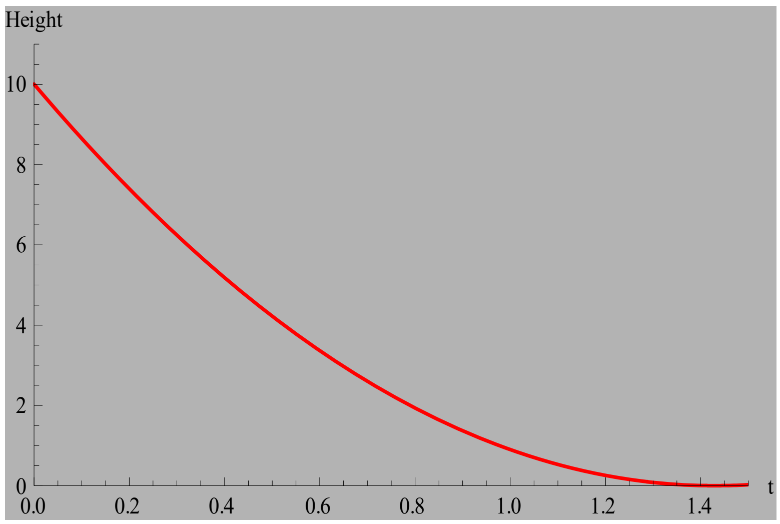
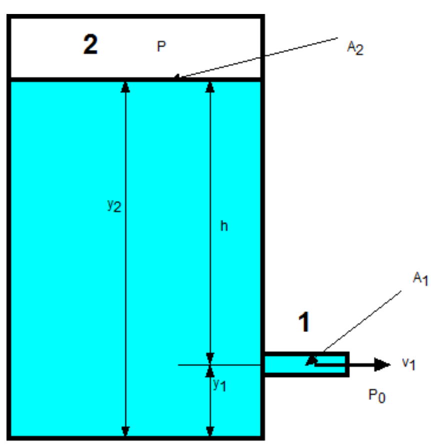


Fig. The time dependence of the level of the water tank. x_0 =1 and $\alpha^2 = 1/\sqrt{2}$

9.2 Example-2

An enclosed tank containing a liquid of density ρ has a hole in its side at a distance y_1 from the tank's bottom. The hole is open to the atmosphere, and its diameter is much smaller than the diameter of the tank. The air above the liquid is maintained at a pressure P.



If we assume that the tank is large in cross section compared to the hole (A_2>>A_1), then the fluid will be approximately at rest at the top, point 2.

Applying Bernoulli's equation to points 1 and 2, and noting that at the hole

$$P_1 = P_0$$

we get

$$P_0 + \frac{1}{2}\rho v_1^{\,2} + \rho g y_1 = P + \rho g y_2$$

But $y_2 - y_1 = h$ and so this reduces to

$$v_1 = \sqrt{\frac{2(P - P_0)}{\rho} + 2gh}$$

This equation is rewritten as

$$v_1 = \sqrt{\frac{2P_0(P/P_0 - 1)}{\rho} + 2gh}$$

Here we use P_0 = 1.013 x 10^5 Pa, ρ = 1.0 x 10^3 kg/m^3, g = 9.8 m/s^2.

