Parity violation

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Space reflection in which all three co-ordinates are inverted can also be thought of as rotation by 180° about some axis, followed by a simple mirror reflection on a plane perpendicular to that axis. For symmetry under rotation parity invariance is, therefore, equivalent to mirror invariance. If a certain process is observed, it must be possible to make the mirror image of that process happen. The mirror image of a spinning ^{60}Co nucleus is a nucleus spinning the other way. If we add to these two sketches the directions of some decay electrons, the mirror symmetry is broken, since electrons are emitted preferentially against the ^{60}Co spin direction.
(**S. Brandt, The Harvest of a Century: Discoveries of Modern Physics in 100 Episode (Oxford, 2009**).

Here we discuss the parity violation in the weak interaction of the β-decay of ^{60}Co.Start with polarized parent particles so that the screw sense is defined initially. Then ask whether the distribution of the final-state particles depends on odd power of cosine of the angle between the initial spin direction and the final momentum direction. A classical example of this method from the weak interactions is the β-decay of ^{60}Co. Wu et al (1957) polarized the ^{60}Co at low temperatures, and observed a $(1+aP\cos\theta)$ angular distribution for the electrons. The ^{60}Co polarization P was estimated from the angular distribution (polar vs equatorial anisotropy) of the subsequent γ emission. The asymmetry parameter was unexpectedly large: $a \approx -(v/c)_{cl}$. This experiment of Wu et al. carried out in December 1956 at the National Bureau of Standards, provided the first conclusive evidence that the parity is not conserved in weak decays. The key observation of the parity violation is the radioactive Co nuclei, oriented according to their nuclear spin, emit beta rays (i.e., electrons) preferentially in the opposite direction.

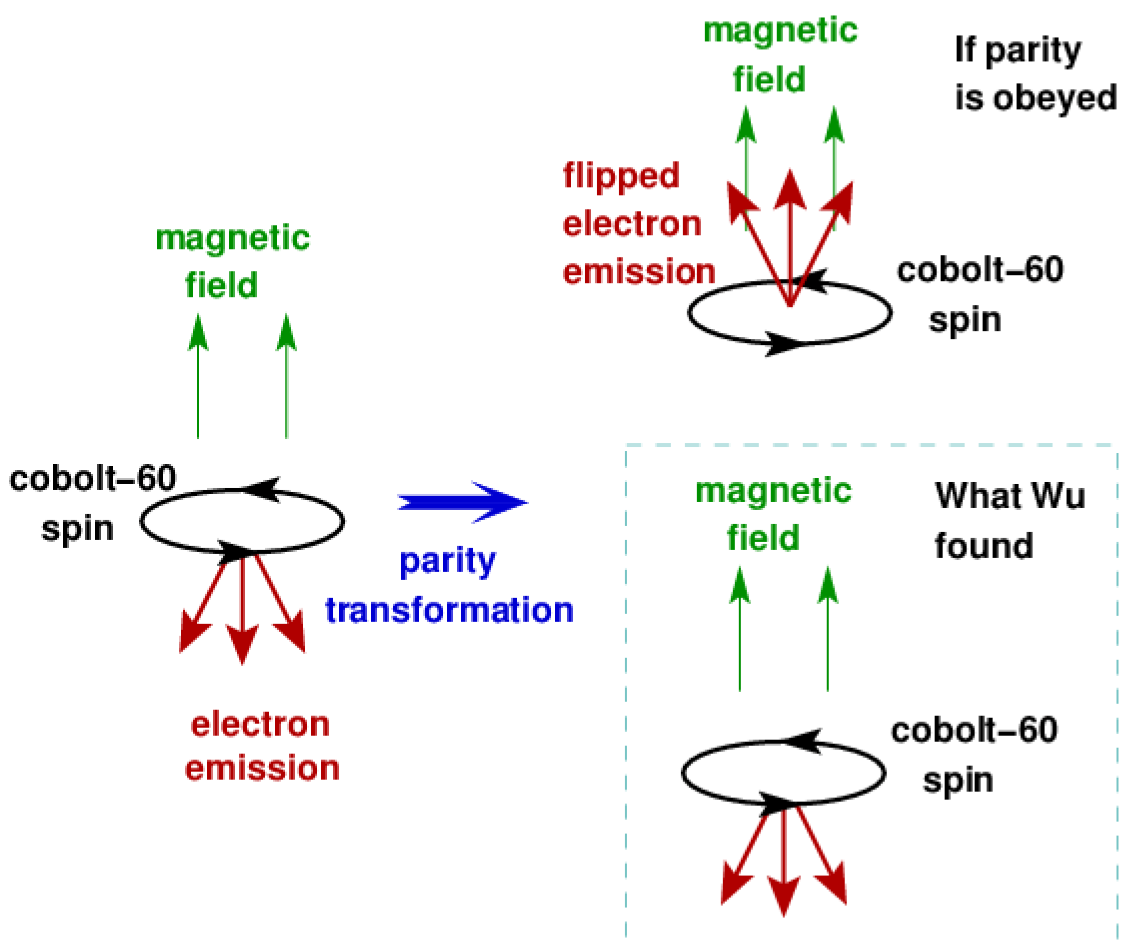


Fig. Experimental result which shows the parity violation in ^{60}Co

Tsung-Dao Lee

Tsung-Dao Lee (**T. D. Lee**, Chinese: 李政道; pinyin: *Lǐ Zhèngdào*) (born November 24, 1926) is a Chinese-born American physicist, known for his work on parity violation, the Lee Model, particle physics, relativistic heavy ion (RHIC) physics, nontopological solitons and soliton stars. He holds the rank of University Professor Emeritus at Columbia University, where he has taught since 1953 and from which he retired in 2012. In 1957, Lee, at the age of 30, won the Nobel Prize in Physics with C. N. Yang[2] for their work on the violation of the parity law in weak interactions, which Chien-Shiung Wu experimentally verified.
https://en.wikipedia.org/wiki/Tsung-Dao_Lee

