Chapter 35
Interference
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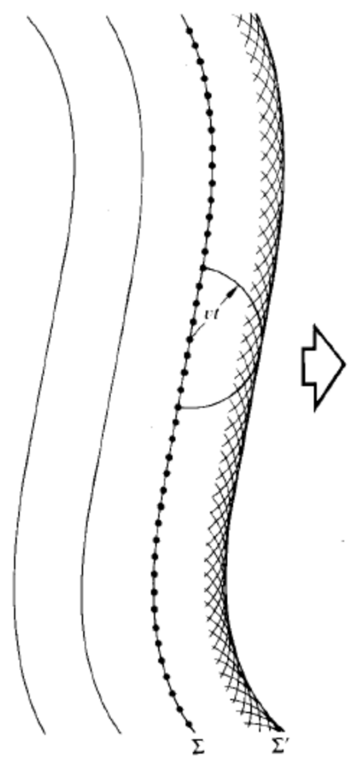
1. Huygens principle

Every point on a primary wavefront serves as the source of spherical secondary wavelets such that the primary wavefront at some later time is the envelope of these wavelets. Moreover, the wavelets advance with a speed and frequency equal to that of the primary wave at each point in space.

If the medium is homogeneous the wavelets may be constructed with finite radii, whereas if it is inhomogeneous the wavelets will have to have infinitesimal radii. Figure shows a view of a wavefront Σ as well as a number of spherical secondary wavelets which, after a time t, have propagated out to a radius of vt. The envelope of all of these wavelets is then asserted to correspond to the advanced primary wave Σ'.



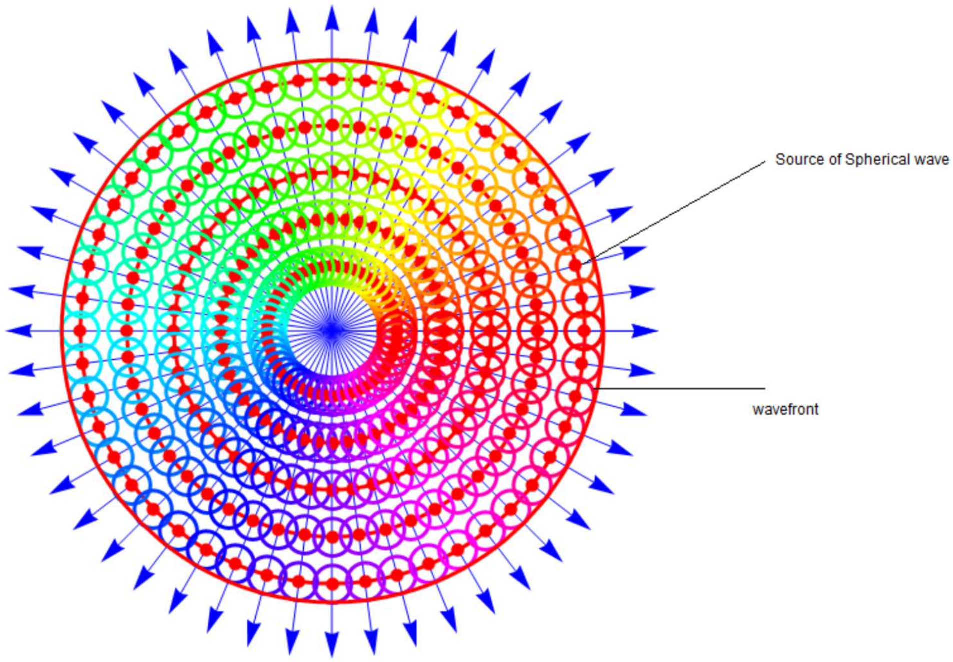
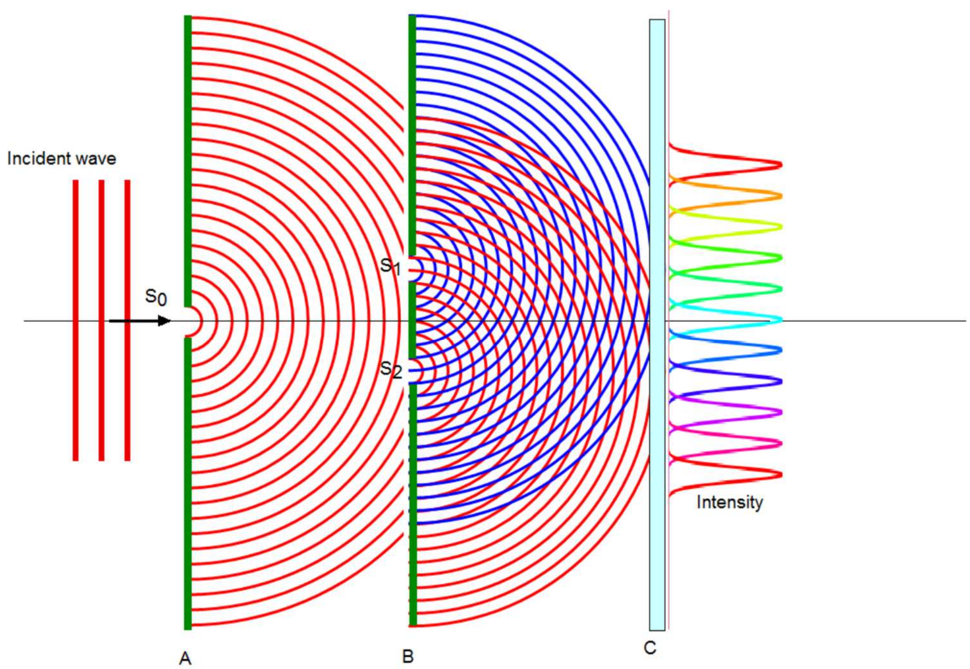
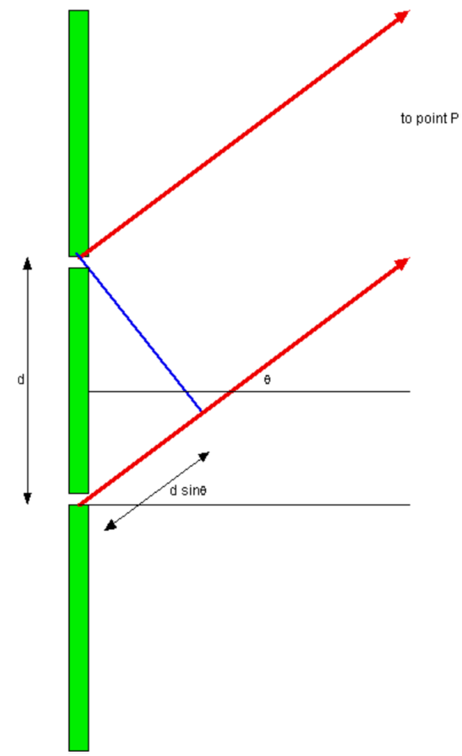


Fig. Huygens principle
Every point (red dot) on a primary wavefront (red circle) serves as the source of spherical secondary wavelets such that the primary wavefront at some later time is the envelope of these wavelets.

2. Diffraction by a double slit





(a) Analysis in the complex plane

The sum of the interfering spherical wavelets yields an electric field at P, given by the real part of

$$
\begin{aligned}
E &= \mathrm{Re}[E_0(r)e^{i(kr_1-\omega t)} + E_0(r)e^{i(kr_2-\omega t)}] \\
&= \mathrm{Re}[E_0(r)e^{i(kr_1-\omega t)}[1+e^{ik(r_2-r_1)}]] \\
&= \mathrm{Re}[E_0(r)e^{i(kr_1-\omega t)}e^{i\phi/2}(e^{i\phi/2}+e^{i\phi/2})] \\
&= \mathrm{Re}[E_0(r)e^{i(kr_1-\omega t)}e^{i\phi/2}2\cos(\frac{\phi}{2})]
\end{aligned}
$$

where

$$k(r_2 - r_1) = \phi = kd\sin\theta = \frac{2\pi d}{\lambda}\sin\theta\ ,$$

and k is the wavenumber, $k = \dfrac{2\pi}{\lambda}$.

If we define R to be the distance from the center of the line of oscillators to the point P, that is

$$kR = \frac{1}{2}\phi + kr_1 .$$

Then we have the form for E as

$$E = E_0(r)e^{i(kR-\omega t)}2\cos\frac{\phi}{2} .$$

The flux-density distribution within the diffraction pattern due to *two* coherent, identical, distant point sources in a linear array is equal to

$$I = I_0\cos^2\left(\frac{\phi}{2}\right).$$

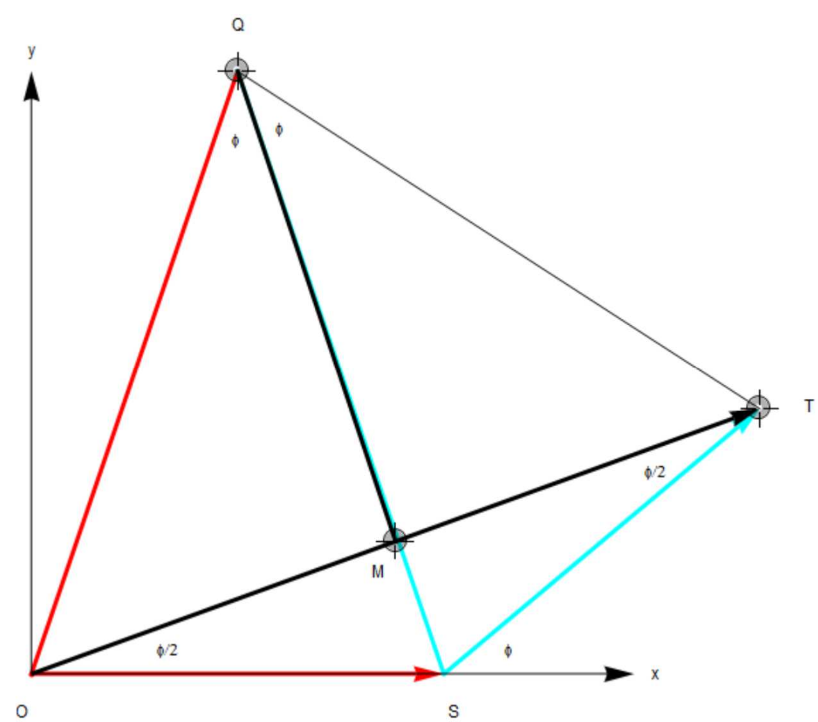
E is the electric field of a light with the wavelength λ. d is the separation distance between the centers of the slits.

