1. Simple cubic structure (conventional lattice)

Structure of simple cubic (sc) lattice,

\[ \mathbf{a}_x = a(1,0,0), \quad \mathbf{a}_y = a(0,1,0), \quad \mathbf{a}_z = a(0,0,1) \]

\[ \mathbf{a}_x \cdot (\mathbf{a}_y \times \mathbf{a}_z) = a^3 \]

The reciprocal lattice vectors;

\[ \mathbf{b}_x = \frac{2\pi (\mathbf{a}_y \times \mathbf{a}_z)}{\mathbf{a}_x \cdot (\mathbf{a}_y \times \mathbf{a}_z)} = \frac{2\pi}{a} (1,0,0) \]
\[ b_y = \frac{2\pi (a_x \times a_y)}{a} \cdot (a_y \times a_z) = \frac{2\pi}{a} (0,1,0) \]

\[ b_z = \frac{2\pi (a_x \times a_y)}{a_x \cdot (a_y \times a_z)} = \frac{2\pi}{a} (0,1,0) \]

Fig. Reciprocal lattice points of sc. The vectors \( b_x, b_y, \) and \( b_z \) are the reciprocal lattice vectors.

2. **Miller indices of planes for the sc lattice**
Note that the reciprocal lattice vectors

\[ G(h, k, l) = nh_x + kb_y + lc_z, \]

are perpendicular to the plane with the Miller indices \((h, k, l)\). Equation for the plane with the Miller indices \((h, k, l)\) can be expressed by

\[ (r - \frac{a_h}{h}) \cdot G(h, k, l) = 0. \]

or

\[(x - n \frac{a}{h}, y, z) \cdot (\frac{2\pi}{a}h, \frac{2\pi}{a}k, \frac{2\pi}{a}l) = 0\]

or

\[hx + ky + lz = na.\]

(a) (100) plane; \(h = 1, k = 0, \text{ and } l = 0.\)

\[G(1,0,0) = b_x = \frac{2\pi}{a} (1,0,0)\]

\[|G(1,0,0)| = \frac{2\pi}{a}\]

The distance between the adjacent cubic (100) plane is

\[d(100) = \frac{2\pi}{|G(1,0,0)|} = a\]
(100) plane. This figure is drawn using the Mathematica (ContourPlot for the planes and Graphics3D).

(a) Plane: \( h=1, k=1, l=0 \).

\[
G(1,1,0) = b_x + b_y = \frac{2\pi}{a} (1,1,0)
\]

\[
|G(1,1,0)| = \frac{2\pi}{a} \sqrt{2}
\]

The distance between the adjacent cubic (110) plane is

\[
d(110) = \frac{2\pi}{|G(1,1,0)|} = \frac{a}{\sqrt{2}}
\]
(e) \((1\bar{1}0)\) plane; \(h = 1, k = -1,\) and \(l = 0.\)

\[
G(1,\bar{1},0) = b_x - b_y = \frac{2\pi}{a} (1, -1, 0)
\]

\[
|G(1,\bar{1},0)| = \frac{2\pi}{a} \sqrt{2}
\]
(d) (111) plane; \( h = 1, k = 1, \) and \( l = 1. \)

\[
G(1,1,1) = b_x + b_y + b_z = \frac{2\pi}{a} (1,1,1)
\]

\[
|G(1,1,1)| = \frac{2\pi}{a} \sqrt{3}
\]

The distance between the adjacent cubic (111) plane is given by

\[
d(111) = \frac{2\pi}{|G(1,1,1)|} = \frac{a}{\sqrt{3}}
\]
(e) (200) plane; \( h = 2, k = 0, \text{ and } l = 0. \)

\[ G(2,0,0) = 2b_x = \frac{2\pi}{a} (2,0,0) \]

\[ |G(2,0,0)| = \frac{2\pi}{a} 2 \]

The distance between the adjacent cubic (200) plane is given by

\[ d(200) = \frac{2\pi}{|G(2,0,0)|} = \frac{a}{2} \]
(f) (210) plane; \( h = 2, \ k = 1, \) and \( l = 0. \)

\[
G(2,1,0) = 2b_x + b_y = \frac{2\pi}{a} (2,1,0)
\]

\[
|G(2,1,0)| = \frac{2\pi}{a} \sqrt{5}
\]

