Chapter 21<br>Electric charge<br>Masatsugu Sei Suzuki<br>Department of Physics<br>(Date: August 15, 2020)

1. Some experimental results
(a) The electric charge can be negative, zero, or positive.

Empirically it was known since ancient times that if amber is rubbed on fur, it acquires the property of attracting light objects such as feathers. This phenomenon was attributed to a new property of matter called "electric charge". (electron is the Greek name for amber) More experiments show that they are two distinct type of electric charge: positive (color code: red), and negative (color code: black). The names "positive" and "negative" were given by Benjamin Franklin.
((Note)) Amber: Wikipedia $\eta \lambda \varepsilon \kappa \tau \rho o v$ (Electron)
The Greek name for amber was $\eta \lambda \varepsilon \kappa \tau \rho o v$ (Electron) and was connected to the Sun God, one of whose titles was Elector or the Awakener. It is discussed by Theophrastus, possibly the first ever mention of the material, and in the 4th century BC. The modern term electron was coined in 1891 by the Irish physicist George Stoney, using the Greek word for amber (and which was then translated as electrum) because of its electrostatic properties and whilst analyzing elementary charge for the first time. The ending -on, common for all subatomic particles, was used in analogy to the word ion.

## ((Note)) Amber

Amber is fossil tree resin, which is appreciated for its color and beauty. Good quality amber is used for the manufacture of ornamental objects and jewelry. Although not mineralized, it is often classified as a gemstone. A common misconception is that amber is made of tree sap; it is not. Sap is the fluid that circulates through a plant's vascular system, while resin is the semi-solid amorphous organic substance secreted in pockets and canals through epithelial cells of the plant. Because it used to be soft and sticky tree resin, amber can sometimes contain insects and even small vertebrates. Semi-fossilized resin or subfossil amber is known as copal.

(a) Uncharged amber rod exerts no force on papers
(b) Amber rod is rubbed against a dry cloth (a fur)
(c) Amber rod becomes charged and attracts the papers.
(1) The electric charge on a glass rod rubbed with silk is positive.
(2) The electric charge on an amber (plastic) rod rubbed with fur is negative.
((Note)) Rubber rubbed with cat fur: rubber becomes negative, while the fur becomes positive.

Amber rod (-)
Plastic rod (-)
Rubber
(-)
Glass rod
(+)


Fig. Plastic rod rubbed with fur
(b) Further experiments on charged objects showed that:

1. Charges of the same type (either both positive or both negative) repel each other.
2. Charges of opposite type on the other hand attract each other.
3. The force direction allows us to determine the sign of an unknown electric charge

(b)

## 2. Charge is quantized

An important experiment in which the charge of small oil droplets was determined was carried out by Millikan. Millikan discovered that the charge on the oil droplets was always a multiple of the charge of the electron ( $e$, the fundamental charge). For example, he observed droplets with a charge equal to $+/-e,+/-2 e,+/-3 e$, etc., but never droplets with a charge equal to $+/-1.45 \mathrm{e},+/-2.28 \mathrm{e}$, etc. The experiments strongly suggested that the electric charge, $q$, is said to be quantized. $q$ is the standard symbol used for charge as a variable. Electric charge exists as discrete packets

$$
q=n e
$$

where $n$ is an integer and $e$ is the fundamental unit of charge.

$$
e=1.602176487 \times 10^{-19} \mathrm{C}
$$

| For electron | $q=-e$ |
| :--- | :--- |
| For proton | $q=+e$ |
| For neutron | $q=0$ |

The SI Unit of charge is the coulomb. How many electrons are there to form 1 C ? The answer is

$$
\begin{aligned}
& \frac{1 C}{e}=\frac{1}{1.602176487 \times 10^{-19}}=6.24 \times 10^{18} \\
& 1 \mu \mathrm{C}=10^{-6} \mathrm{C} \\
& 1 \mathrm{nC}=10^{-9} \mathrm{C} \\
& 1 \mathrm{pC}=10^{-12} \mathrm{C} \\
& 1 \mathrm{fC}=10^{-15} \mathrm{C} \\
& 1 \mathrm{aC}=10^{-18} \mathrm{C} \\
& \text { (n: micro) } \\
& \text { (p: pico) } \\
& \text { (f: femto) } \\
& \text { (a: atto) }
\end{aligned}
$$