(b) Phasor diagram in the *x*-*y* plane

We now consider the superposition of two waves,

$$E_0\sin(kr-\omega t+0) + E_0\sin(kr-\omega t+\phi) ,$$

where $k(r_2 - r_1) = \phi = kd\sin\theta = \frac{2\pi}{\lambda}d\sin\theta$



In the phasor diagram:

$$\overline{OQ} = R$$
$$\overline{OS} = E_0$$
$$E_R = \overline{OT} = 2\overline{OM} = 2\overline{OS}\cos\frac{\phi}{2} = 2E_0\cos\frac{\phi}{2}$$

All the vertices lie on the circle with the radius R. R is related to $\overline{OS}(=E_0)$ by

$$\overline{OS} = E_0 = 2R\sin\frac{\phi}{2}, \qquad R = \frac{E_0}{2\sin\frac{\phi}{2}}.$$

The intensity is obtained as

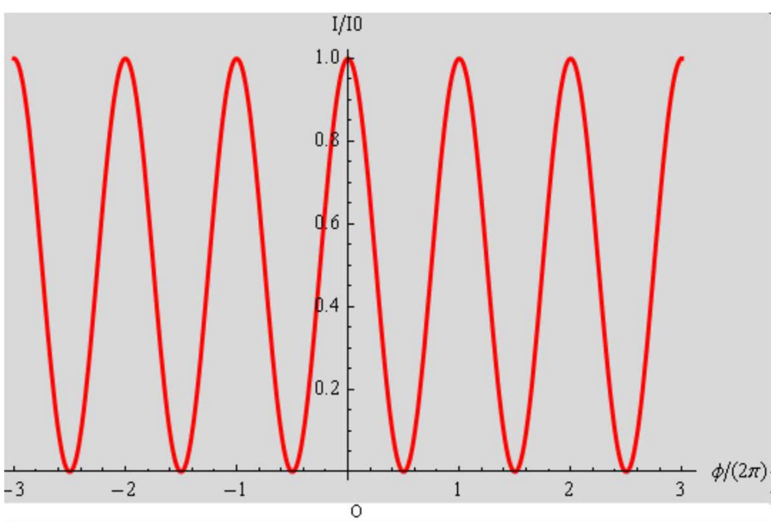
$$I = \frac{\varepsilon_0 c}{2}E_R^{\ 2} = 2\varepsilon_0 c E_0^{\ 2}\cos^2\frac{\phi}{2} = \varepsilon_0 c E_0^{\ 2}(1+\cos\phi),$$

or

$$I = 2I_0(1+\cos\phi),$$

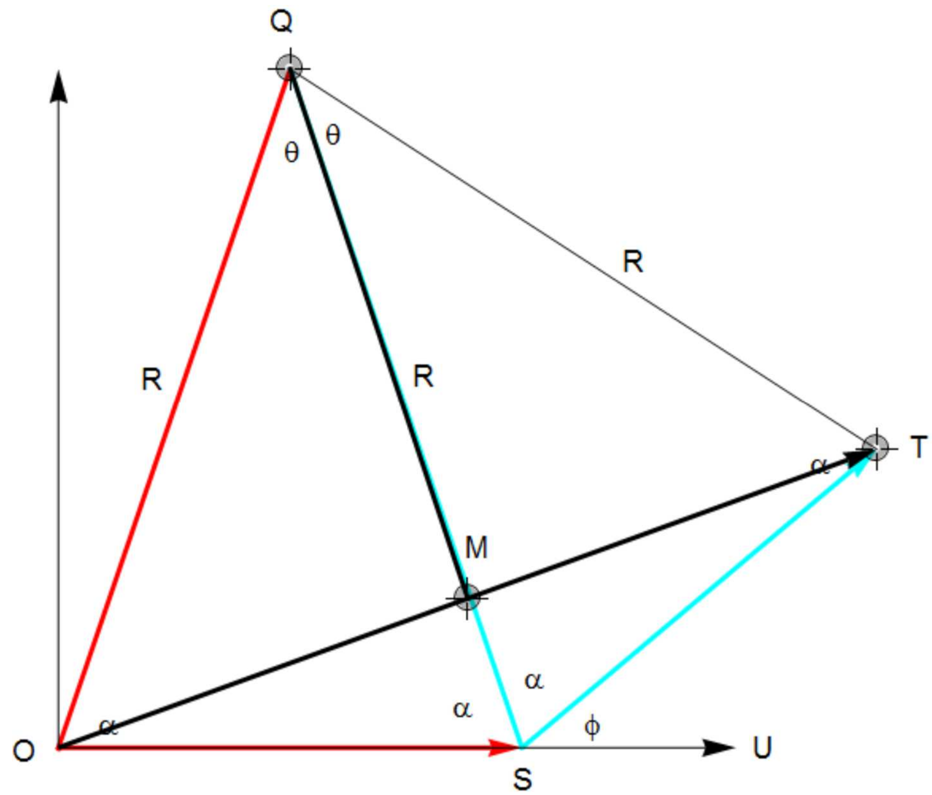
where $I_0 = \dfrac{\varepsilon_0 c E_0^{\ 2}}{2}$, and the phase difference ϕ is given by

$$\phi = 2\pi\frac{d}{\lambda}\sin\theta.$$



((**Note**)) Geometry

Proof of the geometry for the phasor diagram



We consider two isosceles triangles.

$$\overline{QO} = \overline{QS} = \overline{QT} = R$$
$$\angle QOS = \angle QSO = \angle QST = \angle QTS = \alpha$$

In other words, points O, S, and T lie on a circle with radius R (the value of R will be specified later). We assume that

$$\angle TSU = \phi .$$

From the geometry shown in the Fig, we have

$$\theta + 2\alpha = \pi$$
$$2\alpha + \phi = \pi$$

Then we get

$$\theta = \phi .$$

The radius R and the side $\overline{OS}$ are related as

$$\overline{OS} = 2R\sin\frac{\theta}{2} = 2R\sin\frac{\phi}{2}.$$

This means that R is uniquely determined when the angle ϕ and $\overline{OS}$ are given. In Fig., ΔOST is a isosceles triangle with $\overline{OS} = \overline{ST}$. When M is the midpoint of the side $\overline{OT}$, it is found that $\overline{SM}$ is perpendicular to $\overline{OT}$. We also get

$$\angle SOM = \angle STM = \frac{\phi}{2}$$

Finally, we have

$$\overline{OT} = 2\overline{OM} = 2\overline{OS}\sin\frac{\phi}{2} = 4R\sin^2\frac{\phi}{2}$$

((**Note**))

Energy flux of photon: energy of photon passing through area ($\Delta A = 1$) and time ($\Delta t = 1$)

$$2\times\left(\frac{1}{2}\varepsilon_0 E^2\right)c\Delta t\Delta A = c\varepsilon_0 E^2$$

or

$$I = c\varepsilon_0 E^2$$

The time average:

$$\langle I\rangle = I_0 = \frac{c\varepsilon_0}{2}E_0^{\ 2} = c\varepsilon_0 E_{rms}^{\ \ 2}$$

where

$$E_{rms} = \frac{E_0}{\sqrt{2}}.$$

Using the formula of the Poynting vector, we get the same result

$$S = \frac{1}{\mu_0} EB = \frac{\varepsilon_0}{\varepsilon_0 \mu_0} E \frac{E}{c} = c\varepsilon_0 E^2 = I$$

We now consider the Young's double slit experiment: The intensity of the double slit interference is

$$c\varepsilon_0 [2E \cos\frac{\phi}{2}]^2 = 4c\varepsilon_0 E^2 \cos^2\frac{\phi}{2}$$

The time average of the energy flux of photon for the double slits with the same area a,

$$\begin{aligned} 4ac\varepsilon_0 \frac{E_0{}^2}{2} \cos^2\frac{\phi}{2} &= 2ac\varepsilon_0 E_0{}^2 \cos^2\frac{\phi}{2} \\ &= 4ac\varepsilon_0 E_{rms}{}^2 \cos^2\frac{\phi}{2} \\ &= 4I_1 \cos^2\frac{\phi}{2} \end{aligned}$$

where

$$I_1 = ac\varepsilon_0 E_{rms}{}^2$$

3. Young's Interference Experiment

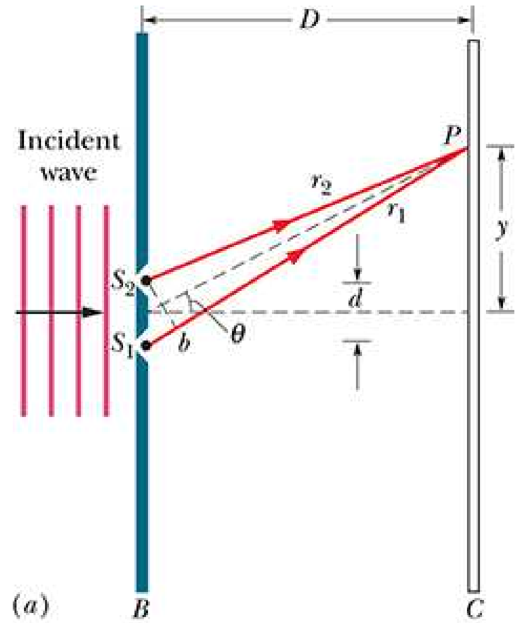
(1) The intensity I has maxima at

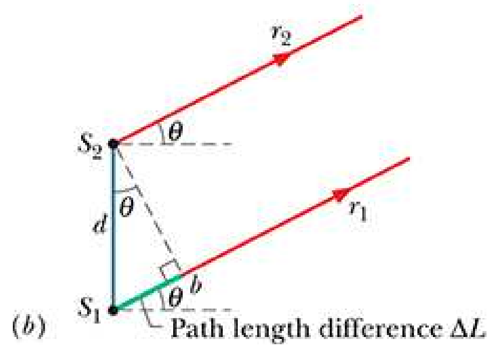
$$\phi = \frac{2\pi d}{\lambda} \sin\theta = 2m\pi \qquad \text{or} \qquad d\sin\theta = \lambda m,$$

where n is integer. The path difference is given by $n\lambda$.