The distance between the adjacent cubic (210) plane is given by

\[
d(210) = \frac{2\pi}{|G(2,1,0)|} = \frac{a}{\sqrt{5}}
\]
((Note))
For scc structure, it is required that

\[ g_x^2 + g_y^2 + g_z^2 = 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 16, 17, 18, 19, 20, 21, 22, 
24, 25, 26, 27, 29, 30, 32, \ldots \]

---

3. **face-centered cubic (fcc) lattice**

The primitive cell by definition has only one lattice point, but the conventional fcc cell contains four lattice points. Note that the lattice point is defined as follows. The atomic arrangement in the crystal looks exactly the same to an observer at \( r' \) (one lattice point) as to an observer at \( r \) (another lattice point).
Top view from the (111) direction in fcc structure. The red line indicates the direction of (111).
The primitive translation vectors of the fcc lattice are expressed by

\[ \mathbf{a}_1 = \frac{1}{2} a(0,1,1), \quad \mathbf{a}_2 = \frac{1}{2} a(1,0,1), \quad \mathbf{a}_3 = \frac{1}{2} a(1,1,0) \]

where there is one atom per this primitive cell. We can generate all the points of the fcc lattice is described by

\[ \mathbf{l} = l_1 \mathbf{a}_1 + l_2 \mathbf{a}_2 + l_3 \mathbf{a}_3 \]

with \( l_1, l_2, \) and \( l_3 \) integers. The volume of the primitive cell is

\[ \mathbf{a}_1 \cdot (\mathbf{a}_2 \times \mathbf{a}_3) = \frac{a^3}{4} \]

The corresponding reciprocal lattice vectors are given by
The reciprocal lattice vector is described by
\[ G = g_1 \mathbf{b}_1 + g_2 \mathbf{b}_2 + g_3 \mathbf{b}_3 = \frac{2\pi}{a} (-g_1 + g_2 + g_3, g_1 - g_2 + g_3, g_1 + g_2 - g_3) \]

where \( g_1, g_2, \) and \( g_3 \) are integers.

The translation vectors of the conventional unit cell (cubic) are expressed by
\[ \mathbf{a}_x = a(1,0,0), \quad \mathbf{a}_y = a(0,1,0), \quad \mathbf{a}_z = a(0,0,1) \]

where there are two atoms per this conventional unit cell. The volume of the cubic cell is
\[ \mathbf{a}_x \cdot (\mathbf{a}_y \times \mathbf{a}_z) = \frac{a^3}{4} \]

The reciprocal lattice vectors are defined by
\[ \mathbf{b}_x = \frac{2\pi (\mathbf{a}_y \times \mathbf{a}_z)}{\mathbf{a}_x \cdot (\mathbf{a}_y \times \mathbf{a}_z)} = \frac{2\pi}{a} (1,0,0) \]
\[ \mathbf{b}_y = \frac{2\pi (\mathbf{a}_z \times \mathbf{a}_x)}{\mathbf{a}_x \cdot (\mathbf{a}_y \times \mathbf{a}_z)} = \frac{2\pi}{a} (0,1,0) \]
\[ \mathbf{b}_z = \frac{2\pi (\mathbf{a}_x \times \mathbf{a}_y)}{\mathbf{a}_x \cdot (\mathbf{a}_y \times \mathbf{a}_z)} = \frac{2\pi}{a} (0,0,1) \]
In general, the reciprocal lattice vector is given by

\[ \mathbf{G} = g_x \mathbf{b}_x + g_y \mathbf{b}_y + g_z \mathbf{b}_z = \frac{2\pi}{a}(g_x, g_y, g_z) \]

with

\[ g_x = g_1 + g_2 + g_3 \]
\[ g_y = g_1 - g_2 + g_3 \]
\[ g_z = g_1 + g_2 - g_3 \]

There are relations between \((g_x, g_y, g_z)\) and \((g_1, g_2, g_3)\). Note that all indices of \((g_x, g_y, g_z)\) are odd or even. There is a selection rule for the indices \((g_x, g_y, g_z)\).

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Selection rule for the indices \((g_1, g_2, g_3)\) for fcc.