## ((Note))

Relation between 1 C (SI units) and 1 esu (cgs gaussian unit of charge, electrostatic unit)
We consider a force between two charges with $q=1 \mathrm{C}$. The separation between two charges is $r=1 \mathrm{~m}$.

$$
F_{S I}=\frac{q^{2}}{4 \pi \varepsilon_{0} r^{2}}=\frac{(1 C)^{2}}{4 \pi \varepsilon_{0}(1 m)^{2}} \quad[\mathrm{~N}] .
$$

In cgs units, the corresponding force between A (esu) $[=1 \mathrm{C}]$ is

$$
F_{c g s}=\frac{q^{2}}{r^{2}}=\frac{(A \text { esu })^{2}}{(100 \mathrm{~cm})^{2}} \quad[\text { dyne }]
$$

Note that $F_{S I}=F_{c g s}$ and $1 \mathrm{~N}=10^{5}$ dyne. Then we have

$$
\frac{1}{4 \pi \varepsilon_{0}} \times 10^{5}=\frac{A^{2}}{10^{4}}, \quad \text { or } \quad A=\sqrt{\frac{1}{4 \pi \varepsilon_{0}} \times 10^{9}}=2.99792 \times 10^{9}
$$

So we have

$$
1 \mathrm{C}=2.99792 \times 10^{9} \mathrm{esu}
$$

The charge of electron is

$$
q_{\mathrm{e}}=1.60217664 \times 10^{-19} \mathrm{C}=4.80320425 \times 10^{-10} \mathrm{esu} .
$$

## 3. Charge is conserved

(a)


Consider a glass rod and a piece of silk cloth (both uncharged) shown in the upper figure. If we rub the glass rod with the silk cloth we know that positive charge appears on the rod (see the figure). At the same time an equal amount of negative charge appears on the silk cloth, so that the net rod-cloth charge is actually zero. This suggests that rubbing does not create charge but only transfers it from one body to the other, thus upsetting the electrical neutrality of each body. Charge conservation can be summarized as follows: In any process the charge at the beginning equals the charge at the end of the process.

The total electric charge in an isolated system, that is, the algebraic sum of the positive and negative charge present at any time, never change.

## ((Example-2))

We consider two identical sphere conductors which are actually well separated from one another. (Hint of HW-12)). The sphere A (with an initial charge of $Q_{1}$ ) is touched to sphere B (with an initial charge of $Q_{2}$ ) and then they are separated.


## (b) Some concepts

Due to the movement of electrons, charge is transferred from one object to another.
Positive ion: the atom that loses an electron is said to be a positive ion;
Negative ion: the atom that receives an extra electron is said to be a negative ion.

| H | (1s) |
| :---: | :---: |
| He | $(1 \mathrm{~s})^{2}$ |
| Li | (1s) ${ }^{2}(2 \mathrm{~s})^{1}$ |
| Ba | (1s) ${ }^{2}(2 \mathrm{~s})^{2}$ |
| B | $(1 \mathrm{~s})^{2} \mid(2 \mathrm{~s})^{2}(2 \mathrm{p})^{1}$ |
| C | $(1 \mathrm{~s})^{2} \mid(2 \mathrm{~s})^{2}(2 \mathrm{p})^{2}$ |
| N | $(1 \mathrm{~s})^{2} \mid(2 \mathrm{~s})^{2}(2 \mathrm{p})^{3}$ |
| O | $(1 \mathrm{~s})^{2} \mid(2 s)^{2}(2 p)^{4}$ |
| F | $(1 \mathrm{~s})^{2} \mid(2 \mathrm{~s})^{2}(2 \mathrm{p})^{5}$ |
| Ne | (1s) ${ }^{2}(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6} \mid$ |
| Na | $(1 \mathrm{~s})^{2}\left\|(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}\right\|(3 \mathrm{~s})^{1}$ |
| Mg | (1s) ${ }^{2}\left\|(2 s)^{2}(2 p)^{6}\right\|(3 s)^{2}$ |
| Al | $(1 \mathrm{~s})^{2}\left\|(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}\right\|(3 \mathrm{~s})^{2}(3 \mathrm{p})^{1}$ |
| Si | $(1 s)^{2}\left\|(2 s)^{2}(2 p)^{6}\right\|(3 s)^{2}(3 p)^{2}$ |
| P | $(1 s)^{2}\left\|(2 s)^{2}(2 p)^{6}\right\|(3 s)^{2}(3 p)^{3}$ |
| S | $(1 s)^{2}\left\|(2 s)^{2}(2 p)^{6}\right\|(3 s)^{2}(3 p)^{4}$ |
| Cl | $(1 s)^{2}\left\|(2 s)^{2}(2 p)^{6}\right\|(3 s)^{2}(3 p)^{5}$ |
| Ar | $(1 s)^{2}\left\|(2 s)^{2}(2 p)^{6}\right\|(3 s)^{2}(3 p)^{6}$ |
| K | (1s) $)^{2}\left\|(2 s)^{2}(2 \mathrm{p})^{6}\right\|(3 \mathrm{~s})^{2}(3 \mathrm{p})^{6}(3 \mathrm{~d})^{1}$ |
| Ca | $(1 \mathrm{~s})^{2}\left\|(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}\right\|(3 \mathrm{~s})^{2}(3 \mathrm{p})^{6}(3 \mathrm{~d})^{2}$ |