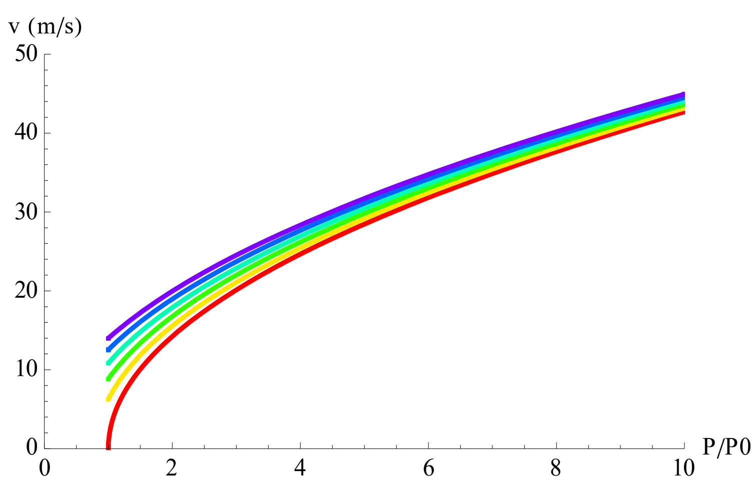
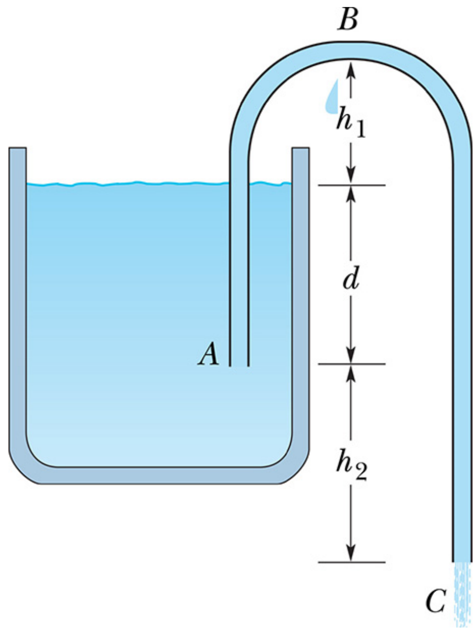


Fig. velocity v vs reduced pressure P/P_0. The height h is changed as a parameter. Red (h = 0).

9.3 Example-3

Problem 14-83 (SP-14) (10-th edition)

Figure shows a siphon, which is a device for removing liquid from a container. Tube ABC must initially be filled, but once this has been done, liquid will flow through the tube until the liquid surface in the container is level with the tube opening at A. The liquid has density 1000 kg/m^3 and negligible viscosity. The distances shown are h_1 = 25 cm, d = 12 cm, and h_2 = 40 cm. (a) With what speed does the liquid emerge from the tube at C? (b) If the atmospheric pressure is 1.0 x 10^5 Pa, what is the pressure in the liquid at the topmost point B? (c) Theoretically, what is the greatest possible height h_1 that a siphon can lift water?



Bernoulli's equation

$$P_0 + \rho g(d + h_2) = P_0 + \rho \cdot g \cdot 0 + \frac{1}{2}\rho v_C^{\,2} = P_B + \rho g(h_1 + h_2 + d) + \frac{1}{2}\rho v_B^{\,2}$$

or

$$P_B = P_0 - \rho g h_1 - \frac{1}{2}\rho v_B^{\,2}$$

Equation of continuity

$v_C = v_B$

When $v_C = 0$ (critical condition), the pressure P_B should be positive.

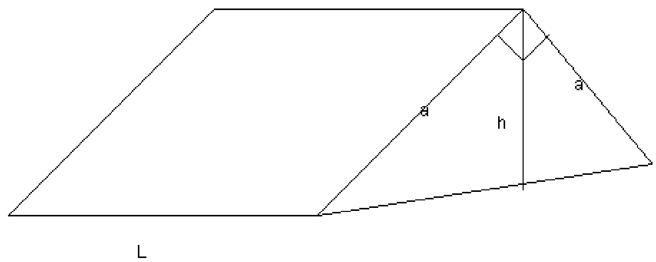
$$P_B = P_0 - \rho g h_1 - \frac{1}{2}\rho v_C^{\,2} = P_0 - \rho g h_1 \geq 0$$