((Note))

For fcc structure, it is required that

\[ g_x^2 + g_y^2 + g_z^2 = 3, 4, 8, 11, 12, 16, 19, 20, 24, 27, 32, 35, 40, 44, 48 \]
The reciprocal lattice vectors, $b_1$, $b_2$, and $b_3$ for the primitive cell and $b_x$, $b_y$, and $b_z$ for the cubic cell (conventional cell) for the fcc lattice. The reciprocal lattice points (denoted by solid blue circles) are located on a bcc lattice with
4. Miller indices of planes for the fcc lattice

(a) (111) cubic plane (conventional)

\[ G = b_x + b_y + b_z = \frac{2\pi}{a} (1,1,1), \quad \text{or} \quad G = b_1 + b_2 + b_3 \]

\[ |G| = \frac{2\pi}{a} \sqrt{3} \]

The distance between the adjacent (111) plane is

\[ d(100) = \frac{2\pi}{|G|} = \frac{a}{\sqrt{3}} \]
We consider a plane which passes through \((n \mathbf{a}_i/1)\) and is perpendicular to \(\mathbf{G}\).

\[(\mathbf{r} - n\mathbf{a}_i/1) \cdot \mathbf{G} = 0\]

where \(n\) is an integer.

\((111)\) cubic plane for fcc

\((\text{Another method})\) Primitive cell

\[\mathbf{G} = \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3\]

We consider a plane which passes through \((n \mathbf{a}_i/1)\) and is perpendicular to \(\mathbf{G}\).

\[(\mathbf{r} - n\mathbf{a}_i/1) \cdot \mathbf{G} = 0\]

where \(n\) is an integer. This plane is the same as that obtained above.
(b) (200) cubic plane

\[
G = 2b_x = -\frac{2\pi}{a} (2,0,0) \quad \text{or} \quad G = b_2 + b_3
\]

\[
|G| = \frac{2\pi}{a} 2
\]

The distance between the adjacent cubic (200) plane is

\[
d(200) = \frac{2\pi}{|G|} = \frac{a}{2}
\]

We consider a plane which passes through \((n a_x/2)\) and is perpendicular to \(G\),

\[
(r - n a_x / 2) \cdot G = 0
\]

where \(n\) is an integer.
((Another method)) Primitive cell

\[ G = b_2 + b_3 \]

We consider a plane which passes through \((n a_3/1)\) and is perpendicular to \(G\).

\[ (r - n a_3/1) \cdot G = 0 \]

where \(n\) is an integer. This plane is the same as that obtained above.

(c) (220) cubic plane

\[ G = 2b_x + 2b_y = \frac{2\pi}{a} (2,2,0) \] or \[ G = b_1 + b_2 + 2b_3 \]

\[ |G| = \frac{2\pi}{a} \frac{2\sqrt{2}}{} \]

The distance between the adjacent cubic (220) plane is

\[ d(220) = \frac{2\pi}{|G|} = \frac{a}{2\sqrt{2}} \]

We consider a plane which passes through \((n a_3/2)\) and is perpendicular to \(G\).

\[ (r - n a_x/2) \cdot G = 0 \]

where \(n\) is an integer.
21 cubic plane for fcc

(220) cubic plane for fcc

((Another method)) Primitive cell

\[ \mathbf{G} = \mathbf{b}_1 + \mathbf{b}_2 + 2\mathbf{b}_3 \]

We consider a plane which passes through \((n\mathbf{a}_1/1)\) and is perpendicular to \(\mathbf{G}\).

\[ (\mathbf{r} - n\mathbf{a}_I/1) \cdot \mathbf{G} = 0 \]

This plane is the same as that obtained above.

5. body-centered cubic (bcc) structure

The primitive cell by definition contains only one lattice point, but the conventional bcc cell contains two lattice points.
The primitive translation vectors of the bcc lattice are expressed by

\[ a_1 = \frac{1}{2} a(-1,1,1), \quad a_2 = \frac{1}{2} a(1,-1,1), \quad a_3 = \frac{1}{2} a(1,1,-1) \]

where there is one atom per this primitive cell. We can generate all the points of the bcc lattice is described by

\[ l = l_1 a_1 + l_2 a_2 + l_3 a_3 \]

with \( l_1, l_2, \) and \( l_3 \) integers. The volume of the primitive cell is

\[ a_1 \cdot (a_2 \times a_3) = \frac{a^3}{2} \]

The corresponding reciprocal lattice vectors are given by
The reciprocal lattice vector is described by

$$G = g_1 b_1 + g_2 b_2 + g_3 b_3 = \frac{2\pi}{a} (g_2 + g_3, g_1 + g_3, g_1 + g_2)$$

where $g_1$, $g_2$, and $g_3$ are integers.