$\mathrm{Na}^{+}$(sodium ion)

| Na | $(1 \mathrm{~s})^{2}\left\|(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}\right\|(3 \mathrm{~s})^{1}$ | $(11$ electrons) |
| :--- | :--- | :--- |
| $\mathrm{Na}^{+}$ | $(1 \mathrm{~s})^{2}\left\|(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}\right\|$ | $(10$ electrons $)$ |

$\mathrm{Cl}^{-}$(chloride ion)
Cl
$(1 \mathrm{~s})^{2}\left|(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}\right|(3 \mathrm{~s})^{2}(3 \mathrm{p})^{5}$
$(1 \mathrm{~s})^{2}\left|(2 \mathrm{~s})^{2}(2 \mathrm{p})^{6}\right|(3 \mathrm{~s})^{2}(3 \mathrm{p})^{6}$
(17 electrons)
(18 electrons)

Na



Chlorine atom

Cl


## 4. Coulomb's law

Charles-Augustin de Coulomb (June 14, 1736, Angoulême, France - August 23, 1806, Paris, France)


He was a French physicist. He is best known for developing Coulomb's law: the definition of the electrostatic force of attraction and repulsion. The SI unit of charge, the coulomb, was named after him.

The interaction between electric charges at rest is described by Coulomb's law. Two stationary electric charges repel or attract one another with a force proportional to the product of the magnitude of the charges and inversely proportional to the square if the distance between them.

We can state this compactly in vector form

$$
\boldsymbol{F}_{12}=k_{e} \frac{q_{1} q_{2}}{r^{2}} \boldsymbol{e}_{12}
$$



Here $q_{1}$ and $q_{2}$ are numbers (scalars) giving the magnitude and sign of the respective charges, $\boldsymbol{e}_{12}$ is the unit vector in the direction from charge 1 to charge 2, and $\boldsymbol{F}_{12}$ is the force acting on charge 2 . Note that

$$
\boldsymbol{F}_{21}=-\boldsymbol{F}_{12} .
$$

The constant of proportionality $\left(k_{e}\right)$ is written as

$$
k_{e}=\frac{1}{4 \pi \varepsilon_{0}}=c^{2} \times 10^{-7}=8.98755 \times 10^{9} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}^{2}(\text { or } \mathrm{V} \mathrm{~m} / \mathrm{C})
$$

where $c$ is the speed of light,

$$
c=2.99792458 \times 10^{8} \mathrm{~m} / \mathrm{s}
$$

Note that $\varepsilon_{0}$ is the permittivity of free space and $\mu_{0}$ is the permeability of free space,

$$
\begin{aligned}
\varepsilon_{0} & =8.8541878176 \times 10^{-12} \frac{C^{2}}{N m^{2}} \\
\mu_{0} & =4 \pi \times 10^{-7}\left(\mathrm{~N} / \mathrm{A}^{2}\right)
\end{aligned}
$$

The coulomb is an extremely large unit. The force between two charges of 1 C each a distance of 1 m apart is

$$
F=\frac{1}{4 \pi \varepsilon_{0}} \frac{1 C \times 1 C}{1 m^{2}}=8.98755 \times 10^{9} \mathrm{~N}
$$

((Note)) It is easy for you to memorize the value of $k_{\mathrm{e}}$.

$$
k_{e}=9 \times 10^{9} \mathrm{~N} \mathrm{~m}^{2} / \mathrm{C}^{2}(\text { or } \mathrm{V} \mathrm{~m} / \mathrm{C})
$$