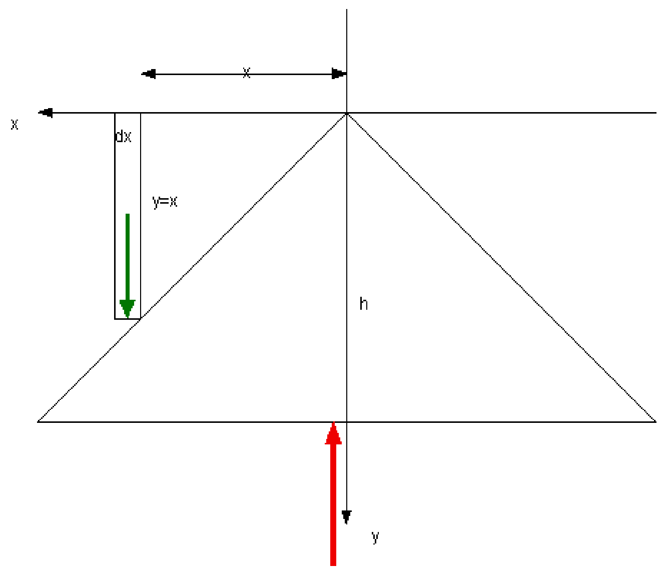
or

$$h_1 \le \frac{P_0}{\rho g} = \frac{1.013\times 10^5}{1.0\times 10^3 \times 9.8} = 10.34 \text{ m}.$$

10 Appendix: The principle of Archimedes is valid for volumes with any shape

We consider a special case for the buoyant force. We show that the Archimedes's principle is also valid for these special cases. $h = a/\sqrt{2}$.





The force at the bottom:

$$F_b = (P_0 + \rho g h)(\sqrt{2}aL) = (P_0 + \rho g \frac{\sqrt{2}a}{2})(\sqrt{2}aL) = (P_0\sqrt{2}aL + \rho g a^2 L)$$

The force from the top part:

$$F_t = 2\int_0^{\sqrt{2}a/2} (P_0 + \rho g x) L dx = P_0\sqrt{2}aL + \frac{1}{2}\rho g a^2 L$$

The buoyant force is

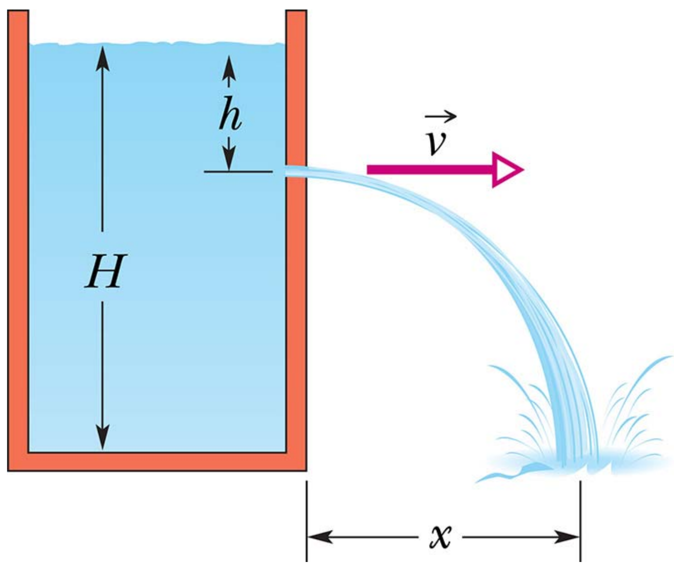
$$F = F_b - F_t$$
$$= \frac{1}{2}\rho g L a^2 = \rho g V$$

11. HW and SP

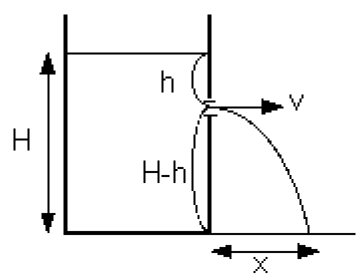
11.1

Problem 14-71 (HW-14) (10-th edition)**

Figure shows a stream of water flowing through a hole at depth h = 10 cm in a tank holding water to height H = 40 cm. (a) At what distance x does the stream strike the floor? (b) At what depth should a second hole be made to give the same value of x? (c) At what depth should a hole be made to maximize x?