(2) The intensity has minima at

$$\phi = \frac{2\pi d}{\lambda} \sin\theta = (2m+1)\pi \qquad \text{or} \qquad d\sin\theta = (m+\frac{1}{2})\lambda.$$





The vertical distance y on the screen (the distance between the slits S_1 and S_2, and the screen is D) is given by

$$y = D\tan\theta \approx D\sin\theta$$

The maximum's vertical distance y_n from the center of the pattern on the screen is

$$y_n = \frac{D\lambda}{d}n. \qquad \text{(bright lines)}$$

The minimum's vertical distance y_n from the center of the pattern on the screen is

$$y_n = \frac{D\lambda}{d}(n+\frac{1}{2}). \qquad \text{(dark lines)}$$

((**Note**))

$$I = I_0 \frac{(1 + \cos\phi)}{2}$$

The phase $\phi = 2\pi \frac{d}{\lambda} \sin\theta$ is expressed in terms of y,

$$\phi = 2\pi \frac{d}{\lambda} \sin\theta \approx 2\pi \frac{d}{\lambda} \tan\theta = \frac{2\pi}{\lambda} \frac{d}{D} y$$

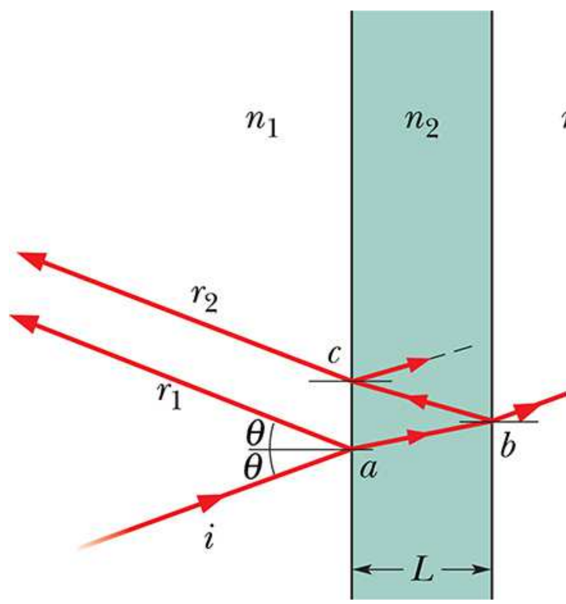
since $y = D\tan\theta$. Since f s proportional to y, the intensity is also a periodic function of y.

5. Interference from thin film

(i) The refraction at an interface never causes a phase change.

(ii) The reflection can, depending on the indexes of refraction on the two sides of the interface. When light traveling in a medium of the index of refraction (n_1) is reflected from a medium of index n_2, it undergoes a π phase shift if $n_2>n_1$, and it undergoes no phase change.

(iii) When light of the wavelength λ travels in a medium (with the index of refraction n), its wavelength is $\lambda_n = \lambda/n$.



We assume that $n_2>n_1$ and $n_2>n_3$. The phase of ray 1, $\phi_1 = \pi$. The phase of ray 2,

$$\phi_2 = \frac{2\pi(2L)}{(\lambda / n_2)}$$

where λ / n_2 is the wavelength in the medium with the index of refraction n_2.

Then the phase difference is

$$\Delta\phi = \phi_2 - \phi_1 = \frac{4\pi L}{\lambda} n_2 - \pi$$

Maximum reflection (bright film))

$$\frac{4\pi L}{\lambda} n_2 - \pi = 2m\pi$$

$$2n_2 L = (m + \frac{1}{2})\lambda$$

Minimum reflection (dark film)

$$\frac{4\pi L}{\lambda} n_2 - \pi = (2m-1)\pi$$

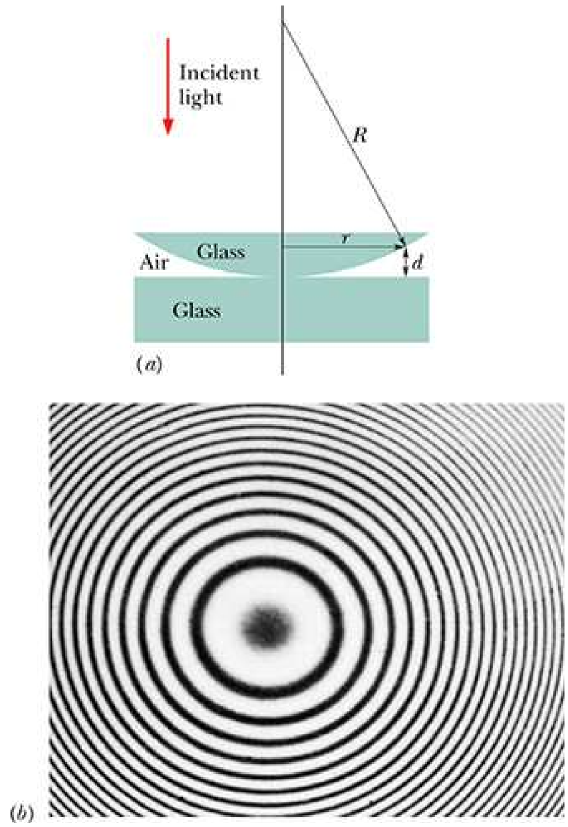
$$2n_2 L = m\lambda$$

6. Newton ring

((**Wikipedia**))

The phenomenon was first described by Robert Hooke in his 1664 book Micrographia, although its name derives from the physicist Sir Isaac Newton, who was the first to analyze it.

The pattern is created by placing a very slightly convex curved glass on an optical flat glass. The two pieces of glass make contact only at the center, at other points there is a slight air gap between the two surfaces, increasing with radial distance from the center to the microscope. The diagram at right shows a small section of the two pieces, with the gap increasing right to left. Light from a monochromatic (single color) source shines through the top piece and reflects from both the bottom surface of the top piece and the top surface of the optical flat, and the two reflected rays combine and superpose. However the ray reflecting off the bottom surface travels a longer path. The additional path length is equal to twice the gap between the surfaces. In addition the ray reflecting off the bottom piece of glass undergoes a π phase reversal, while the internal reflection of the other ray from the underside of the top glass causes no phase reversal. The brightness of the reflected light depends on the difference in the path length of the two rays. https://en.wikipedia.org/wiki/Newton%27s_rings



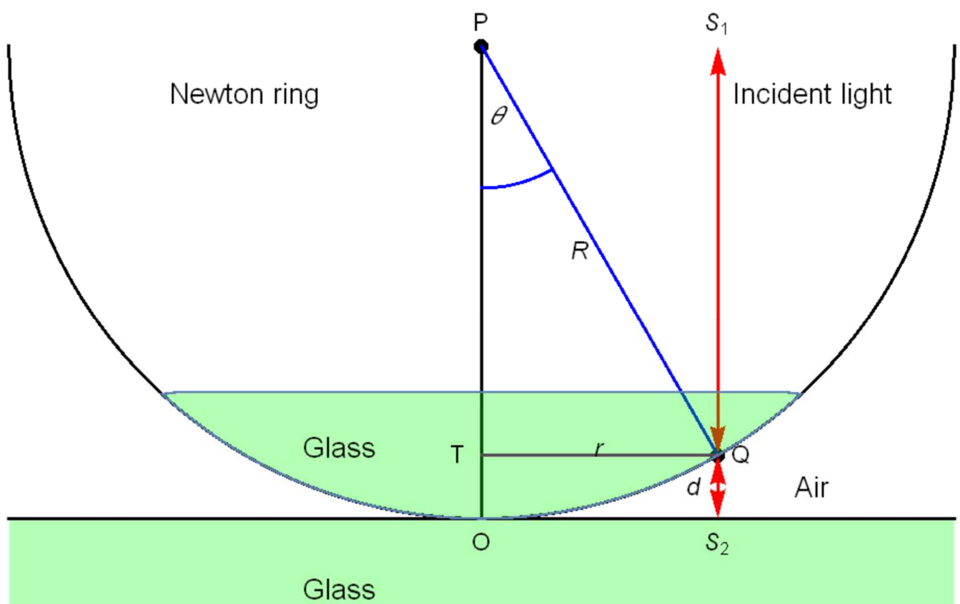


Fig. Newton ring. Note that the phase changes by π at the reflection from air to glass.

From the geometry, we have

$$d = R(1-\cos\theta) = 2R\sin^2\left(\frac{\theta}{2}\right) \approx 2R\left(\frac{\theta}{2}\right)^2 = \frac{R\theta^2}{2}$$
$$r = R\sin\theta \approx R\theta$$
$$d = \frac{r^2}{2R}$$

or

$$r = \sqrt{2dR}.$$

The phase of the ray 1 reflecting at the lower glass

$$\phi_1 = \frac{2\pi}{\lambda}(2d) + \pi \ .$$

The phase of the ray 2 reflecting at the upper glass

$$\phi_2 = 0 \ .$$

The phase difference between two rays is

$$\Delta\phi = \phi_2 - \phi_1 = \frac{2\pi}{\lambda}(2d) + \pi \ .$$

The condition for the bright ring:

$$\Delta\phi = \frac{2\pi}{\lambda}(2d) + \pi = 2\pi(m+1)$$
$$2d = (m + \frac{1}{2})\lambda$$
$$r = \sqrt{2dR} = \sqrt{(m + \frac{1}{2})R\lambda}$$

where $m = 0, 1, 2, \ldots\ldots$

The condition for the dark rings:

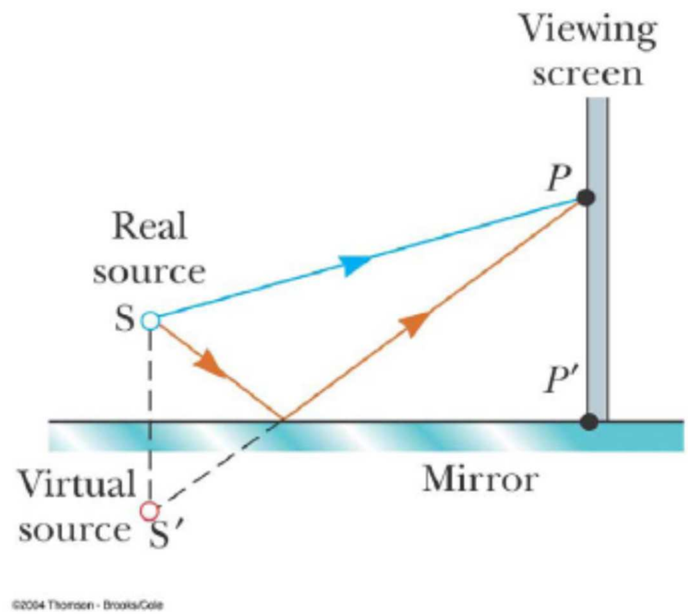
$$\Delta\phi = \frac{2\pi}{\lambda}(2d) + \pi = 2\pi(m + \frac{1}{2})$$
$$2d = m\lambda$$
$$r = \sqrt{2dR} = \sqrt{mR\lambda}$$

where m = 0, 1, 2, 3,….