The quantity $\varepsilon_{0}$ is called the permittivity constant.

$$
\varepsilon_{0}=8.854187817 \times 10^{-12} \mathrm{C}^{2} /\left(\mathrm{N} \mathrm{~m}^{2}\right) .
$$

((Note))

$$
\begin{aligned}
& \frac{N m^{2}}{C^{2}}=\frac{N m}{C} \frac{m}{C}=\frac{J}{C} \frac{m}{C}=\frac{V A s}{A s} \frac{m}{C}=\frac{V m}{C} \\
& \mathrm{Nm}=\mathrm{J}, \quad C=A s \\
& W=V A \quad J=W s=V A s
\end{aligned}
$$

where
J (Joule), A (Ampere), V (Volt), C (Coulomb), s (second), N (Neuton), and W (Watt).

## ((Note))

The SI unit of charge is coulomb. The coulomb unit is derived from the SI unit A (Ampere) for the electric current $i$. The current $i$ is the rate $\mathrm{d} q / \mathrm{d} t$ at which the amount of charge ( $\mathrm{d} q$ ) moves past a point or through a region in time $\mathrm{d} t$ (second).

$$
i=\frac{d q}{d t} .
$$

This relation implies that.

$$
1 \mathrm{C}=(1 \mathrm{~A})(1 \mathrm{~s})
$$

## 5. Bohr model

We now consider the Bohr model shown in this figure. The system consists of a proton and an electron. These two particles are coupled with an attractive Coulomb interaction.


The electrical force between the electron (charge $q_{1}=-e$ ) and proton (charge $q_{2}=e$ ) is found from Coulomb's law,

$$
F_{e}=\frac{k_{e} q_{1} q_{2}}{r_{B}{ }^{2}}=8.19 \times 10^{-8} \mathrm{~N}
$$

where $e=1.602176487 \times 10^{-19} \mathrm{C}$ and $r_{\mathrm{B}}$ is the Bohr radius given by

$$
r_{\mathrm{B}}=5.2917720859 \times 10^{-11}(\mathrm{~m})=0.52917720859 \AA .
$$

This can be compared with the gravitational force between the electron and proton

$$
F_{g}=\frac{G m_{e} m_{p}}{r_{B}{ }^{2}}=3.63153 \times 10^{-47} \mathrm{~N}
$$

What is the angular frequency $\omega$ for electrons rotating the circular orbit?

$$
\begin{aligned}
& F_{e}=\frac{1}{4 \pi \varepsilon_{0}} \frac{e^{2}}{r_{B}^{2}}=m \frac{v^{2}}{r_{B}}=m r_{B} \omega^{2} \\
& \omega=\sqrt{\frac{1}{4 \pi \varepsilon_{0}} \frac{e^{2}}{m r_{B}^{3}}}=4.13414 \times 10^{16} \mathrm{rad} / \mathrm{s}
\end{aligned}
$$

where $m$ is the mass of electron, $m=9.1093821545 \times 10^{-31} \mathrm{~kg}$.
The period is

$$
T=\frac{2 \pi}{\omega}=1.51983 \times 10^{-16} \mathrm{~S}
$$

((Note))
An important difference between the electric force and the gravitational force is that the gravitational force is always attractive, while the electric force can be repulsive, or attractive, depending on the charges of the particles.
((Mathematica))

## Clear["Global`*"];

rule1 $=\left\{g \rightarrow 9.80665, G \rightarrow 6.674286710^{-11}\right.$,
me $\rightarrow 9.1093821545 ■ 10^{-31}$, qe $\rightarrow 1.602176487 ■ 10^{-19}$, $r B \rightarrow 0.52917720859 ■ 10^{-10}, \quad c \rightarrow 2.99792458 ■ 10^{8}$, $\mu 0 \rightarrow 12.566370614 ■ 10^{-7}, \quad \epsilon 0 \rightarrow 8.854187817 ■ 10^{-12}$, $\left.\mathrm{mp} \rightarrow 1.672621637 ■ 10^{-27}\right\}$;
$k=\frac{1}{4 \pi \in 0} /$. rule1
$8.98755 \times 10^{9}$
$\mathrm{Fe}=\frac{1}{4 \pi \epsilon 0} \frac{\mathrm{qe}^{2}}{\mathrm{rB}^{2}} / /$. rule1
$8.23872 \times 10^{-8}$
$\mathrm{Fg}=\mathrm{G} \frac{\mathrm{mpme}}{\mathrm{rB}^{2}} / /$ rule1
$3.63153 \times 10^{-47}$