((**Solution**))



h = 10 cm, H=40 cm

$$x = vt$$
$$y = (H-h) - \frac{1}{2}gt^2$$

When $y = 0$,

$$t = \sqrt{\frac{2(H-h)}{g}}$$

$$x = v\sqrt{\frac{2(H-h)}{g}}$$

Bernoulli's equation

$$P_0 + \rho g H = P_0 + \rho g(H-h) + \frac{1}{2}\rho v^2$$

or

$$v = \sqrt{2gh}$$

$$x = 2\sqrt{h(H-h)} = 0.35m$$

$$h^2 - Hh + \frac{x^2}{4} = 0$$

We assume that α and β are two roots of the quadratic equation.

$$\alpha + \beta = H$$

$$\alpha\beta = \frac{x^2}{4}$$

11.2

Problem 14-41 (SP-14) (10-th edition)**

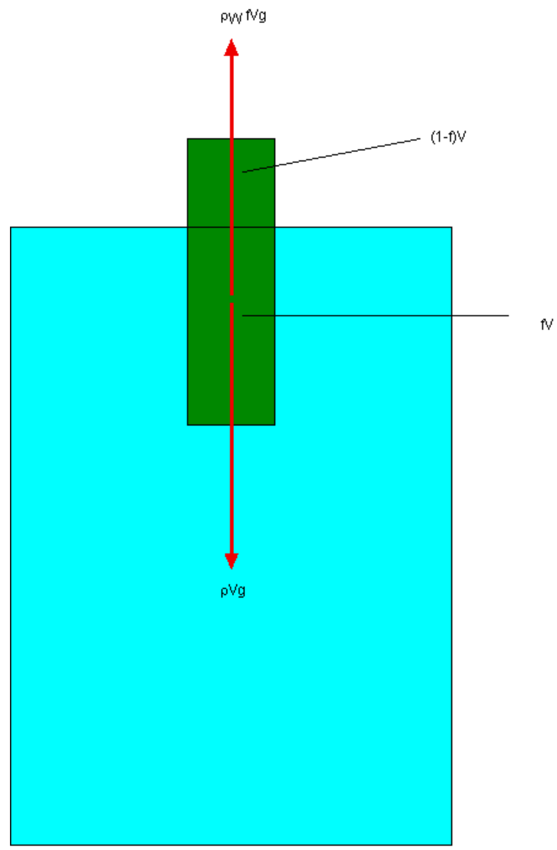
What fraction of the volume of an iceberg (density 917 kg/m^3) would be visible of the iceberg floats (a) in the ocean (salt water, density 1024 kg/m^3) and (b) in a river (fresh water, density 1000 kg/m^3)? (When salt water freezes to form ice, the salt is excluded. So, an iceberg could provide fresh water to a community.)

((**Solution**))

$\rho_{\text{salt water}}$= 1024 kg/m^3

ρ_{iceberg}= 917 kg

$\rho_{\text{pure water}}$ = 1000 kg



$$\rho_W fVg = \rho_{iceberg}Vg$$

$$f = \frac{\rho_{iceberg}}{\rho_W}$$

(a) Salt water

$$f = \frac{\rho_{iceberg}}{\rho_{salt-water}} = \frac{917}{1024} = 0.896 \qquad 1\text{-}f = 0.104$$

(b) Pure water

$$f = \frac{\rho_{iceberg}}{\rho_{Pure-water}} = \frac{917}{1000} = 0.917 \qquad 1\text{-}f = 0.083$$