Chen-Ning Franklin Yang

Chen-Ning Franklin Yang (born October 1, 1922), also known as **Yang Jhenning**, is a Chinese-born American physicist who works on statistical mechanics and particle physics. He and Tsung-dao Lee received the 1957 Nobel Prize in Physics[2] for their work on parity nonconservation of weak interaction. The two proved theoretically that one of the basic quantum-mechanics laws, the conservation of parity, is violated in the so-called weak nuclear reactions, those nuclear processes that result in the emission of beta or alpha particles.
https://en.wikipedia.org/wiki/Chen-Ning_Yang

1. Experiments by Wu et al. (1957) on ^{60}Co

In 1920's it had been suggested by several scientists that parity might not be conserved, but without solid evidence these suggestions were not considered important. Then, in 1956, a careful review and analysis by theoretical physicists Tsung Dao Lee and Chen Ning Yang went further, showing that while parity conservation had been verified in decays by the strong or electromagnetic interactions, it was untested in the weak interaction. They proposed several possible direct experimental tests. They were mostly ignored, but Lee was able to convince his Columbia colleague Chien-Shiung Wu to try it. In 1956 C. S. Wu, E. Ambler, R. W. Hayward, D.

D. Hoppes, and R. P. Hudson found a clear violation of parity conservation in the beta decay of ^{60}Co.

A crystal of ^{60}Co at very low temperature is located at the center of a solenoid, represented in the figure. The magnetic field of the solenoid orients the spins of the ^{60}Co nuclei in a direction normal to the plane. The low temperature is needed to avoid this alignment of the spins being disturbed by thermal motions. Because the spin behave like axial vectors (being rotation-like quantities), they are symmetrical with respect to the plane.

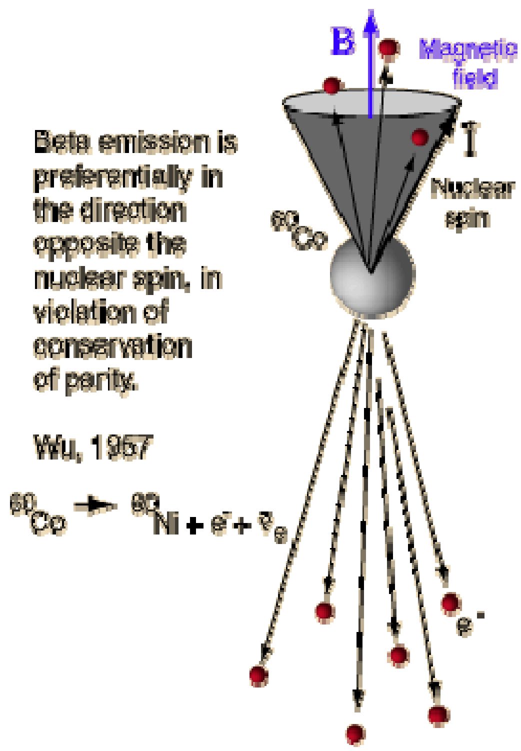


Fig. Experiment by Wu's experiment on ^{60}Co (1957).
http://hyperphysics.phy-astr.gsu.edu/hbase/quantum/parity.html

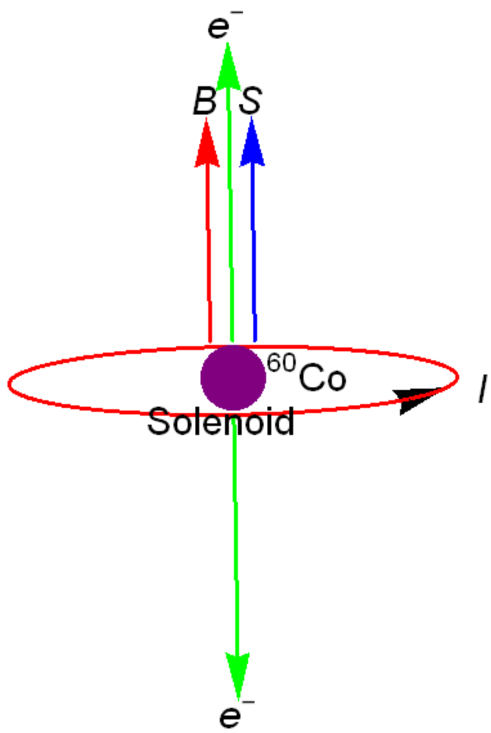


Fig. Prediction from the parity conservation

The parity conservation. radioactive Co nuclei, oriented according to their nuclear spin, emit beta rays (i.e., electrons) isotropically (directions parallel to the magnetic field and antiparallel to the magnetic field).

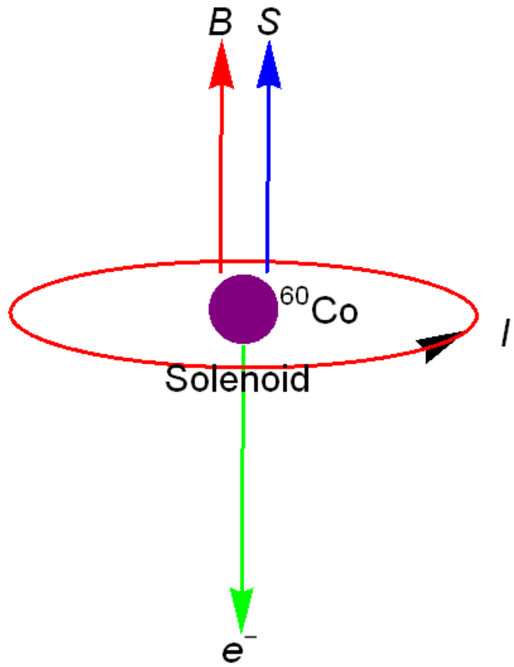


Fig. The parity violation (experimental result). The radioactive Co nuclei, oriented according to their nuclear spin, emit beta rays (i.e., electrons) preferentially in the opposite direction.