Fe / Fg // Simplify
$2.26867 \times 10^{39}$
$\omega 1=\sqrt{\frac{\mathrm{qe}^{2}}{4 \pi \epsilon 0 \mathrm{merB}^{3}}} /$. rule1
$4.13414 \times 10^{16}$
$\mathrm{T} 1=\frac{2 \pi}{\omega 1}$
$1.51983 \times 10^{-16}$

## 6. Conductors and insulators

## (a) Conductors

A conductor is a material that permits the motion of electric charge through its volume. Examples of conductors are copper, aluminum and iron. An electric charge placed on the end of a conductor will spread out over the entire conductor until an equilibrium distribution is established.

## (b) Insulators

In contrast, electric charge placed on an insulator stays in place: an insulator (like glass, rubber and mylar) does not permit the motion of electric charge.

## (c) Superconductors

Superconductors are materials that are perfect conductors, allowing charge to move without any hindrance. In these chapters we discuss only conductors and insulators.

## 7. Principle superposition

When there are more than two charges present - the only really interesting times-we must supplement the Coulomb's law with one other fact of nature: the force on any charge is the vector sum of the Coulomb forces from each of the other charges. This fact is called "the principle of superposition." That is all there is to electrostatics. If we combine the Coulomb's law and the principle of superposition, there is nothing else.

Suppose we have some arrangement of charges $q_{1}, q_{2}, q_{3}, \ldots, q_{\mathrm{N}}$, fixed in space. From the principle of superposition, the resultant force on the charge $q_{0}$ is expressed by

$$
\boldsymbol{F}_{0}=\sum_{j=1}^{N} \boldsymbol{F}_{j 0}=\sum_{j=1}^{N} \frac{1}{4 \pi \varepsilon_{0}} \frac{q_{0} q_{j}}{r_{j 0}{ }^{2}} \boldsymbol{e}_{j 0}
$$

((Example))


The resultant force $\boldsymbol{F}_{0}$ on the charge $q_{0}$ is given by

$$
\boldsymbol{F}_{0}=\boldsymbol{F}_{10}+\boldsymbol{F}_{20}+\boldsymbol{F}_{30}+\boldsymbol{F}_{40}
$$

## 8. Typical example

8.1 Problem 21-8 (SP-21)

In Fig., four particles form a square. The charges are $q_{1}=q_{4}=Q$ and $q_{2}=q_{3}=q$. (a) What is $Q / q$ if the net electrostatic force on particles 1 and 4 is zero? (b) Is there any value of $q$ that makes the net electrostatic force on each of the four particles zero? Explain.

((Solution))
$q_{1}=q_{4}=Q$
$q_{2}=q_{3}=q$


$$
\begin{aligned}
& F_{3}=F_{2}=\frac{Q q}{4 \pi \varepsilon_{0} a^{2}} \\
& F_{4}=\frac{Q^{2}}{4 \pi \varepsilon_{0}(\sqrt{2} a)^{2}}=\frac{Q^{2}}{8 \pi \varepsilon_{0} a^{2}}
\end{aligned}
$$

We find that $\boldsymbol{F}_{3}+\boldsymbol{F}_{2}$ has only the diagonal component from the symmetry.

$$
\begin{aligned}
& \left(\boldsymbol{F}_{3}+\boldsymbol{F}_{2}\right)_{\text {diagonal }}=\frac{Q q}{4 \pi \varepsilon_{0} a^{2}}\left(2 \cos 45^{\circ}\right)=\frac{\sqrt{2} Q q}{4 \pi \varepsilon_{0} a^{2}} \\
& \left(\boldsymbol{F}_{4}\right)_{\text {diagonal }}=\frac{Q^{2}}{8 \pi \varepsilon_{0} a^{2}}
\end{aligned}
$$

If the net electrostatic force on particle is zero, we have

$$
\frac{\sqrt{2} Q q}{4 \pi \varepsilon_{0} a^{2}}+\frac{Q^{2}}{8 \pi \varepsilon_{0} a^{2}}=0
$$

or

$$
\begin{aligned}
& 2 \sqrt{2} Q q+Q^{2}=0 \\
& Q(2 \sqrt{2} q+Q)=0
\end{aligned}
$$

(a)