12. Applcation

12.1

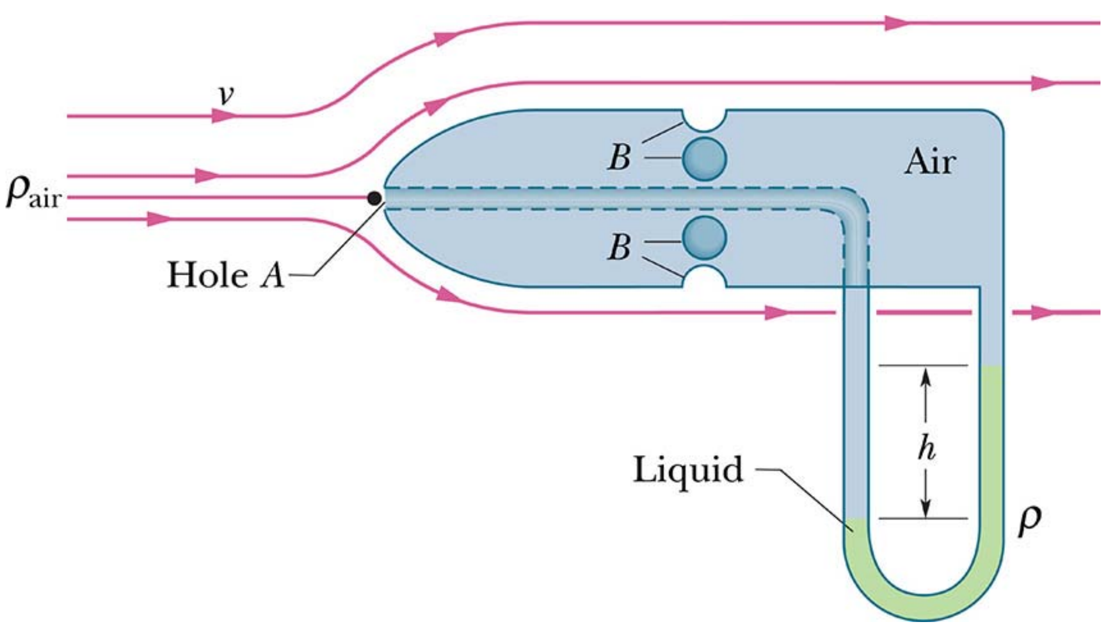
Problem 14-62 (HW-14) (10-th edition)**

A pitot tube (Fig.) is used to determine the airspeed of an airplane. It consists of an outer tube with a number of small holes B (four are shown) that allow air into the tube;

that tube is connected to one arm of a U-tube. The other arm of the U-tube is connected to hole A at the front end of the device, which points in the direction the plane is headed. At A the air becomes stagnant so that $v_A = 0$. At B, however, the speed of the air presumably equals the airspeed v of the plane. (a) Use Bernoulli's equation to show that

$$v = \sqrt{\frac{2\rho g h}{\rho_{air}}}$$

where ρ is the density of the liquid in the U-tube and h is the difference in the liquid levels in that tube. (b) Suppose that the tube contains alcohol and the level difference h is 26.0 cm. What is the plane's speed relative to the air? The density of the air is 1.03 kg/m^3 and that of alcohol is 810 kg/m^3.



((**Solution**))

ρ_{air} = 1.03 kg/m^3, ρ = 810 kg/m^3, h = 0.26 m.