The translation vectors of the conventional unit cell are expressed by

$$a_x = a(1,0,0), \quad a_y = a(0,1,0), \quad a_z = a(0,0,1)$$

where there are two atoms per this conventional unit cell. The reciprocal lattice vectors are defined by

$$b_x = \frac{2\pi(a_x \times a_y)}{a_x \cdot (a_x \times a_y)} = \frac{2\pi}{a} (1,0,0)$$

$$b_y = \frac{2\pi(a_y \times a_z)}{a_y \cdot (a_y \times a_z)} = \frac{2\pi}{a} (0,1,0)$$

$$b_z = \frac{2\pi(a_z \times a_x)}{a_z \cdot (a_z \times a_x)} = \frac{2\pi}{a} (0,1,0)$$

The reciprocal lattice vector is given by

$$G = g_x b_x + g_y b_y + g_z b_z = \frac{2\pi}{a} (g_x, g_y, g_z)$$
with
\[ g_x = g_2 + g_3 \]
\[ g_y = g_1 + g_3 \]
\[ g_z = g_1 + g_2 \]

There are relations between \((g_x, g_y, g_z)\) and \((g_1, g_2, g_3)\). Note that

\[ g_x + g_y + g_z = 2(g_1 + g_2 + g_3) \]

which is even.

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Selection rule for the indices \((g_x, g_y, g_z)\) for the cubic bcc

For bcc structure, it is required that

\[ g_x^2 + g_y^2 + g_z^2 = 2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, ' \]
The reciprocal lattice vectors, \( b_1, b_2, \) and \( b_3 \) for the primitive cell and \( b_x, b_y, \) and \( b_z \) for the cubic cell (conventional cell) for the bcc lattice. The reciprocal lattice points (denoted by solid blue circles) are located on a fcc lattice with
6. Miller indices of planes for the bcc lattice

(a) (110) cubic plane

\[ \mathbf{G} = b_x + b_y = \frac{2\pi}{a} (1,1,0), \quad \text{or} \quad \mathbf{G} = b_3 \]

\[ |\mathbf{G}| = \frac{2\pi}{a} \sqrt{2} \]

The distance between the adjacent cubic (110) plane is

\[ d(110) = \frac{2\pi}{|\mathbf{G}|} = \frac{a}{\sqrt{2}} \]

We consider a plane which passes through \((n a_x/1)\) and is perpendicular to \(\mathbf{G}\).

\[ (\mathbf{r} - n a_x/1) \cdot \mathbf{G} = 0 \]

where \(n\) is an integer.
(110) cubic plane for bcc

(Another method)) Primitive cell

\[ \mathbf{G} = \mathbf{b}_3 \]

We consider a plane which passes through \((n \, \mathbf{a}_3/1)\) and is perpendicular to \(\mathbf{G}\).

\[ (\mathbf{r} - n\mathbf{a}_3/1) \cdot \mathbf{G} = 0 \]

where \(n\) is an integer. This plane is the same as that obtained above.

(b) (200) cubic plane

\[ \mathbf{G} = 2\mathbf{b}_x = \frac{2\pi}{a} (2,0,0), \quad \text{or} \quad \mathbf{G} = -\mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3 \]

\[ |\mathbf{G}| = \frac{2\pi}{a} \frac{2}{2} \]
The distance between the adjacent cubic (200) plane is

\[ d(200) = \frac{2\pi}{|G|} = \frac{a}{2} \]

We consider a plane which passes through \((n \, a_x/2)\) and is perpendicular to \(G\).

\[ (r - n a_x/2) \cdot G = 0 \]

where \(n\) is an integer.
((Another method)) Primitive cell

\[ G = -b_1 + b_2 + b_3 \]

We consider a plane which passes through \((n a_3/1)\) and is perpendicular to \(G\).

\[ (r - na_3/1) \cdot G = 0 \]

where \(n\) is an integer. This plane is the same as that obtained above.