(b) The net electrostatic force on the charge $q_{2}$ is

$$
\begin{aligned}
& \left(\boldsymbol{F}_{1}+\boldsymbol{F}_{4}\right)_{\text {diagonal }}=\frac{Q q}{4 \pi \varepsilon_{0} a^{2}}\left(2 \cos 45^{\circ}\right)=\frac{\sqrt{2} Q q}{4 \pi \varepsilon_{0} a^{2}} \\
& \left(\boldsymbol{F}_{3}\right)_{\text {diagonal }}=\frac{q^{2}}{4 \pi \varepsilon_{0} a^{2}}
\end{aligned}
$$

The net electrostatic force on the change $q_{2}$ is equal to zero,

$$
\begin{aligned}
& \frac{\sqrt{2} Q q}{4 \pi \varepsilon_{0} a^{2}}+\frac{q^{2}}{4 \pi \varepsilon_{0} a^{2}}=0 \\
& q(\sqrt{2} Q+q)=0
\end{aligned}
$$

or


This ratio $Q / q$ is inconsistent with that obtained previously. So it is impossible to have such a given situation.

### 8.2 Problem 21-35 (S-21)

In crystals of the salt cesium chloride $(\mathrm{CsCl})$, cesium ions $\mathrm{Cs}^{+}$form the eight corners of a cube and a chlorine ion $\mathrm{Cl}^{-}$is at the cubes (Fig.). The edge length of the cube is 0.40 nm . The $\mathrm{Cs}^{+}$ions are each deficient one electron (and thus each has a charge of $-e$ ), and the $\mathrm{Cl}^{-}$ion has one excess electron (and thus has a charge of $-e$ ). (a) What is the magnitude of the net electrostatic force exerted on the $\mathrm{Cl}^{-}$ion by the eight $\mathrm{Cs}+$ ions at the corners of the cube? (b) If one of the Cs+ ions is missing, the crystal is said to have a defect; what is the magnitude of the net electrostatic force exerted on the $\mathrm{Cl}^{-}$ion by the seven remaining $\mathrm{Cs}^{+}$ ions?


((WileyPlus))
35. (a) Every cesium ion at a corner of the cube exerts a force of the same magnitude on the chlorine ion at the cube center. Each force is a force of attraction and is directed toward the cesium ion that exerts it, along the body diagonal of the cube. We can pair every cesium ion with another, diametrically positioned at the opposite corner of the cube. Since the two ions in such a pair exert forces that have the same magnitude but are oppositely directed, the two forces sum to zero and, since every cesium ion can be paired in this way, the total force on the chlorine ion is zero.
(b) Rather than remove a cesium ion, we superpose charge $-e$ at the position of one cesium ion. This neutralizes the ion, and as far as the electrical force on the chlorine ion is concerned, it is equivalent to removing the ion. The forces of the eight cesium ions at the cube corners sum to zero, so the only force on the chlorine ion is the force of the added charge.

The length of a body diagonal of a cube is $\sqrt{3} a$, where $a$ is the length of a cube edge. Thus, the distance from the center of the cube to a corner is $d=(\sqrt{3} / 2) a$. The force has magnitude

$$
F=k \frac{e^{2}}{d^{2}}=\frac{k e^{2}}{(3 / 4) a^{2}}=\frac{\left(8.99 \times 10^{9} \mathrm{~N} \cdot \mathrm{~m}^{2} / \mathrm{C}^{2}\right)\left(1.60 \times 10^{-19} \mathrm{C}\right)^{2}}{(3 / 4)\left(0.40 \times 10^{-9} \mathrm{~m}\right)^{2}}=1.9 \times 10^{-9} \mathrm{~N}
$$

Since both the added charge and the chlorine ion are negative, the force is one of repulsion. The chlorine ion is pushed away from the site of the missing cesium ion.

## 9. Hint of HW-21

### 9.1 Problem 21-21 (Hint)***

In Fig., particles 1 and 2 of chare $q_{1}=q_{2}=+3.20 \times 10^{-19} \mathrm{C}$ are on a $y$ axis at distance $d$ $=17.0 \mathrm{~cm}$ from the origin. Particle 3 of charges $q_{3}=+6.40 \times 10^{-19} \mathrm{C}$ is moved gradually along the $x$ axis from $x=0$ to $x=+5.0 \mathrm{~m}$. At what values of $x$ will the magnitude of the electrostatic force on the third particle from the other two particles be (a) minimum and (b) maximum? What are the (c) minimum and (d) maximum magnitudes?