((Note)) Beautiful Newton ring; see the Wekipedia

http://en.wikipedia.org/wiki/Newton%27s_rings

7. Lloyd's mirror

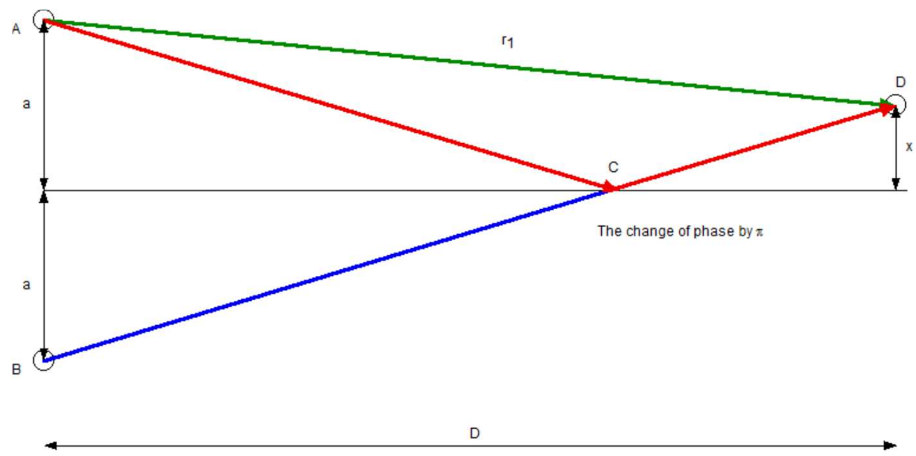


An arrangement for producing an interference pattern with a single light source. Waves reach point P either by a direct path or by reflection. The reflected ray can be treated as a ray from the source S' behind the mirror. This arrangement can be thought of as a double slit source with the distance between points S and S' comparable to length d.

An interference pattern is formed. The positions of the dark and bright fringes are reversed relative to pattern of two real sources. This is because there is a 180° phase change produced by the reflection

((Example)) Problem 35-87

In Fig., a microwave transmitter at height a above the water level of a wide lake transmits microwaves of wavelength λ toward a receiver on the opposite shore, a distance x above the water level. The microwaves reflecting from the water interfere with the microwaves arriving directly from the transmitter. Assuming that the lake width D is much greater than a and x, and that $\lambda \geq a$, find an expression that gives the values of x for which the signal at the receiver is maximum.



$$r_1 = \overline{AD} = \sqrt{(a-x)^2 + D^2} = D\sqrt{1 + \frac{(x-a)^2}{D^2}}$$

$$\approx D[1 + \frac{(x-a)^2}{2D^2}]$$

$$r_2 = \overline{AC} + \overline{CD} = \overline{BD} = \sqrt{(a+x)^2 + D^2} = D\sqrt{1 + \frac{(x+a)^2}{D^2}}$$

$$\approx D[1 + \frac{(x+a)^2}{2D^2}]$$

Then the path difference Δr ($= r_2 - r_1$) is obtained as

$$\Delta r = r_2 - r_1 = D[1 + \frac{(x+a)^2}{2D^2}] - D[1 + \frac{(x-a)^2}{2D^2}]$$

$$= \frac{1}{2D}[(x+a)^2 - (x-a)^2]$$

$$= \frac{1}{2D} 4ax = \frac{2ax}{D}$$

The phase difference between two rays is

$$\Delta\phi = \frac{2\pi}{\lambda}(r_2 - r_1) + \pi = 2\pi(m+1) \qquad \text{(Constructive)}$$

where m is an integer. Here we take into account of the phase change by π on the reflection of the ray 2 at the point C. Then we have

$$\frac{2\pi}{\lambda}\frac{2ax}{D} = 2\pi(m+\frac{1}{2})$$

$$x = \frac{\lambda D}{2a}(m+\frac{1}{2})$$

8. Michelson's interferometer

8.1 Method

The basic design of the Michelson interferometer is shown in Fig.1. The light beam from the arc source (Hg or Na) is separated in two parts by the partly silvered surface P. At this point, half the beam is reflected to M_1 and the other half is transmitted to M_2. Mirrors M_1 and M_2 reflect the light back to the half silvered plate, and half of each beam reaches the observer, the remainder being directed back to the source and lost. The mirror M_1 can be translated toward or away from the observer by means of micrometer. The calibration of the micrometer reading is necessary.

Suppose that the distance between M_1 and P is d_1 and the distance between M_2 and P is d_2. When the condition

$$2(d_1 - d_2) = n\lambda \text{ (}n\text{: integer).}$$

the circular fringe can be observed. Since d_2 is fixed and d_1 is varied, this equation is rewritten as

$$2\Delta d_1 = \Delta n\, \lambda \quad \text{or} \quad \Delta d_1 = \Delta n\, \lambda/2 \qquad (1)$$

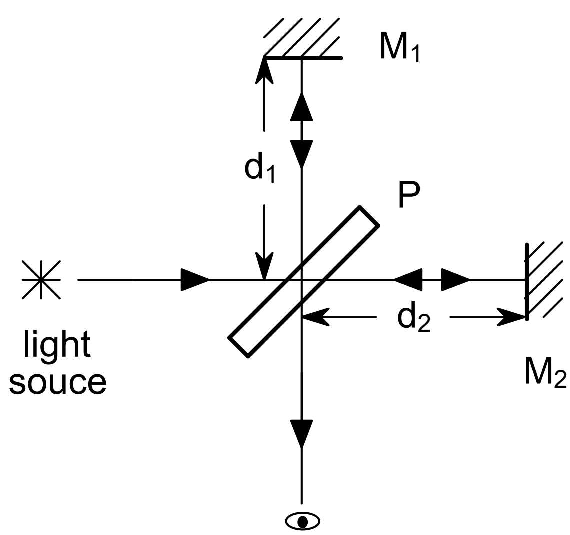
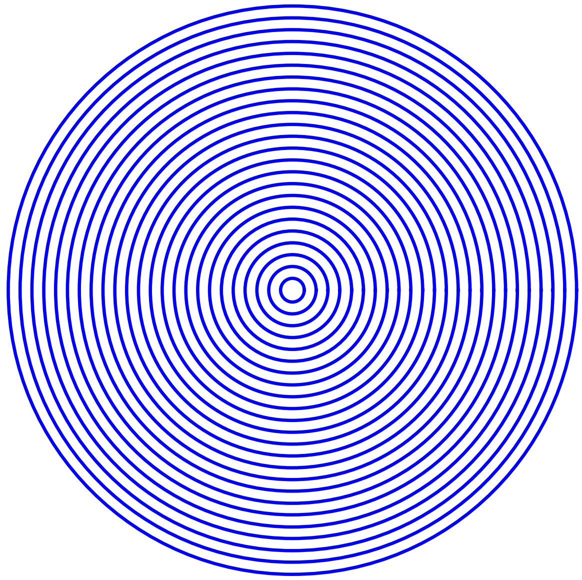


Figure 1



8.2. Determine the difference of wavelength λ_1 and λ_2 of Na D lines.

The wavelength difference between two close lines such as the components of the sodium D lines is determined from their average wavelength and the visibility of fringes. At certain positions of mirror M_1, it is found that the fringes are clear and sharp whereas at intermediate positions, they are very indistinct. The reason is that there are two sets of fringes which are not identical, and at some positions, the two sets are in step and the overall fringe pattern sharp, whereas at the intermediate positions the two sets overlap thus washing out the overall pattern. **Figure 2** shows the situation schematically. The separation of the positions of maximum (or minimum) visibility of the fringe pattern determines the wavelength difference. Let the two wavelengths be described by λ_1 and $\lambda_2(<\lambda_1)$. Let d_a and d_b represent points where both set of fringes are in step (maximum visibility) for d_1. Then we have

$$2(d_a - d_2) = m\lambda_1, \qquad 2(d_b - d_2) = n\lambda_1$$

$$2(d_a - d_2) = m'\lambda_2, \qquad 2(d_b - d_2) = n'\lambda_1$$

where $n - m = n' - m' - 1$ and d_2 is fixed. When $\Delta d = d_b - d_a$, we have

$$n - m = \frac{2(d_b - d_a)}{\lambda_1} = \frac{2\Delta d}{\lambda_1}$$

$$n' - m' = \frac{2(d_b - d_a)}{\lambda_2} = \frac{2\Delta d}{\lambda_2}$$

which leads to

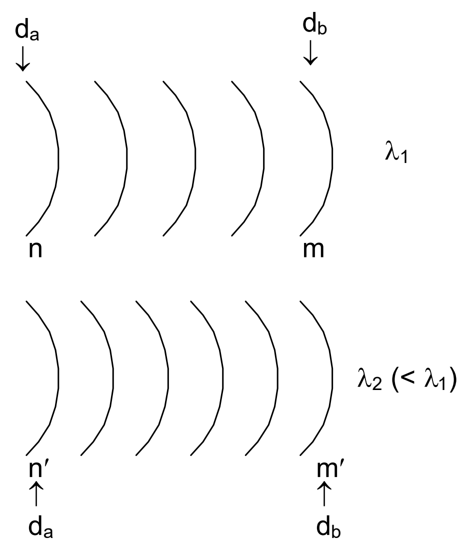
$$\frac{2\Delta d}{\lambda_1} = \frac{2\Delta d}{\lambda_2} - 1$$

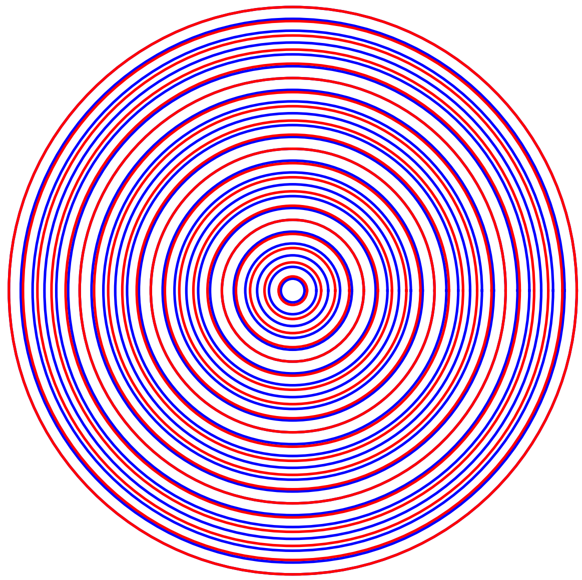
or

$$\Delta\lambda = \lambda_1 - \lambda_2 = \frac{\lambda_1\lambda_2}{2\Delta d} \approx \frac{\lambda_1^{\,2}}{2\Delta d}, \qquad (3)$$

where $\lambda_1 \approx \lambda_2$.

Obtain Δd. Using λ_1 = 589 nm and Δd, determine the difference $\Delta\lambda$ for the Na D lines.





