Aharonov-Casher effect
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1. Aharonov-Casher effect

The **Aharonov–Casher effect** is a quantum mechanical phenomenon predicted in 1984 in which a traveling magnetic dipole is affected by an electric field. It is dual to the Aharonov–Bohm effect, in which the quantum phase of a charged particle depends upon which side of a magnetic flux tube it comes through. In the Aharonov–Casher effect, the particle has a magnetic moment and the tubes are charged instead. It was observed in a gravitational neutron inferometer in 1989 (Cimmino et al.) and later by fluxon interference of magnetic vortices in Josephson junctions (Elion et al.). It has also been seen with electrons and atoms. In both effects the particle acquires a phase shift (φ) while traveling along some path *P*. In the Aharonov–Bohm effect it is

$$\phi_{AB} = \frac{q}{\hbar}\int \boldsymbol{A}\cdot d\boldsymbol{r}$$

While for the Aharonov–Casher effect it is

$$\phi_{AC} = \frac{1}{\hbar c^2}\int (\boldsymbol{E}\times\boldsymbol{\mu})\cdot d\boldsymbol{r}$$

where q is its charge and $\boldsymbol{\mu}$ is the magnetic moment.

2. Feynman path integral

According to Aharonov-Chasee, the Lagrangian of the magnetic moment which moves in the presence of an electric field produced by a long charged line (with the line charge density λ) is given by

$$L = \frac{1}{2}M\boldsymbol{v}^2 - \frac{1}{c}\boldsymbol{v}\cdot(\boldsymbol{E}\times\boldsymbol{\mu})$$

where $\boldsymbol{E}$ is the electric field. We now apply the Feynman path integral

$$\psi = \exp(\frac{i}{\hbar}\int Ldt) \approx \exp[-\frac{i}{c\hbar}\int \boldsymbol{v}\cdot(\boldsymbol{E}\times\boldsymbol{\mu})dt]$$

We note that

$$\boldsymbol{B} = \frac{1}{c}(\boldsymbol{E} \times \boldsymbol{v})$$

$$\frac{1}{c}\boldsymbol{v} \cdot (\boldsymbol{E} \times \boldsymbol{\mu}) = -\frac{1}{c}\boldsymbol{v} \cdot (\boldsymbol{\mu} \times \boldsymbol{E}) = -\frac{1}{c}\boldsymbol{\mu} \cdot (\boldsymbol{E} \times \boldsymbol{v}) = -\boldsymbol{\mu} \cdot \boldsymbol{B}$$

$$\boxed{\Delta\phi = \frac{1}{\hbar}\int \boldsymbol{\mu} \cdot \boldsymbol{B} dt}$$