(e) (211) plane (cubic)

\[ G = 2b_x + b_y + b_z = \frac{2\pi}{a}(2,1,1), \quad \text{or} \quad G = b_2 + b_3 \]

\[ |G| = \frac{2\pi}{a} \sqrt{5} \]

The distance between the adjacent cubic (211) plane is

\[ d(211) = \frac{2\pi}{|G|} = \frac{a}{\sqrt{5}} \]

We consider a plane which passes through \((n a_3/2)\) and is perpendicular to \(G\).

\[ (r - na_3/2) \cdot G = 0 \]

where \(n\) is an integer.
(211) cubic plane for bcc

\textbf{((Another method))} Primitive cell

\[ G = b_2 + b_3 \]

We consider a plane which passes through \((n a_3/1)\) and is perpendicular to \(G\).

\[(r - n a_3 /1) \cdot G = 0\]

where \(n\) is an integer.

\textbf{(d) (222) cubic plane}

\[ G = 2b_1 + 2b_2 + 2b_3 = \frac{2\pi}{a}(2,2,2) \quad \text{or} \quad G = b_1 + b_2 + b_3 \]
\[ |G| = \frac{2\pi}{a} \cdot 3\sqrt{2} \]

The distance between the adjacent cubic (222) plane is

\[ d(222) = \frac{2\pi}{|G|} = \frac{a}{3\sqrt{2}} \]

We consider a plane which passes through \((n a_x/2)\) and is perpendicular to \(G\).

\[
(r - n a_x/2) \cdot G = 0
\]

where \(n\) is an integer.
((Another method)) Primitive cell

\[ \mathbf{G} = \mathbf{b}_1 + \mathbf{b}_2 + \mathbf{b}_3 \]

We consider a plane which passes through \((n \mathbf{a}_1/1)\) and is perpendicular to \(\mathbf{G}\).

\[(r - na_1/1) \cdot \mathbf{G} = 0\]

where \(n\) is an integer. This plane is the same as that obtained above.

REFERENCES

Appendix
Miller indices and reciprocal lattice vector

The reciprocal lattice vector \(\mathbf{G}\) is expressed by

\[ \mathbf{G} = k\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3. \]

Then we have

\[ \mathbf{G} \cdot \mathbf{T} = (h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3) \cdot (u_1\mathbf{a}_1 + u_2\mathbf{a}_2 + u_3\mathbf{a}_3) = 2\pi(hu_1 + ku_2 + lu_3) \]

Index of planes

\((hkl)\) plane

Consider the \((hkl)\) plane.

\((hkl)\) are the smallest three integers (Miller indices).

(1) The reciprocal lattice vector is defined by
\[ \mathbf{G} = h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3. \]

\(G\) is perpendicular to the \((hkl)\) plane.

((Proof))

Fig.12 Definition of \((hkl)\) plane where \(h\), \(k\), and \(l\) are the smallest three integers.

First we find the intercepts on the axes in terms of the lattice constants \(a_1\), \(a_2\), and \(a_3\): \(a_1/h\), \(a_2/k\), \(a_3/l\) (see the above figure). We take the reciprocals of these numbers and then reduces to three integers having the same ratio, usually the smallest three integers: \((hkl)\). These indices \((hkl)\) may denote a single phase or a set of parallel planes. If a plane cuts an axis on the negative side of the origin, the corresponding index is negative, indicated by placing a minus sign above the index \((h\bar{k}\bar{l})\).

Take the reciprocal
These two vectors are perpendicular to \( \mathbf{G} \).

\[
\mathbf{HK} = \frac{\mathbf{a}_2}{k} - \frac{\mathbf{a}_1}{h}
\]

\[
\mathbf{HK} = \frac{\mathbf{a}_1}{l} - \frac{\mathbf{a}_2}{k}
\]

by using the relations

\[
\mathbf{a}_1 \cdot \mathbf{b}_1 = \mathbf{a}_2 \cdot \mathbf{b}_2 = \mathbf{a}_3 \cdot \mathbf{b}_3 = 2\pi
\]

Then the \((hkl)\) plane is perpendicular to \( \mathbf{G} \).

(2) The distance between two parallel adjacent \((hkl)\) planes is

\[
d(hkl) = \frac{2\pi}{|\mathbf{G}|} \quad (\text{nearest neighbor distance})
\]

where \((hkl)\) indices are the smallest integers.
Fig. 13 Adjacent \((hkl)\) planes.

Fig. 14 The nearest neighbor distance between the adjacent \((hkl)\) planes.