8.3. Physical meaning of Na D lines

Consider a sodium atom. From standard atomic spectroscopy notation, the ground-state configuration is $(1s)^2(2s)^2(2p)^6(3s)$. The inner 10 electrons can be visualized to form a spherically symmetrical electron cloud. We are interested in the excitation of the eleventh electron from 3s to a possible higher state. The nearest possibility is excitation to 3p. Because the central potential is no longer of the pure Coulomb form, 3s and 3p are now split. The fine structure brought by spin orbit coupling (V_{LS}) refers to even a finer split within 3p, between 3p1/2 and 3p3/2, where the subscript refers to j. Experimentally, we observe two closely separated yellow lines- known as the sodium D lines- one at 589.6 nm, the other at 589.0 nm.

$$\Delta\lambda = 0.6\,\text{nm} = 6\ \text{Å}.$$

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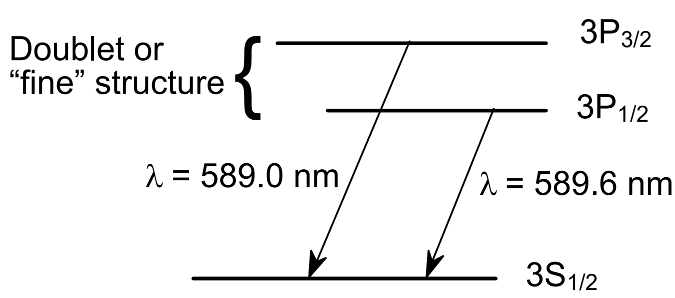


9. Fabry-Perot Interferometer (Physics Department, Junior Laboratory

The Fabry-Perot interferometer, devised by C. Fabry and A. Perot in 1899, employs multiple-beam interference. It is used to measure wavelengths with high precision and to study the fine structure of spectrum lines. A Fabry-Perot interferometer consists essentially of two optically flat, partially reflecting plates of glass or quartz with their reflecting surfaces held accurately parallel. If the plate spacing can be mechanically varied, the device is called an interferometer, whereas if the plates are held fixed by spacers, it is called an etalon. The surfaces must be extremely flat and parallel in order to obtain the maximum fringe sharpness. We consider the case of the Fabry-Perot etalon (Fig.1). In this case the interference occurs when the condition

$$2t = n\lambda, \qquad (1)$$

is satisfied, where t is the thickness of parallel plate, λ is the wavelength of the light source, and n is an integer.

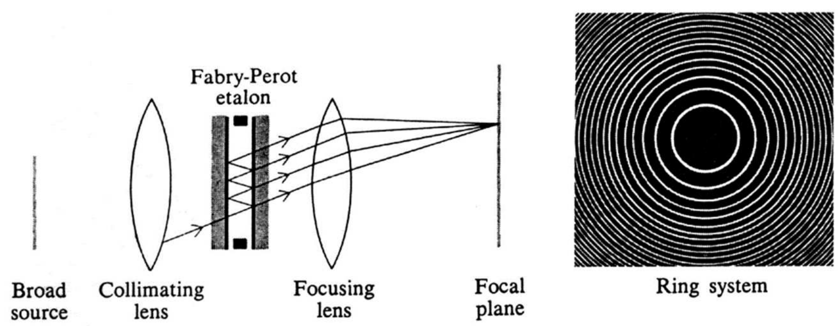


Fig. Arrangements for the Fabry-Perot etalon

((Note))
The Fabry-Perot interferometer

https://en.wikipedia.org/wiki/Fabry%E2%80%93P%C3%A9rot_interferometer

In optics, a Fabry–Pérot interferometer (FPI) or etalon is an optical cavity made from two parallel reflecting surfaces (i.e.: thin mirrors). Optical waves can pass through the optical cavity only when they are in resonance with it. It is named after Charles Fabry and Alfred Perot, who developed the instrument in 1899. Etalon is from the French étalon, meaning "measuring gauge" or "standard". Etalons are widely used in telecommunications, lasers and spectroscopy to control and measure the wavelengths of light. Recent advances in fabrication technique allow the creation of very precise tunable Fabry–Pérot interferometers. The device is called an interferometer when the distance between the two surfaces (and with it the resonance length) can be changed, and an etalon when the distance is fixed (however, the two terms are often used interchangeably).

((**Jenkins** and **White**))

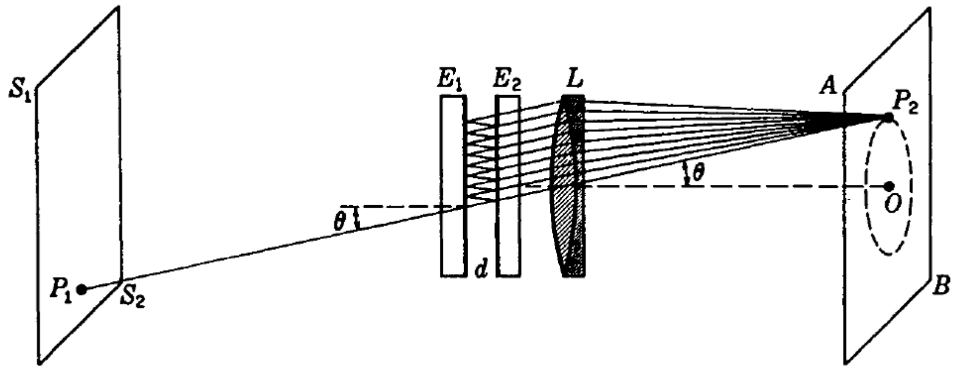
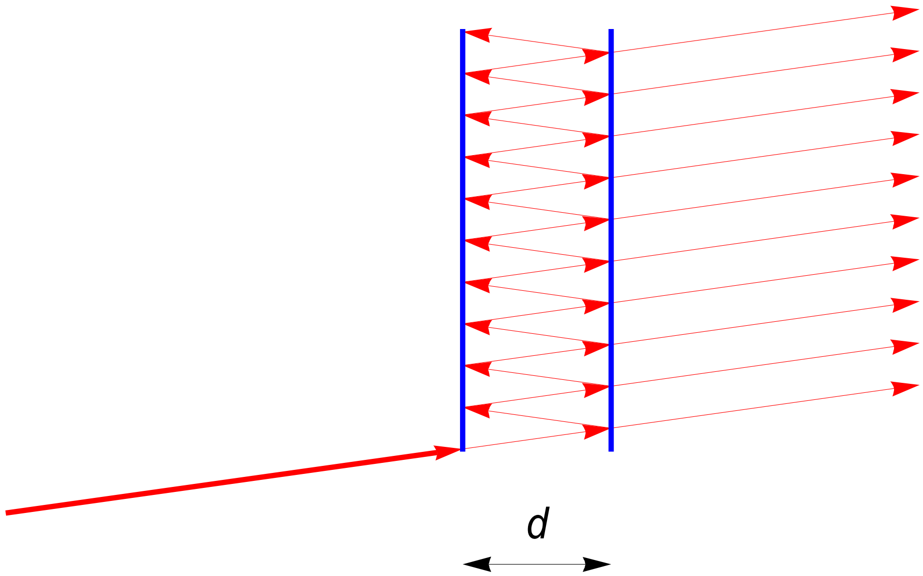


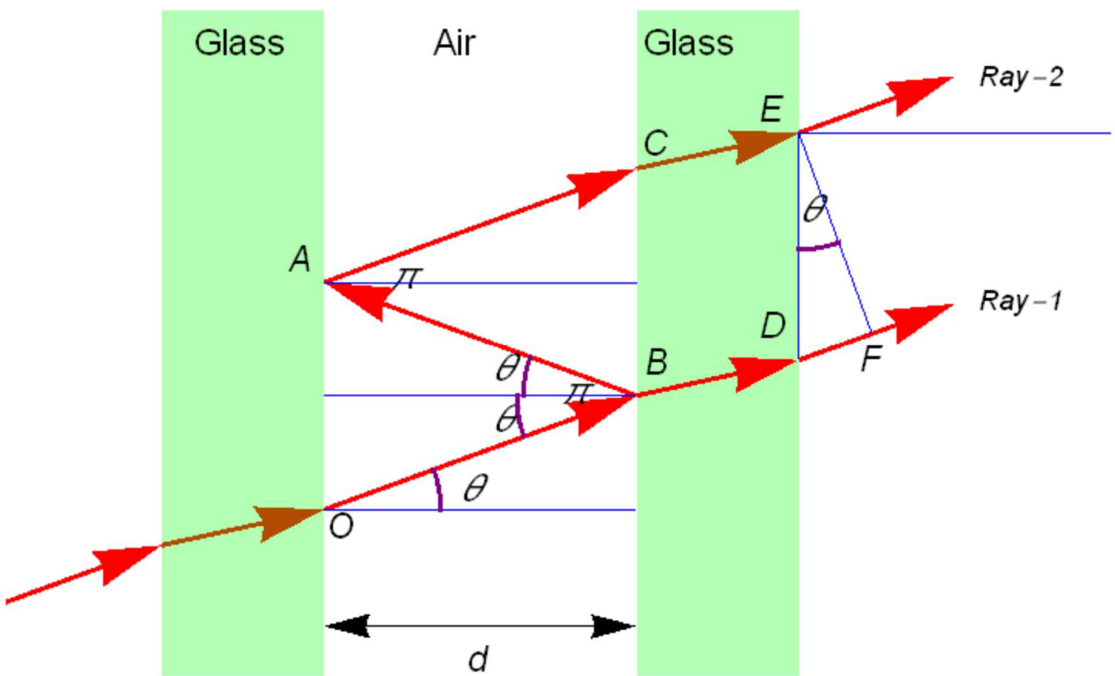
Fig. Fabry-Perot interferometer E_1E_2 set up to show the formation of circular interference fringes from multiple reflections.

This instrument utilizes the fringes produced in the transmitted light after multiple reflection in the air film between two plane plates thinly silvered on the inner surfaces. Since the separation *d* between the reflecting surfaces is usually fairly large (from 0.1 to 10 cm) and observations are made near the normal direction, the fringes come under the class of fringes of equal inclination. To observe the fringes, the light from a broad source (S_1S_2) of monochromatic light is allowed to traverse the interferometer plates E_1E_2. Since any ray incident on the first silvered surface is broken by reflection into a series of parallel transmitted rays, it is essential to use a lens *L*, which may be the lens of the eye, to bring these parallel rays together for interference. A ray from the point P_1 on the source is incident at the angle θ, producing a series of parallel rays at the same angle, which are brought together at the point P_2 on the screen AB. It is to be noted that P_2 is not an image of P_1. The condition for the reinforcement of the transmitted rays is given by

$$2d\cos\theta = m\lambda .$$

Here we derive this condition with the aid of the Mathematica.