((Solution))
$q_{1}=q_{2}=q=3.20 \times 10^{-19} \mathrm{C}$
$q_{3}=6.40 \times 10^{-19} \mathrm{C}$
$d=17 \mathrm{~cm}$
$0 \leq x \leq 5.0 \mathrm{~m}$
From the symmetry, $F_{y}=0$.

$$
\begin{aligned}
& F_{x}=\frac{2 q q_{3}}{4 \pi \varepsilon_{0}} \frac{1}{\left(x^{2}+d^{2}\right)} \frac{x}{\sqrt{x^{2}+d^{2}}}=\frac{2 q q_{3}}{4 \pi \varepsilon_{0}} \frac{x}{\left(x^{2}+d^{2}\right)^{3 / 2}} \\
& =\frac{2 q q_{3}}{4 \pi \varepsilon_{0} d^{2}} \frac{\frac{x}{d}}{\left(\frac{x^{2}}{d^{2}}+1\right)^{3 / 2}}=\frac{2 q q_{3}}{4 \pi \varepsilon_{0} d^{2}} \frac{t}{\left(t^{2}+1\right)^{3 / 2}}
\end{aligned}
$$

where $t=x / d$.

### 9.2 Problem 21-44 (Hint)

Figure shows a long, nonconducting, massless rod of length $L$, pivoted at its center and balanced with a block of weight $W$ at a distance $x$ from the left end. At the left and right ends of the rod are attached small conducting spheres with positive charges $q$ and $2 q$, respectively. A distance $h$ directly beneath each of these spheres is a fixed sphere with positive charge $Q$. (a) Find the distance $x$ when the rod is horizontal and balanced. (b) What value should $h$ have so that the rod exerts no vertical force on the bearing when the rod is horizontal and balanced?


Free-body diagram
We set up the equations from the conditions, $\sum F_{x}=0, \sum F_{y}=0$, and $\sum \tau=0$ around the origin.

### 9.3 Problem 21-60 (HW-21, Hint) SSM

In Fig., what are the (a) magnitude and (b) direction of the net electrostatic force on particle 4 due to the other three particles? All four particles are fixed in the $x y$ plane, and $q_{1}=-3.20 \times 10^{-19} \mathrm{C}, q_{2}=+3.20 \times 10^{-19} \mathrm{C}, q_{3}=+6.40 \times 10^{-19} \mathrm{C}, q_{4}=+3.20 \times 10^{-19} \mathrm{C}, q_{1}=$ $35.0^{\circ}, d_{1}=3.00 \mathrm{~cm}$, and $d_{2}=d_{3}=2.00 \mathrm{~cm}$.


## REFERENCES

R.A. Ford, Homemade Lightning: Creative Experiments in Electricity $3^{\text {rd }}$ edition, McGraw-Hill, 2001).

## APPENDIX:

Experimental equipment for electrostatics. You can find interesting explanation of equipment how it works, in Wikipedia.

## 1. Electroscope

https://en.wikipedia.org/wiki/Electroscope
2. Electrophorous
https://en.wikipedia.org/wiki/Electrophorus
3. Faraday cage
https://en.wikipedia.org/wiki/Faraday_cage
4. Van der Graaf generator
https://en.wikipedia.org/wiki/Van de Graaff_generator
5. Leyden jar
https://en.wikipedia.org/wiki/Leyden_jar

APPENDIX-II

|  | Submultiples |  |
| :---: | :---: | :---: |
| Value | SI symbol | Name |
| $10^{-1} \mathrm{C}$ | dC | decicoulomb |
| $10^{-2} \mathrm{C}$ | cC | centicoulomb |
| $10^{-3} \mathrm{C}$ | mC | millicoulomb |
| $10^{-6} \mathrm{C}$ | $\mu \mathrm{C}$ | microcoulomb |
| $10^{-9} \mathrm{C}$ | nC | nanocoulomb |
| $10^{-12} \mathrm{C}$ | pC | picocoulomb |
| $10^{-15} \mathrm{C}$ | fC | femtocoulomb |
| $10^{-18} \mathrm{C}$ | aC | attocoulomb |
| $10^{-21} \mathrm{C}$ | zC | zeptocoulomb |
| $10^{-24} \mathrm{C}$ | yC | yoctocoulomb |