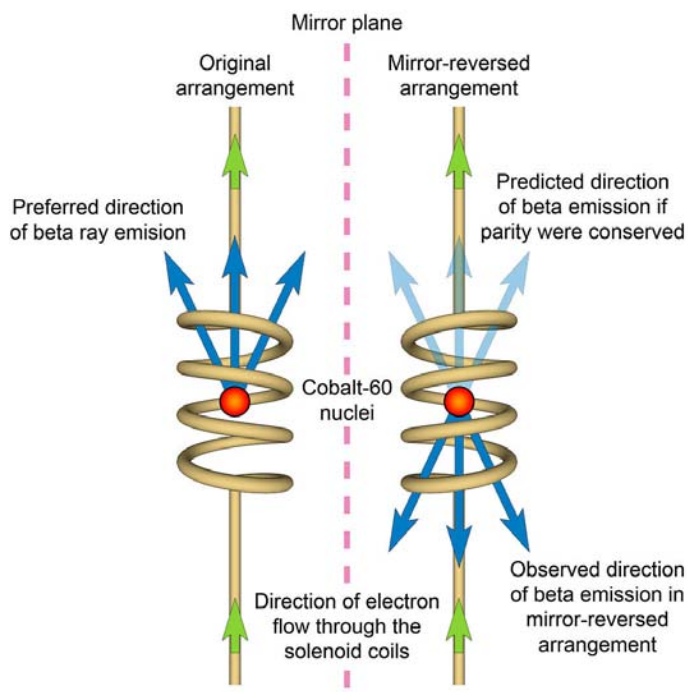


Fig. Schematic diagram of C-S. Wu's experiment on the beta decay of ^{60}Co nuclei, which are located at the center of a current-carrying wire. The electrons are preferentially emitted opposite to the direction of the magnetic field, because of the parity violation.

2. *β*-decay

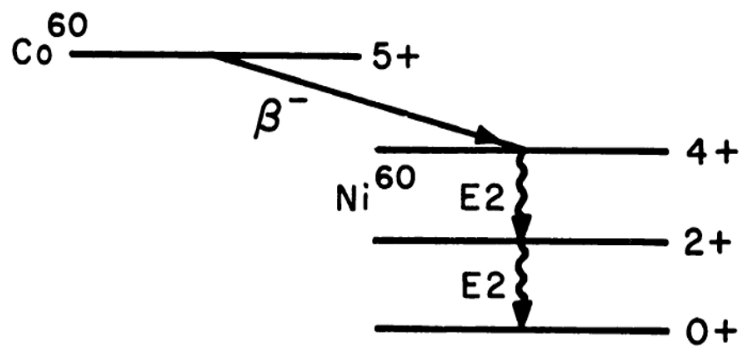


Fig. The decay scheme of ^{60}Co.

The experiment itself monitored the decay of ^{60}Co atoms, cooled to near absolute zero and aligned in a uniform magnetic field. ^{60}Co is an unstable isotope of cobalt that decays by *β*-decay to the stable isotope ^{60}Ni. During this decay, one of the neutrons in the ^{60}Co nucleus decays to a proton by emitting an electron (e^-) and an electron antineutrino (ν_e). This changes the ^{60}Co nucleus into a ^{60}Ni nucleus. The resulting nickel nucleus, however, is in an excited state and promptly decays to its ground state by emitting two gamma rays (γ). Hence the overall nuclear equation of the reaction is:

$$ {}^{60}_{27}Co \rightarrow {}^{60}_{28}Ni + e^- + \overline{\nu}_e + 2\gamma \, . $$

Gamma rays are photons, and their release from the ^{60}Ni nuclei is an electromagnetic (EM) process. This is important because EM was known to respect P-conservation. Hence, the distribution of the emitted gamma rays acted as a control for the polarization of the emitted electrons *via* the weak interaction, as well as an indicator of the uniformity of the cobalt-60 atoms. Wu's experiment compared the distribution of gamma and electron emissions with the nuclear spins in opposite orientations. If the electrons were always found to be emitted in the same direction and in the same proportion as the gamma rays, the parity-conservation would be true. If there were a bias in the direction of decays, that is, if the distribution of electrons did not follow the distribution of the gamma rays, then the parity-violation would be established.

3. Space inversion for the axial vector and polar vector

In quantum mechanics it is known that

$$\hat{\pi}\hat{\boldsymbol{A}}\hat{\pi} = \hat{\boldsymbol{A}}, \qquad \hat{\pi}\hat{\boldsymbol{P}}\hat{\pi} = -\hat{\boldsymbol{P}}$$

where $\hat{\pi}$ is the parity operator, $\hat{\boldsymbol{A}}$ is the axial vector and $\hat{\boldsymbol{P}}$ is the polar vector. First we show the change of the axial vector and the polar vector on the space inversion (the parity) by using the following figure.