Using the above figure, the condition for the constructive and destructive interferences can be obtained as follows.

$$\overline{OB} = r$$

For the ray-2 (along the path BACE)

Path (BAC) length $=2r$

For the ray-1 (along the path BDF)

Path (DF) length $= 2r\sin^2\theta$

The phase difference between the ray-1 and ray-2 is

$$\begin{aligned}\Delta\phi &= \frac{2\pi}{\lambda}(2r - 2r\sin^2\theta) + 2\pi \\ &= \frac{2\pi}{\lambda}2r\cos^2\theta + 2\pi \\ &= \frac{2\pi}{\lambda}2d\cos\theta + 2\pi\end{aligned}$$

since $r\cos\theta = d$, where λ is the wavelength. The constructive interference occurs when the condition is satisfied

$$\frac{2\pi}{\lambda}2d\cos\theta = 2m\pi, \qquad \text{or} \qquad 2d\cos\theta = m\lambda.$$

The destructive interference occurs when

$$2d\cos\theta = (m + \frac{1}{2})\lambda.$$

A typical circular fringe observed using the Fabry-Perot experiment, is shown below.

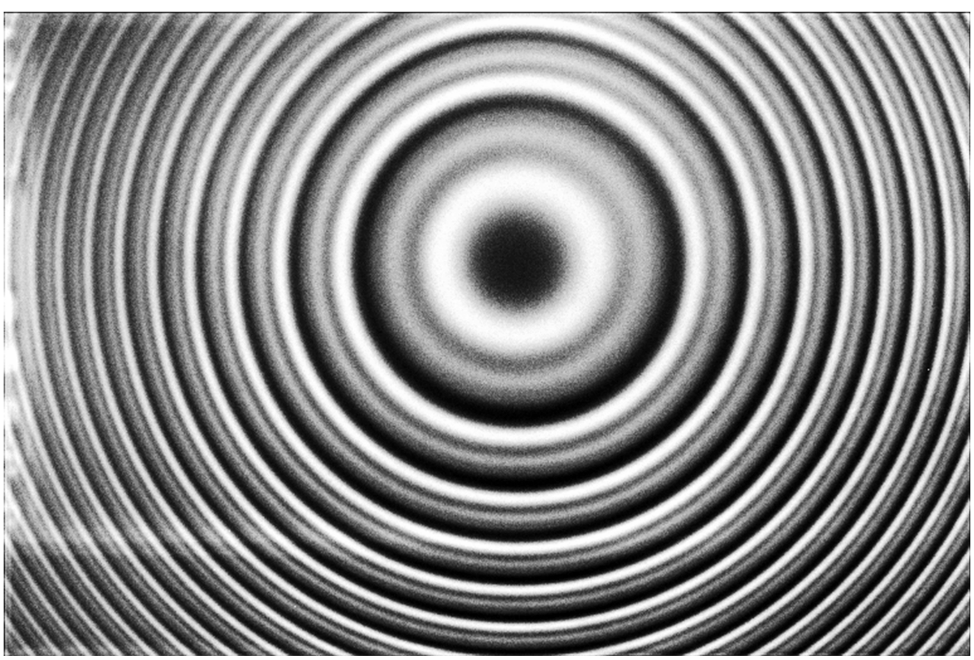


Fig. Interference fringes, showing fine structure, from a Fabry–Pérot etalon. The source is a cooled deuterium lamp. (Wikipedia)
https://en.wikipedia.org/wiki/Fabry%E2%80%93P%C3%A9rot_interferometer#/media/File:Fabry_Perot_Etalon_Rings_Fringes.png

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10. Typical problems

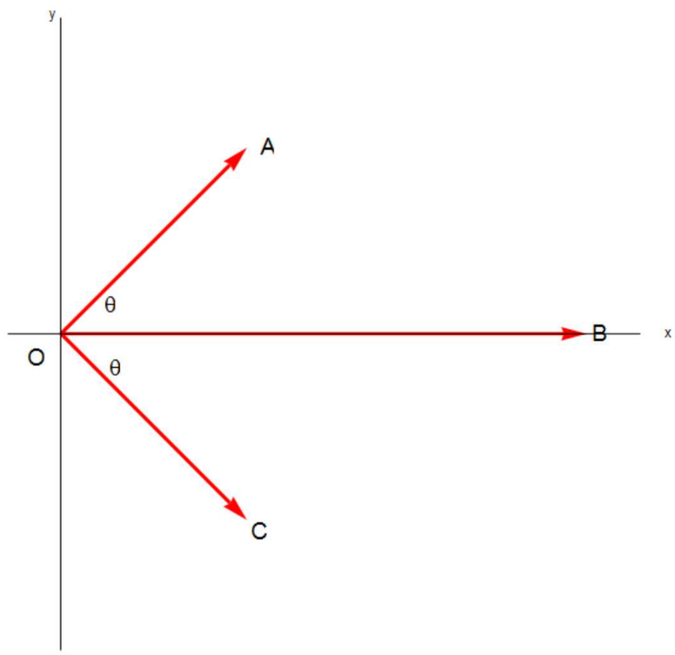
10.1 Problem 35-31

Three electromagnetic waves travel through a certain point P along an *x* axis. They are polarized parallel to a *y* axis, with the following variations in their amplitudes. Find their resultant at P.

$$E_1 = (10.0\mu V/m)\sin[(2.0\times 10^{14} rad/s)t]$$
$$E_2 = (5.00\mu V/m)\sin[(2.0\times 10^{14} rad/s)t + 45.0°]$$
$$E_3 = (5.00\mu V/m)\sin[(2.0\times 10^{14} rad/s)t - 45.0°]$$

((**Solution**))
ω = 2.0 x 10^{14} ead/s
OA = 5
OB = 10
OC = 5
θ = 45°

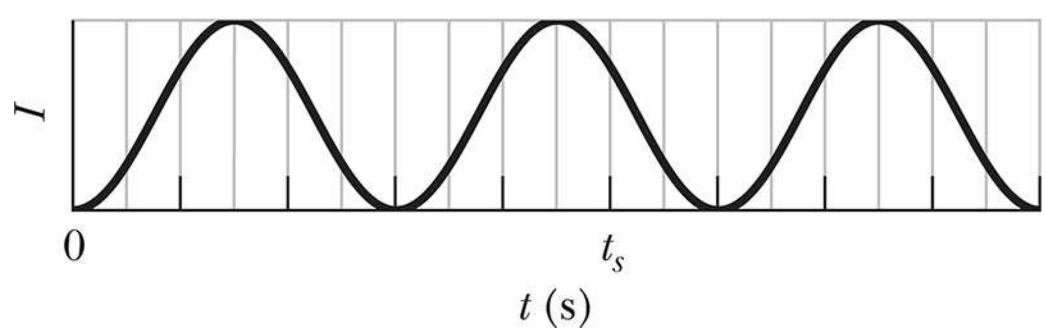
We use the phasor diagram.



From the symmetry, the resultant y-component is equal to zero.
The resultant x-component = 2 x (5cos45°) + 10 = $10 + 5\sqrt{2}$

$$E_{total} = (10 + 5\sqrt{2})\sin(\omega t + 0°) = 17.1\sin(\omega t + 0°)$$

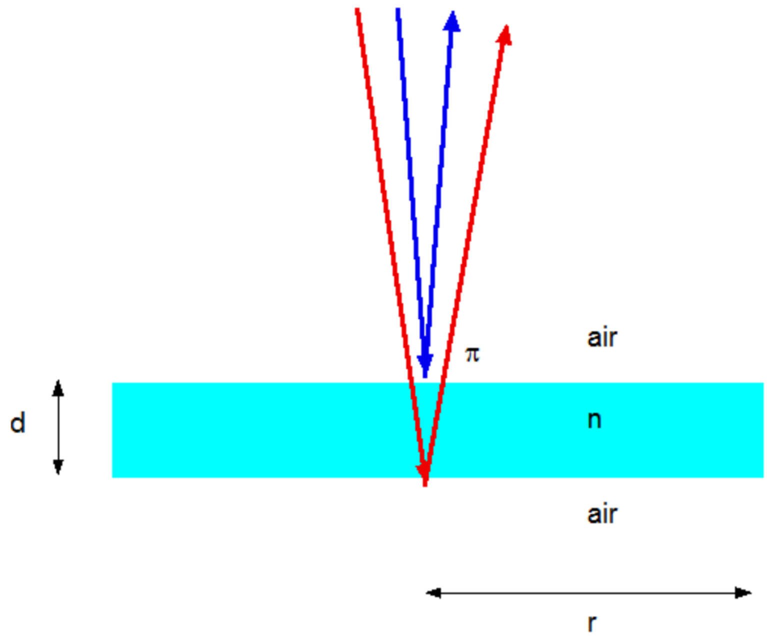
10.2 Problem 35-78



A thin film of liquid is held in a horizontal circular ring, with air on both sides of the film. A beam of light at wavelength 550 nm is directed perpendicularly onto the film, and the intensity I of its reflection is monitored. Figure gives intensity I as a function of time t; the horizontal scale is set by t_s = 20.0 s. The intensity changes because of evaporation from the the two sides of the film. Assume that the film is flat and has parallel sides, a radius of 1.80 cm, and an index of refraction of 1.40. Also assume that the film's volume decreases at a constant rate. Find the rate.

((**Solution**))

$\lambda = 550$ nm
$r = 1.80$ cm
$n = 1.40$
period = 12 s.



The constructive interference:

$$\phi = \frac{2\pi}{\lambda}(2nd) + \pi = 2\pi(m+1)$$

$$d = \frac{1}{2n}(m+\frac{1}{2})\lambda = \frac{550}{2\times1.40}(m+\frac{1}{2}) = 196.43(m+\frac{1}{2}) \text{ [nm]}$$

or

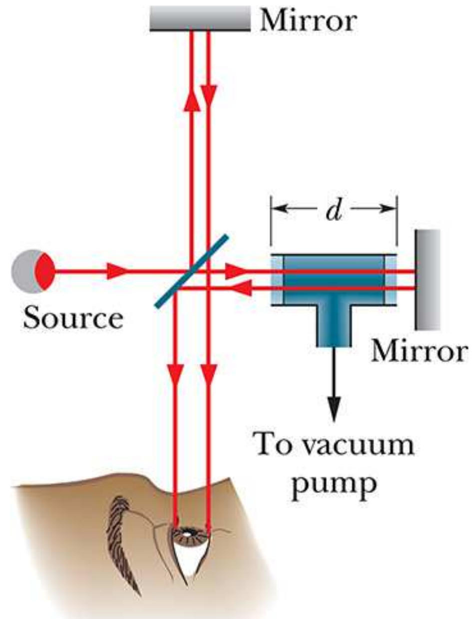
$$\Delta d = 196.43nm$$

The rate of the volume change:

$$\frac{\Delta V}{\Delta t} = \frac{\pi r^2 \Delta d}{\Delta t} = 1.66\times10^{-11} m^3/s$$

10.3 Problem 35-81 Michael interferometer

In Fig., an airtight chamber of length $d = 5.0$ cm is placed in one of the arms of a Michelson interferometer. (The glass window on each end of the chamber has negligible thickness.) Light of wavelength source $\lambda = 500$ nm is used. Evacuating the air from the chamber causes a shift of 60 bright fringes. From these data evaluate the six significant refraction of air at atmospheric pressure.