\(n (hkl)\) plane
\[
\frac{1}{n} : \frac{1}{n} : \frac{1}{n} = \frac{h}{n} : \frac{k}{n} : \frac{l}{n}
\]

adjacent \((n+1)\ (hkl)\) plane

\[
\frac{1}{n+1} : \frac{1}{n+1} : \frac{1}{n+1} = \frac{h}{n+1} : \frac{k}{n+1} : \frac{l}{n+1}
\]

\[
\mathbf{n} = \frac{\mathbf{G}}{G}
\]

Since \((hkl)\) plane is perpendicular to \(\mathbf{G}\),

\[
d(hkl) = \frac{1}{h} \mathbf{a}_h \cdot \mathbf{n} = \frac{1}{h} \mathbf{a}_h \cdot \frac{\mathbf{G}}{G} = \frac{2\pi}{G}
\]

or

\[
d(hkl) = \frac{2\pi}{G}
\]

What is the separation distance between the \(n(hkl)\) plane and \((n+m)\ (hkl)\) plane?
Fig. 15 Two \((hkl)\) planes.

\[
d_m = \frac{1}{h} m a_i \cdot n = \frac{1}{h} m a_i \cdot \frac{G}{G} = \frac{2m}{G} = md(hkl)\).
\]
(i) Pick up one of three vectors as a vector $\mathbf{a}$.

$$\mathbf{a} = n \frac{\mathbf{a}_1}{h}, \quad \mathbf{a} = n \frac{\mathbf{a}_2}{k}, \quad \mathbf{a} = n \frac{\mathbf{a}_3}{l},$$

(ii) Set up an equation for the plane with the Miller indices $(h, k, l)$.

$$(\mathbf{r} - \mathbf{a}) \cdot \mathbf{G}(h, k, l) = 0$$

APPENDIX

Consideration-1

(100) plane and (200) plane

(a) Intercept $\mathbf{a}_1 \rightarrow \mathbf{a} \times (1/1 \ 0 \ 0) \rightarrow (100)$ plane with $n = 1$
What is another (100) plane?

Intercept $2\mathbf{a}_1 \rightarrow 2 \times \mathbf{a}_1 \rightarrow$ (100) plane with $n = 2$

Intercept $3\mathbf{a}_3 \rightarrow 3 \times \mathbf{a}_1 \rightarrow$ (100) plane with $n = 3$

(b) Intercept $\mathbf{a}_{1/2} \rightarrow \frac{1}{2} \times (1/(1/2) 0 0) \rightarrow (200)$ plane with $n = 1$

What is another (200) plane?

Intercept $\mathbf{a}_1 \rightarrow 2 \times \mathbf{a}_{1/2}$

(200) plane with $n = 2$.

(100) plane with $n = 1$

Intercept $3\mathbf{a}_{1/2} \rightarrow 3 \times \mathbf{a}_{1/2}$

(200) plane with $n = 3$.

Intercept $2\mathbf{a}_1 \rightarrow 4 \times \mathbf{a}_{1/2}$

(200) plane with $n = 4$.

(100) plane with $n = 2$
Intercept $5a_1/2 \rightarrow 5 \times a_1/2$

(200) plane with $n = 5$.

((Consideration-2))

(001) plane and (002) plane
Intercept $a_3 \rightarrow 1 \times (00\text{ }1/1) \rightarrow (001) \text{ plane with } n = 1$

What is another (001) plane?

Intercept $2a_3 \rightarrow 2 \times a_3 \rightarrow (001) \text{ plane with } n = 2$

Intercept $3a_3 \rightarrow 3 \times a_3 \rightarrow (001) \text{ plane with } n = 3$

Intercept $a_3/2 \rightarrow (00\text{ }2) \rightarrow (002) \text{ plane}$

What is another (002) plane?

Intercept $a_1 \rightarrow 2 \times a_3/2$
(002) plane with $n = 2$.
(001) plane with $n = 1$

Intercept $3a_{3/2} \rightarrow 3 \times a_{3/2}$

(002) plane with $n = 3$.

Intercept $2a_3 \rightarrow 4 \times a_{3/2}$

(002) plane with $n = 4$.
(001) plane with $n = 2$

Intercept $5a_{3/2} \rightarrow 5 \times a_{3/2}$

(002) plane with $n = 5$. 