(a) Axial vector

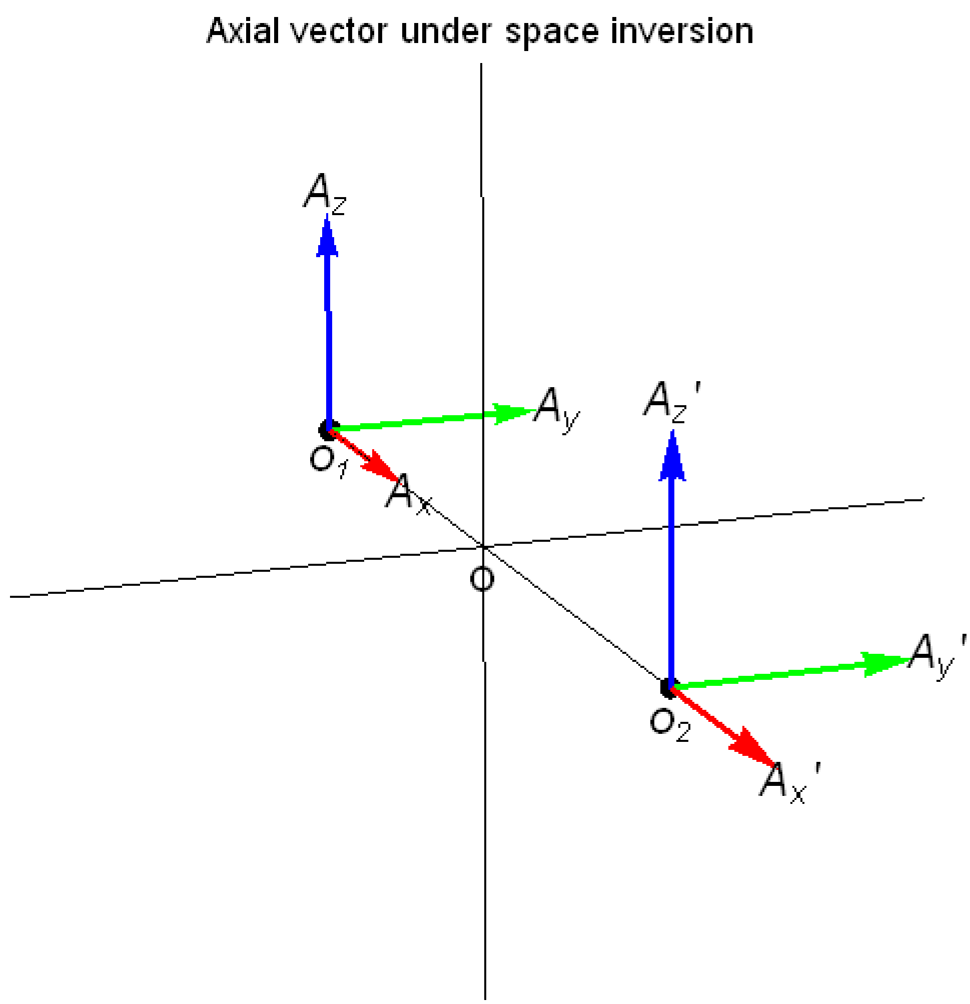


Fig. Axial vector on the space inversion. O_1 (real). O_2 (image).