((**Solution**))
$d = 5.0$ cm. $\lambda = 500$ nm.

$$\Delta\phi_{air} = \frac{2\pi}{\lambda}(2dn_{air} - 2d) + \frac{2\pi}{\lambda}\Delta L = 2\pi m_1$$

$$\Delta\phi_{vacuum} = \frac{2\pi}{\lambda}(2dn_{vacuum} - 2d) + \frac{2\pi}{\lambda}\Delta L = 2\pi m_2$$

where ΔL is the geometrical path difference between two rays, and m_1 and m_2 are positive integers. Then we have

$$\Delta\phi_{air} - \Delta\phi_{vacuum} = \frac{2\pi}{\lambda}2d(n_{air} - n_{vacuum}) = 2\pi(m_1 - m_2)$$

Since $n_{vacuum} = 1$, we have

$$n_{air} = 1 + (m_1 - m_2)\frac{\lambda}{2d},$$

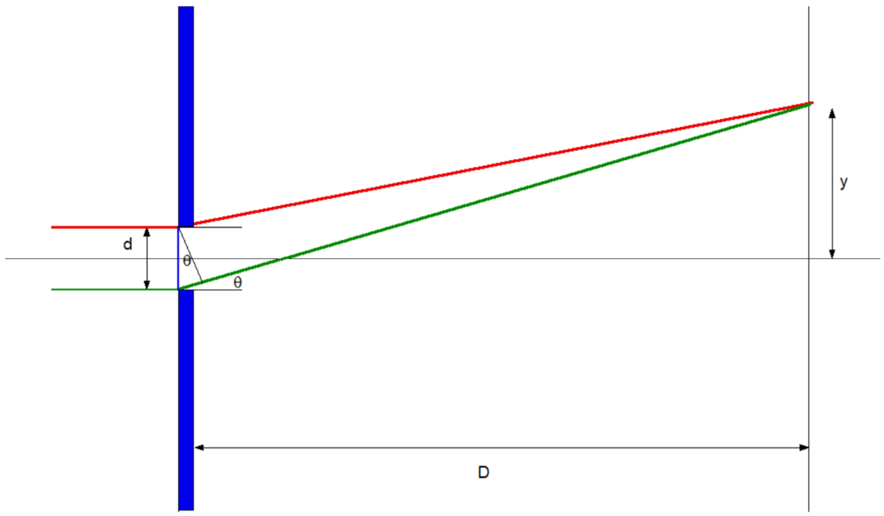
or

$$n_{air} = 1 + 60\frac{\lambda}{2d} = 1 + 3.0\times 10^{-4} = 1.00030 \ .$$

10.4 Problem 35-89

A double-slit arrangement produces bright interference fringes for sodium light (λ = 589 nm) that are angularly separated by 0.30º near the center of the pattern. What is the angular fringe separation if the entire arrangement is immersed in water, which has an index of refraction of 1.33?

((**Solution**))
λ = 589 nm
n = 1.33



$$\phi = \frac{2\pi}{\lambda}(d \sin\theta) = 2\pi m \tag{1}$$
$$d \sin\theta = m\lambda$$

$$\phi_n = \frac{2\pi}{\lambda}(nd \sin\theta') = 2\pi m' \tag{2}$$
$$nd \sin\theta' = m'\lambda$$

From Eq.(1),

$$d \cos\theta\Delta\theta = \lambda\Delta m$$
$$d\Delta\theta = \lambda\Delta m$$

where $\cos\theta \approx 1$.

From Eq,(2)

$$nd \cos\theta'\Delta\theta' = \lambda\Delta m'$$
$$nd\Delta\theta' = \lambda\Delta m'$$

where $\cos\theta' \approx 1$.

Then we have

$$\Delta\theta = \frac{\lambda}{d}\Delta m = \frac{\lambda}{d} = 0.30°,$$

for $\Delta m = 1$.

$$\Delta\theta' = \frac{\lambda}{nd}\Delta m' = \frac{\lambda}{nd} = 0.30° \times \frac{1}{1.33} = 0.23°,$$

for $\Delta m' = 1$.

APPENDIX: Josephson junction and DC SQUID

A1. Introduction

For superconducting tunnel junctions with extremely thin insulating layers (10 – 15 Å) (weak link between the superconductors), the electron pair correlations extend through the insulating barrier. In this situation, it has been predicted by Josephson that paired electrons (Cooper pairs) can tunnel without dissipation from one superconductor to the other superconductor on the opposite side of the insulating layer [B.D. Josephson, Phys. Lett. 1, 251 (1962). The direct supercurrent of pairs, for currents less that I_c, flows with zero voltage drop across the junction (DC Josephson effect). The width of the insulating barrier of the junction limits the maximum that can flow across the junction, but introduce no resistance in the flow. Josephson also predicted that in the case a constant finite voltage V is established across the junction, an alternating supercurrent $I_c \sin(\omega_J t+\phi_0)$ flows with frequency $\omega_J = 2eV/\hbar$ (AC Josephson effect

Tunneling if Cooper pairs form a superconductor through a layer of insulator into another superconductor. Such a junction is called a weak link.

(i) DC Josephson effect
A DC current flows across the junction in the absence of any electric or magnetic field.

(ii) AC Josephson effect
A DC voltage applied across the junction causes rf (radio frequency) current oscillation across the junction.

(iii) Macroscopic long range quantum interference
A DC magnetic field applied through a superconducting circuit containing two junctions causes the maximum supercurrent to show interference effects as a function of magnetic field intensity.

((Brian D. Josephson))

Josephson, Pippard's graduate student at Cambridge, attending Philip Anderson's lectures there in 1961 to 1962, became fascinated by the concept of the pgase of the BCS-GL order parameter as a manifestation of the quantum theory on a macroscopic scale. Playing with the theory of Giaver tunneling, Josephson found a phase-dependent term in the current; he then worked out all the consequences in a series of papers, private letters, and a privately circulated fellowship thesis. In particular, Jpsephson predicted that a direct current should flow, without any applied voltage, between two superconductors separated by a thin insulating layer. This current would come as a *cones quence of the tunneling of electron pairs between the superconductors, and the current would be proportional to the sine of the phase difference between the superconductors. At a finite applied voltage V, an alternating supercurrent of frequency 2eV/h should flow between the superconductors. Josephson's work established the phase as a fundamental variable in superconductivity.*(Book edited by Hoddeson et al.[12]).

A2. DC Josephson junction[3]

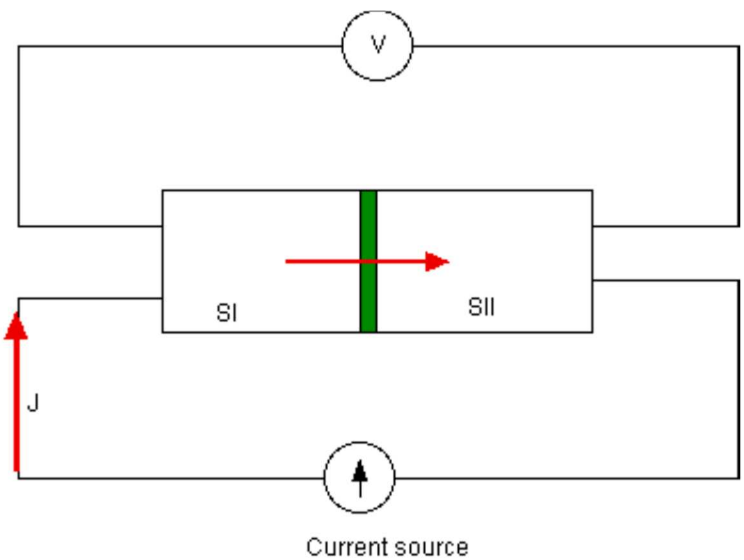


Fig.1 Schematic diagram for experiment of DC Josephson effect. Two superconductors SI and SII (the same metals) are separated by a very thin insulating layer (denoted by green). A DC Josphson supercurrent (up to a maximum value I_c) flows without dissipation through the insulating layer.

A3. SQUID (superconducting quantum interference device)[3]

A.3.1 Current density and flux quantization

In quantum mechanics, the current density is defined as

$$\boldsymbol{J} = \frac{q\hbar}{2mi}[\psi^*\nabla\psi - \psi\nabla\psi^*] - \frac{q^2|\psi|^2}{mc}\boldsymbol{A},$$

where q (=-2e, e>0) is a charge for electron pairs, m is a mass, $\boldsymbol{A}$ is a vector potential, and ψ is a wavefunction. When the wavefunction is given by the amplitude $|\psi(\boldsymbol{r})|$and the phase $\theta(\boldsymbol{r})$ as

$$\psi = |\psi(\boldsymbol{r})|e^{i\theta(r)},$$

then $\boldsymbol{J}$ can be rewritten as

$$J = \frac{q\hbar}{m}|\psi|^2 (\nabla\theta - \frac{q}{c\hbar}A)\,.$$

Note that this current density is invariant under the gauge transformation. $\mathbf{A}' = \mathbf{A} + \nabla\chi$ and $\theta' = \theta + q\chi / c\hbar$,

$$J' = \frac{q\hbar}{m}|\psi|^2 (\nabla\theta' - \frac{q}{c\hbar}A') = \frac{q\hbar}{m}|\psi|^2 (\nabla\theta' - \frac{q}{c\hbar}A') = \frac{q\hbar}{m}|\psi|^2 (\nabla\theta - \frac{q}{c\hbar}A)\,,$$

Where $\psi'(\boldsymbol{r}) = e^{iq\chi/c\hbar}\psi(\boldsymbol{r}) = |\psi(\boldsymbol{r})| e^{i[\theta(\boldsymbol{r}) + q\chi/c\hbar]}$.

If we consider now a cylinder which may become superconductor in an external magnetic field and if we take a path from a surface at a distance which is larger than the penetration depth λ, then $\boldsymbol{J} = 0$. When $q = -2e$, we have

$$J = -\frac{2e\hbar}{m}|\psi|^2 (\nabla\theta + \frac{2e}{c\hbar}A) = 0\,,$$

or

$$\nabla\theta = -\frac{2e}{c\hbar}A\,,$$

$$\oint \nabla\theta\cdot d\boldsymbol{l} = -\frac{2e}{c\hbar}\oint \boldsymbol{A}\cdot d\boldsymbol{l} = -\frac{2e}{c\hbar}\oint \nabla\times\boldsymbol{A}\cdot d\boldsymbol{a} = -\frac{2e}{c\hbar}\oint \boldsymbol{B}\cdot d\boldsymbol{a} = -\frac{2e}{c\hbar}\Phi = -2\pi\frac{\Phi}{\Phi_0}\,,$$

where Φ is the magnetic flux inside the ring and $\Phi_0 = 2\pi\hbar c/(2e)$ (=2.06783372 x 10^{-7} Gauss cm^2) is a quantum fluxoid. In the last equation we apply the Stoke's theorem.