$$P_A + \frac{1}{2}\rho_{air} \cdot 0^2 = P_B + \frac{1}{2}\rho_{air} v^2$$

$$P_A = P_B + \rho g h$$

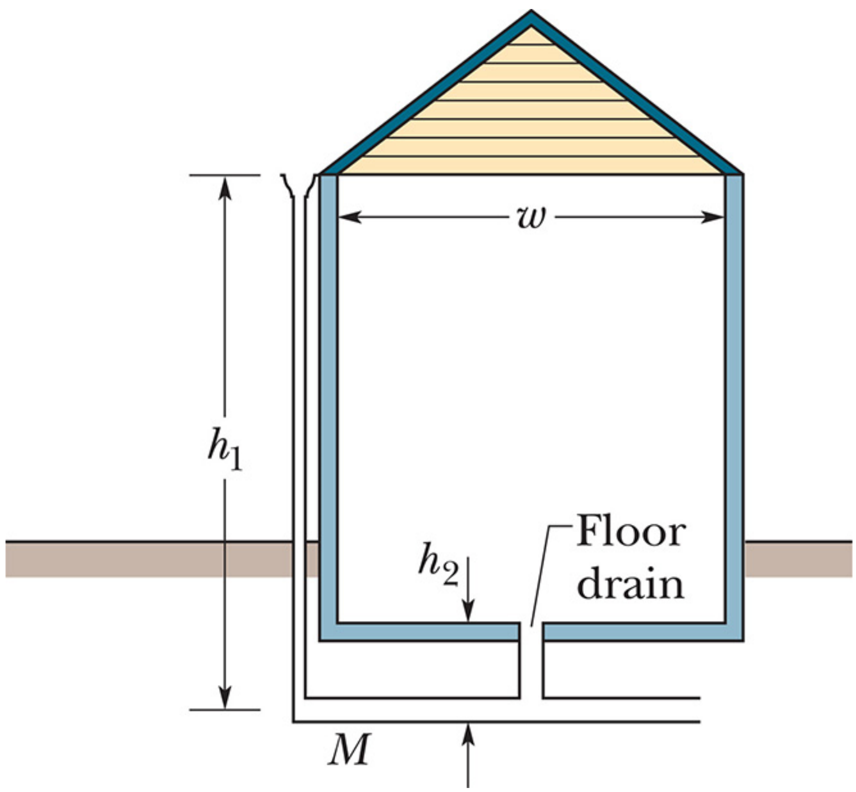
12.2

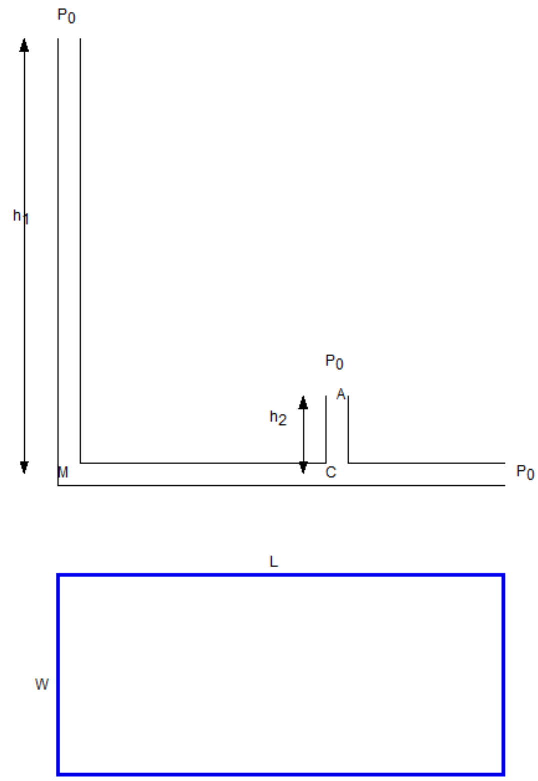
Problem 14-72 (SP-14)*** **(10-th edition)**

A very simplified schematic of the rain drainage system for a home is shown in Fig. Rain falling on the slanted roof runs off into gutters around the roof edge; it then drains through downspouts (only one is shown) into a main drainage pipe M below the basement, which carries the water to an even larger pipe below the street. In Fig., a floor drain in the basement is also connected to drainage pipe M. Suppose the following apply:

1. the downspouts have height $h_1 = 11$ m.,
2. the floor drain has height $h_2 = 1.2$ m
3. pipe M has radius 3.0 cm,
4. the house has side width $w = 30$ m and front length $L = 60$ m.
5. all the water striking the roof goes through pipe M,
6. the initial speed of the water in a downspout is negligible,
7. the wind speed is negligible (the rain falls vertically).

At what rainfall rate, in centimeters per hour, will water from pipe M reach the height of the floor drain and threaten to flood the basement?





$w = 30\text{m}$, $L = 60$ m, $h_1 = 11$ m, $h_2 = 1.2$ m,
$A = \pi r^2$, $r = 0.03\text{m}$

Bernoulli's equation

$$\rho g h_1 + P_0 = P_C + \frac{1}{2} v_C{}^2$$
$$P_C = P_0 + \rho g h_2$$

or

$$v_C = \sqrt{2\rho g (h_1 - h_2)} = 13.86 m/s$$

The equation of continuity

Suppose that V is the rainfall rate

$$A v_C = wLV$$

$$V = \frac{A}{wL} v_C = \frac{A}{wL} \sqrt{2\rho g (h_1 - h_2)} = 2.18 \times 10^{-5} m/s$$

13. Link

Blaise Pascal
http://en.wikipedia.org/wiki/Blaise_Pascal

Archimedes
http://en.wikipedia.org/wiki/Archimedes

Bernoulli's equation
http://en.wikipedia.org/wiki/Bernoilli_equation

Daniel Bernoulli
http://en.wikipedia.org/wiki/Daniel_Bernoulli

Bernoulli ball
http://www.youtube.com/watch?v=fgHvC55AKig

WileyPlus
http://edugen.wiley.com/edugen/shared/alerts/timeout.uni