(b) Polar vector

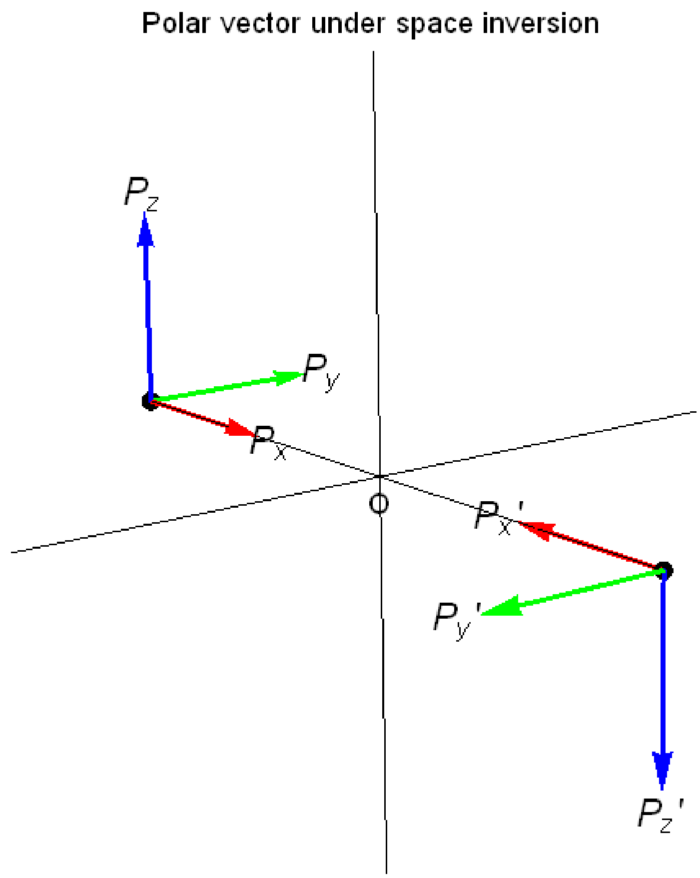


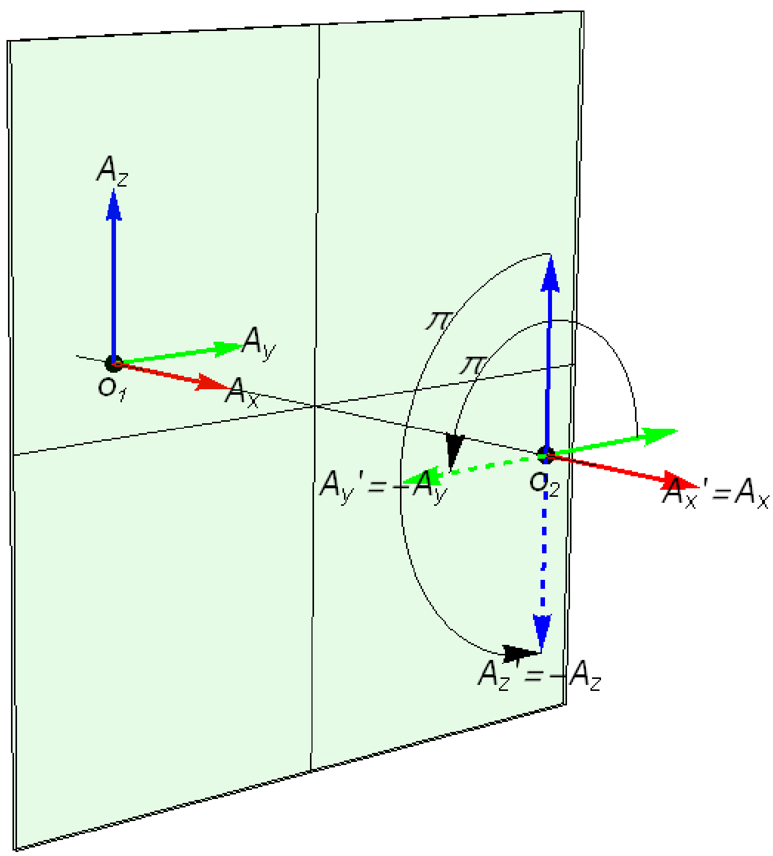
Fig. Polar vector on the space inversion. O_1 (real). O_2 (image).

4. Mirror reflection

Here we discuss the change of the axial vector and the polar vector on the mirror reflection using the following figures. The space inversion is equivalent, in the case of our three spatial dimensions, to a *mirror reflection* with respect to an arbitrary plane, followed by a π rotation with respect to an axis orthogonal to this plane. Since rotational symmetry, linked to the isotropy of space, is always assumed to hold, symmetry under mirror reflections in a plane actually corresponds to parity symmetry.

Vectors are defined by their transformation properties with respect to the rotation group, and classified according to their transformation properties with respect to parity: *polar vectors* are reversed by a parity transformation, while *axial vectors* (or *pseudo-vectors*) are not. Correspondingly, under reflection in a plane polar vectors have their component orthogonal to the plane reversed and the components parallel to the plane unaffected, while for axial vectors the opposite occurs (see Fig.); a π rotation around the axis orthogonal to the reflection plane then turns the reflected image of an axial vector into itself (unaffected by the parity transformation), while a polar vector turns into a reversed copy of itself (inverted by the parity transformation). The above can also be understood by considering that axial vectors can be expressed as vector products of two polar vectors. Chirality (a term coined by Lord Kelvin, from the Greek *χειρ* for hand) is the property of an object not superposable to its mirror image, an example being

(a) Axial vector



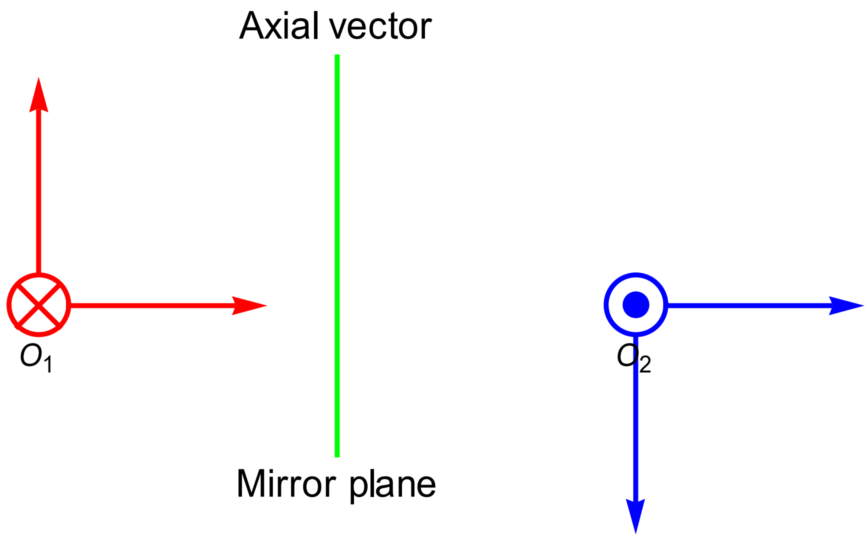
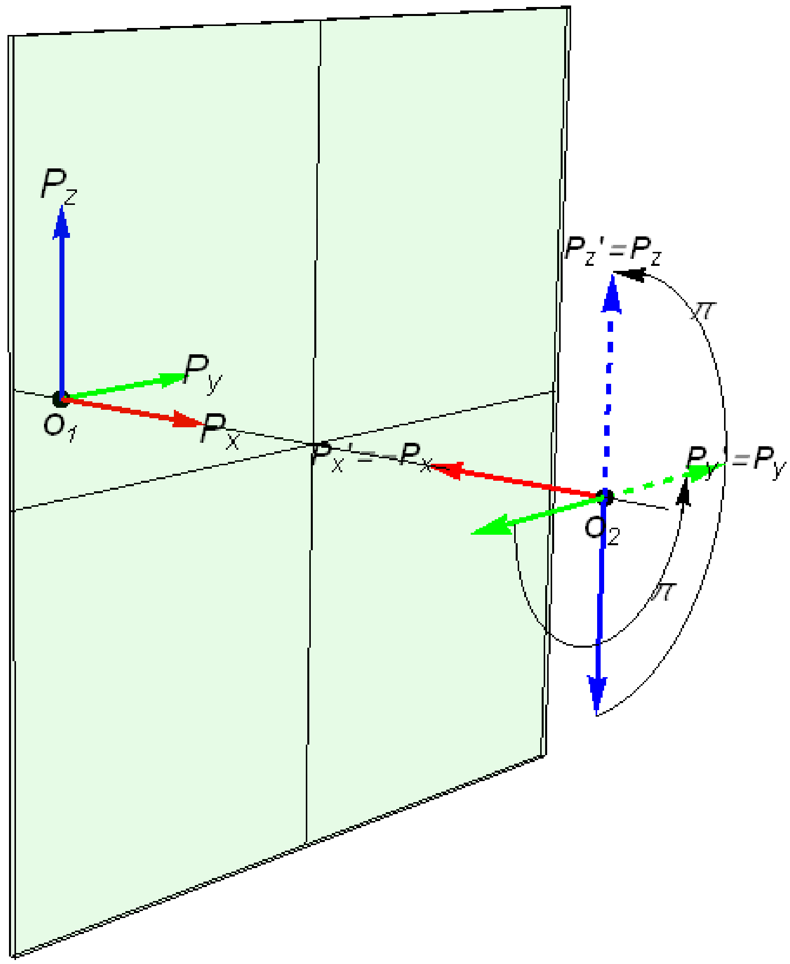