((**Note**))
The current flows along the ring. However, this current flows only on the surface boundary (region from the surface to the penetration depth λ). Inside of the system (region far from the surface boundary), there is no current since $\nabla\times\boldsymbol{H} = 4\pi\boldsymbol{J}/c$ and $\boldsymbol{H} = 0$.

A.3.2 DC SQUID (double junctions): quantum mechanics

DC SQUID consists of two points contacts in parallel, forming a ring. Each contact forms a Josephson junctions of superconductor 1, insulating layer, and superconductor 2 (S_1-I-S_2). Suppose that a magnetic flux Φ passes through the interior of the loop.

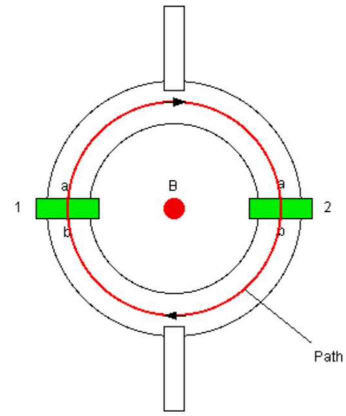


Fig. Schematic diagram of DC SQUID (superconducting quantum interference device). δ_1 and δ_2 refer to two point-contact weak links. The rest of the circuit is strongly superconducting.

Here we have

$$\oint \nabla\theta \cdot d\boldsymbol{l} = \theta_{2a} - \theta_{1a} + \theta_{1b} - \theta_{2b}.$$

or

$$\theta_{2a} - \theta_{1a} + \theta_{1b} - \theta_{2b} = 2\pi \frac{\Phi}{\Phi_0}$$

or

$$\delta_1 - \delta_2 = 2\pi \frac{\Phi}{\Phi_0}$$

where $\delta_1 (= \theta_{1b} - \theta_{1a})$ is the phase difference between the superconductors *a* and *b* through the junction 1 and $\delta_2 = (\theta_{2b} - \theta_{2a})$ are is the phase difference between the superconductors *a* and *b* through the junction 2.

When $\boldsymbol{B} = 0$ (or $\Phi = 0$), we have $\delta_1 - \delta_2 = 0$. In general, we put the form

$$\delta_1 = \delta_0 + \frac{e}{\hbar c}\Phi , \qquad \delta_2 = \delta_0 - \frac{e}{\hbar c}\Phi .$$

The total current is given by

$$I = I_1 + I_2 = I_c[\sin(\delta_1) + \sin(\delta_2)]$$
$$= I_c[\sin(\delta_0 + \frac{e}{\hbar c}\Phi) + \sin(\delta_0 - \frac{e}{\hbar c}\Phi)]$$
$$= 2I_c \sin(\delta_0)\cos(\frac{e}{\hbar c}\Phi)$$

or

$$I = 2I_c \sin(\delta_0)\cos(\pi\frac{\Phi}{\Phi_0}).$$

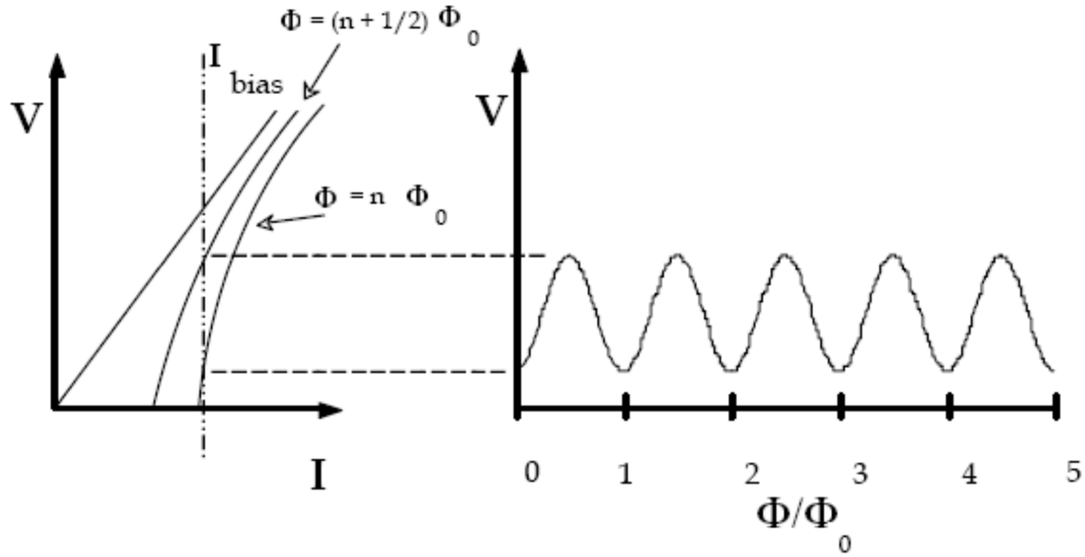
The current varies with Φ and has a maximum of $2I_c$ when $\frac{e}{\hbar c}\Phi = s\pi$ (s: integers), or

$$\Phi = \frac{\hbar c\pi}{e}s = \frac{hc}{2e}s = \Phi_0 s.$$

The simple two point contact device corresponds to a two-slit interference pattern, for which the physically interesting quantity is the modulus of the amplitude rather than the square modulus, as it is for optical interference patterns.

((**Note**)) **Use of DC SQUID to measure the magnetic flux**

We find that the critical current periodically changes as a function of Φ/Φ_0. The critical current is equal to $2I_c$ for $\Phi/\Phi_0 = n$, while it equal to zero for $\Phi/\Phi_0 = n+1/2$. In the real system, the critical current is finite for $\Phi/\Phi_0 = n+1/2$. As shown in Fig., the I-V curve for $\Phi/\Phi_0 = n$ is rather different from that for $\Phi/\Phi_0 = n+1/2$. When the total current I is fixed, it follows that the voltage across the DC SQUID periodically changes as a function of Φ/Φ_0. Using this principle, the value of Φ can be exactly determined by $n\Phi_0$ within the error of $\Delta n = \pm 1$, where n is an integer.



A.3.3 Analogy of the diffraction with double slits and single slit

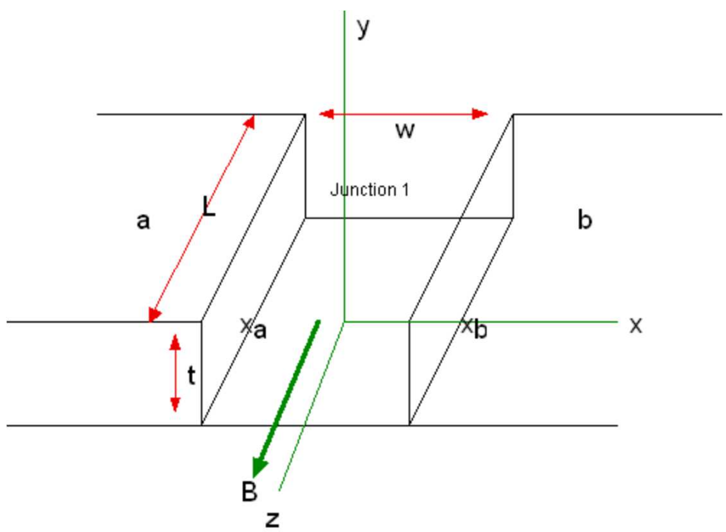


Fig. Diffraction effect of Josephson junction. A magnetic field ***B*** along the z direction, which is penetrated into the junction (in the normal phase).

We consider a junction (1) of rectangular cross section with magnetic field ***B*** applied in the plane of the junction, normal to an edge of width *w*.

$$J = J_0 \sin[\delta_1 + \frac{q}{\hbar c}\int_1^2 \boldsymbol{A}\cdot d\boldsymbol{l}],$$

weith $q = -2e$. We use the vector potential ***A*** given by

$$\boldsymbol{A} = \frac{1}{2}(\boldsymbol{B} \times \boldsymbol{r}) = (-\frac{By}{2}, \frac{Bx}{2}, 0), \qquad \boldsymbol{A}' = \boldsymbol{A} + \nabla\chi = (-By, 0, 0)$$

Where

$$\chi = -\frac{Bxy}{2}.$$

Then we have

$$J = J_0 \sin[\delta_1 + \frac{q}{\hbar c}\int_{x_a}^{x_b}(-By)dx] = J_0 \sin[\delta_1 - \frac{qB}{\hbar c} yW],$$

$$dI_1 = JLdy = J_0 L \sin[\delta_1 - \frac{qB}{\hbar c} yW]dy,$$

or

$$I_1 = J_0 L \int_{-t/2}^{t/2} \sin[\delta_1 - \frac{qB}{\hbar c} yW]dy.$$

Then we have

$$I_1 = J_0 L \frac{2\hbar c}{BWq} \sin(\delta_1) \sin(\frac{qB}{\hbar c}\frac{t}{2}W).$$

Here we introduce the total magnetic flux passing through the area Wt ($\Phi_W = BWt$), $I_c = J_0 L t$, and

$$\frac{\Phi_W}{\Phi_0} = \frac{BWt}{\frac{2\pi\hbar c}{2e}} = \frac{eBWt}{\pi\hbar c}, \qquad \text{or} \qquad \pi\frac{\Phi_W}{\Phi_0} = \frac{eBWt}{\hbar c}.$$

Therefore we have

$$I_1 = I_c \sin(\delta_1) \frac{\sin(\frac{\pi\Phi_W}{\Phi_0})}{\left(\frac{\pi\Phi_W}{\Phi_0}\right)}.$$

The total current is given by

$$I = I_1 + I_1 = I_c[\sin(\delta_1) + \sin(\delta_1)]\frac{\sin(\frac{\pi\Phi_W}{\Phi_0})}{\left(\frac{\pi\Phi_W}{\Phi_0}\right)},$$

or

$$I = I_1 + I_1 = 2I_c \sin(\delta_0)\cos(\pi\frac{\Phi}{\Phi_0})\frac{\sin(\frac{\pi\Phi_W}{\Phi_0})}{\left(\frac{\pi\Phi_W}{\Phi_0}\right)}.$$

The short period variation is produced by interference from the two Josephson junctions, while the long period variation is a diffraction effect and arises from the finite dimensions of each junction. The interference pattern of $|I|^2$ is very similar to the intensity of the Young's double slits experiment. If the slits have finite width, the intensity must be multiplied by the diffraction pattern of a single slit, and for large angles the oscillations die out.

A.4 Application of SQUID

Magnetic Fields