We note that

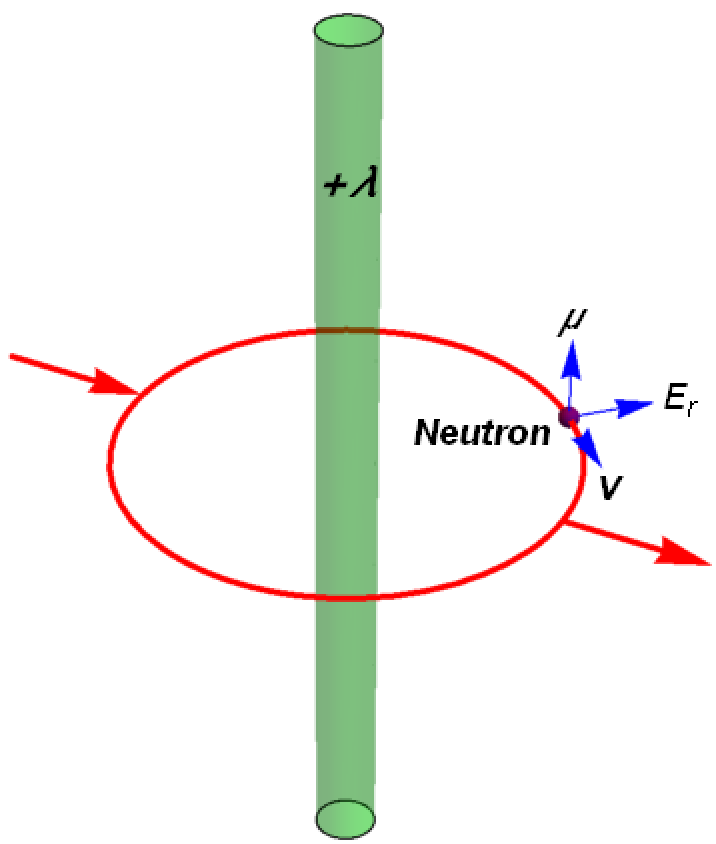


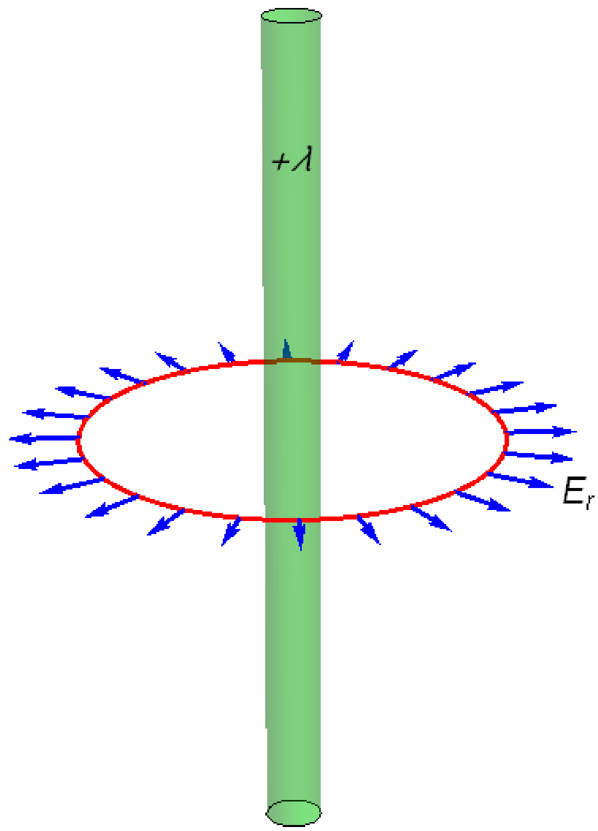
Fig. Aharonov-Casher effect.

The electric field due to the charged line line with the line charge density λ, is derived using the Gauss's law as

$$E_r(2\pi r)\Delta h = 4\pi\lambda\Delta h$$

or

$$E_\phi = \frac{4\pi\lambda}{2\pi r} = \frac{2\lambda}{r}$$



The interaction of the neutron magnetic moment with an electric field is given by the Schwinger interaction as

$$H' = -\boldsymbol{\mu} \cdot \boldsymbol{B}'$$

where the magnetic field $\boldsymbol{B}'$ is given by

$$\boldsymbol{B}' = \frac{1}{c} \boldsymbol{E} \times \boldsymbol{v}$$

which permits the definition of an effective magnetic field since the neutron moves relative to the electric field $\boldsymbol{E}$. The Schwinger interaction is nearly identical to the spin–orbit interaction in atomic physics. For atoms there is a factors of 1/2 which comes from Thomas precession in the accelerated (circular) frame of the electron in an atom. From the analogy of the movement of a charged particle (electron) around a magnetic flux tube and the movement of a magnetic dipole (neutron) around a line of electric charge, Aharonov and Casher (1984) introduced a gauge-invariant Lagrangian which yields for the two polarization states an additional phase shift. It can be calculated from a proper time integral or the path integral of the canonical momentum, namely

$$(\Delta\phi)_{AC} = \frac{1}{\hbar}\oint \boldsymbol{\mu}\cdot\boldsymbol{B}dt$$

Here we note that

$$E_r = \frac{2\lambda}{r}$$

and

$$\begin{aligned}\boldsymbol{B} &= \frac{1}{c}E_r\boldsymbol{e}_r \times (\dot{r}\boldsymbol{e}_r + r\dot{\phi}\boldsymbol{e}_\phi + \dot{z}\boldsymbol{e}_z)\\ &= \frac{1}{c}(E_r r\dot{\phi}\boldsymbol{e}_z - E_r\dot{z}\boldsymbol{e}_\phi)\end{aligned}$$

$$\boldsymbol{\mu}\cdot\boldsymbol{B} = \frac{\mu}{c}E_r r\dot{\phi} = \frac{\mu}{c}\frac{2\lambda}{r}r\dot{\phi} = \frac{2\lambda\mu}{c}\dot{\phi}$$

Then we have

$$(\Delta\phi)_{AC} = \frac{1}{\hbar}\oint \boldsymbol{\mu}\cdot\boldsymbol{B}dt = \frac{1}{\hbar}\oint\frac{2\lambda\mu}{c}\dot{\phi}dt = \frac{2\lambda\mu}{c\hbar}\int_0^{2\pi}d\phi = \frac{4\pi\lambda\mu}{c\hbar}$$

REFERENCES

H. Rauch and S.A. Werner, Neutron Interferometry: Lessons in Experimental Quantum Mechanics, Wave-Particle Duality, and Entanglement, second edition (Oxford, 2000).

Y. Aharonov and A. Casher, Phys. Rev. Letts. 53, 319 (1984).

APPENDIX

Feynman path integral for the Aharonov-Casher effect

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$$
\begin{aligned}
\Psi(\boldsymbol{r}, T) =&(\frac{m}{\mathrm{i}h})^{5/2}\frac{2v}{r_0 r\sqrt{T}}\exp\{\frac{\mathrm{i}}{\hbar}[\frac{mv^2}{2}T-\frac{mgT(z+|z_0|)}{2} \\
&-\frac{mg^2T^3}{12}]\}\cos[\frac{mvd\sin\theta}{2\hbar}+\frac{mgTd\sin\phi}{2\hbar} \\
&-\frac{1}{2\hbar c^2}\oint(\mu\times\boldsymbol{E})\cdot\mathrm{d}\boldsymbol{r}]
\end{aligned}
$$