Fig. Change of the polar vector on the mirror reflection [from O_1 (real) to O_2 (image)]

(b) Polar vector



Pz
Py
Px
o1
Pz'=Pz
π
Px'=-Px
Py'=Py
o2
π

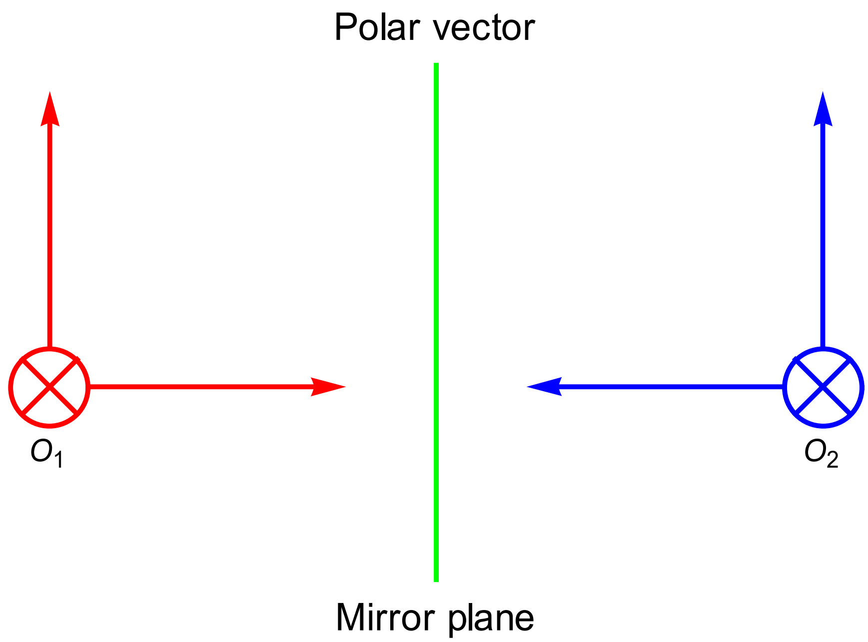


Fig. Change of the polar vector on the mirror reflection [from O_1 (real) to O_2 (image)]

5. Direction of spins on the mirror reflection

The direction of the current flowing in the solenoid is opposite to the direction of electrons moving in the solenoid. The velocity vector of electron is a polar vector. Based on the rule derived for the above discussion, the current direction in the image (O_2) is determined from the mirror reflection of the current direction in the real (O_1).

(a) The case where the axis of the solenoid is perpendicular to the mirror plane

As shown in the figure below, the direction of the current in the image is the same as that of the current in the real. Thus the magnetic field produced by the current is parallel to the axis of the solenoid. Since the direction of $\boldsymbol{S}$ is the same as the magnetic field, the direction of the spin vector in the image is the same as that in the real. This is consistent with the mirror reflection of the axial vector (the spin vector $\boldsymbol{S}$).

Fig.

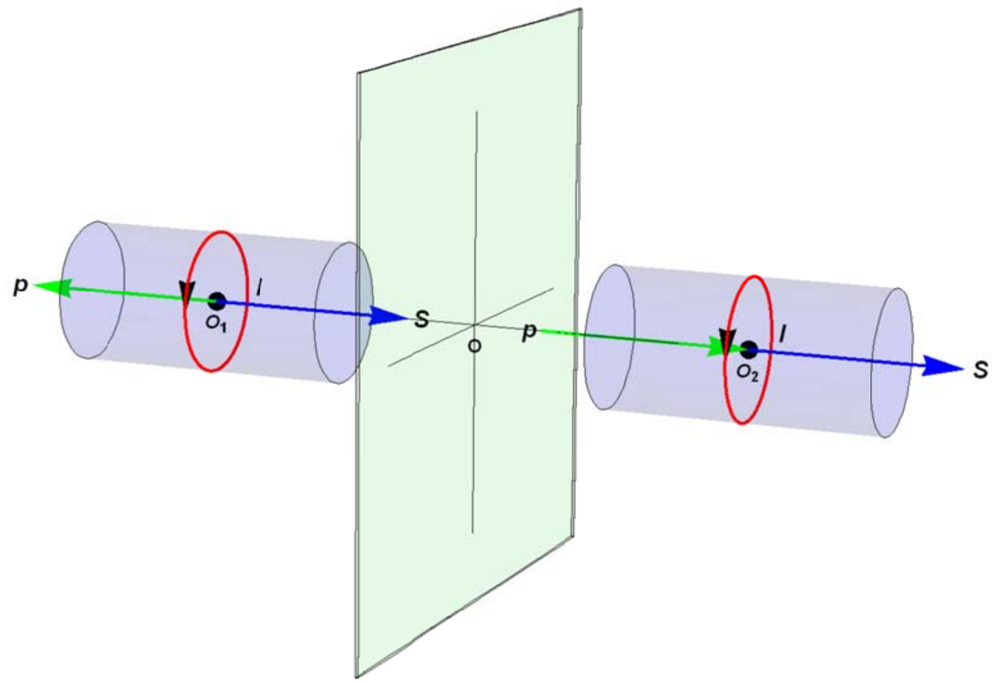


Fig. Axial vectors (***S***) are symmetric with respect to reflection through a perpendicular plane. The direction of the current in the solenoid in the real is the same as that in the image, since the velocity vector of electron is a polar axis vector. The direction of the momentum ***p*** in the real is opposite to that of ***p*** in the image. (The parity is conserved.)

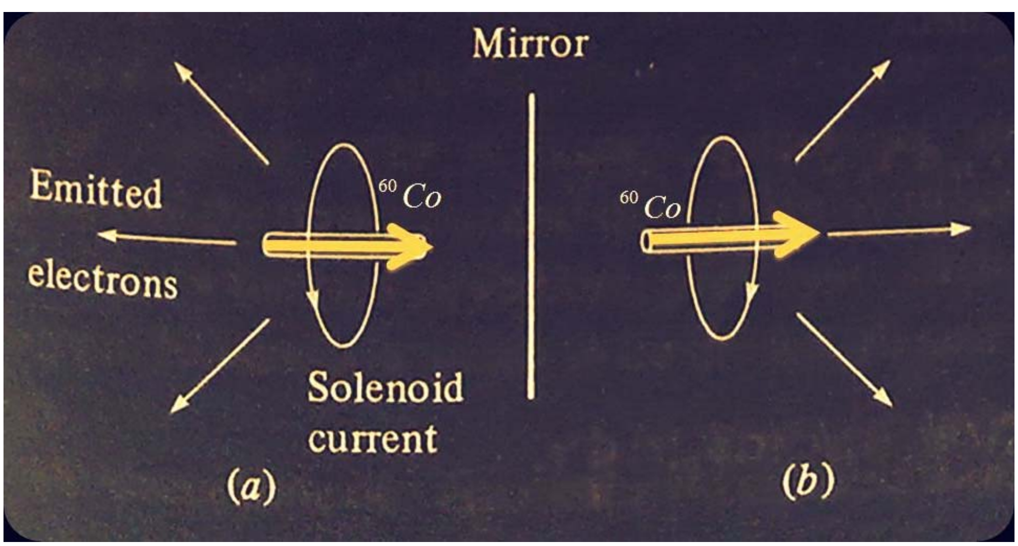


Fig. Mirror reflection of axial vectors under the parity conservation

(b) The case where the axis of the solenoid is parallel to the mirror plane

As shown in the figure below, the direction of the current in the image (O_2) is opposite to that of the current in the real (O_1). Thus the magnetic field produced by the current in the image is antiparallel to that in the real (O_1). Since the direction of the spin vector ***S*** is the parallel to the magnetic field, the direction of the spin vector ***S*** in the image is antiparallel to in the real. This is consistent with the mirror reflection of the axial vector (the spin vector ***S***).

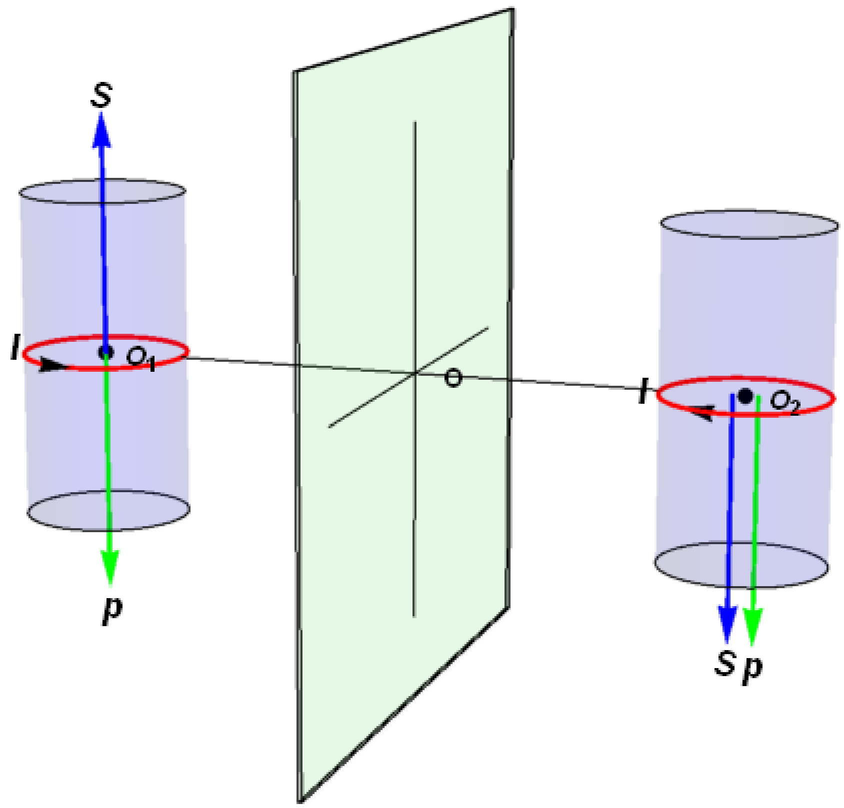


Fig. The direction of spin ***S*** in the real is antiparallel to that of spin in the image. The direction of the momentum ***p*** in the real is antiparallel to that of momentum in the image. The parity is conserved.

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6. Prediction from the parity conservation

The spin vector is an axial vector, while the velocity of electron is the polar vector. The magnetic field vector is an axial vector.

(a) Configuration where the spin vector is parallel to the mirror plane

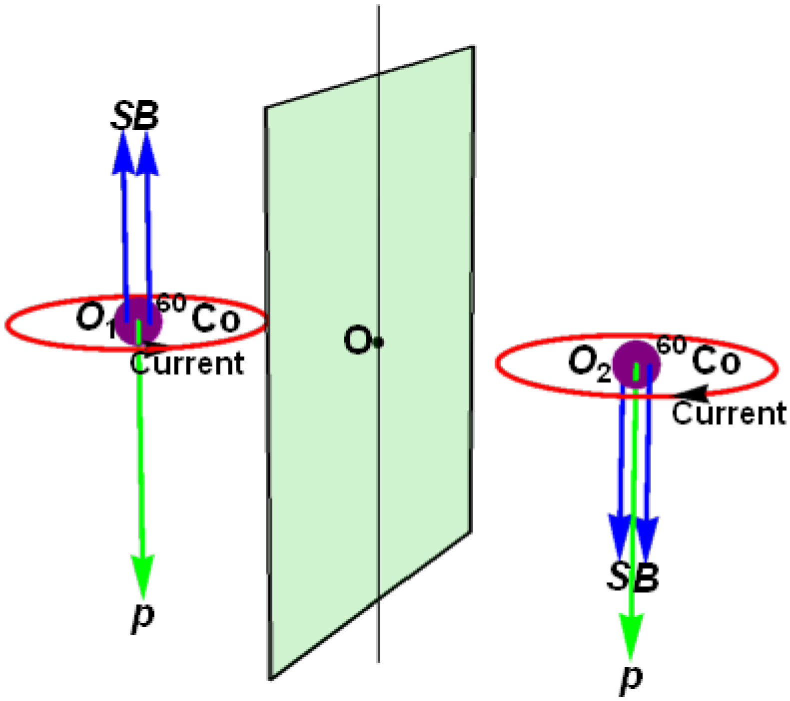

Fig. Actual experiment and mirror-reflected experiment where the parity is conserved. O_1 (real). O_2 (image). The parity is conserved.

((**Real**))

Suppose that a crystal of ^{60}Co at very low temperature is located at the center of a solenoid, represented in the above figure. The direction of the current flowing in the solenoid is opposite to that of electrons moving in the solenoid. The magnetic field ***B*** (directed upward) is generated by this current. The spin direction of ^{60}Co is parallel to the direction of ***B***. The direction of electrons due to the β-decay is antiparallel the spin direction.

((**Image**))

We note that ***S*** and ***B*** are the axial vector. The momentum ***p*** of electron is the polar vector. Then the spin ***S*** and the magnetic field ***B*** in the image (O_2) are antiparallel to those in the real (O_1). The momentum ***p*** in the image is parallel to that in the real.

This property can be alternatively explained by the nature of the mirror reflection. We start with the image of the current. The clockwise direction of current in the image (O_2) is a result of the mirror reflection from the counterclockwise direction of current in the real (O_1). This current induces the magnetic field *B* downward, leading to the direction of spin downward (parallel to ***B***). The momentum of the electron in the image is parallel to that in the real.

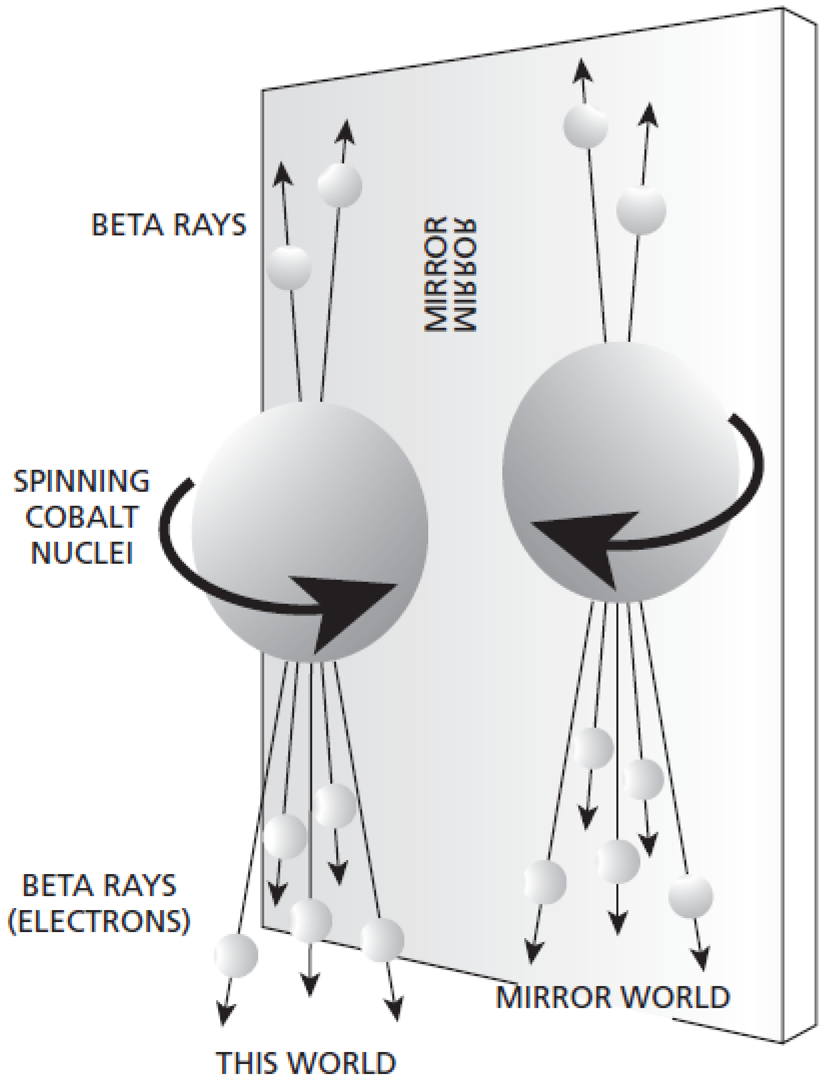

Fig. Mirror reflection of axial vector under the parity conservation.

(b) Configuration where the spin vector is normal to the mirror plane

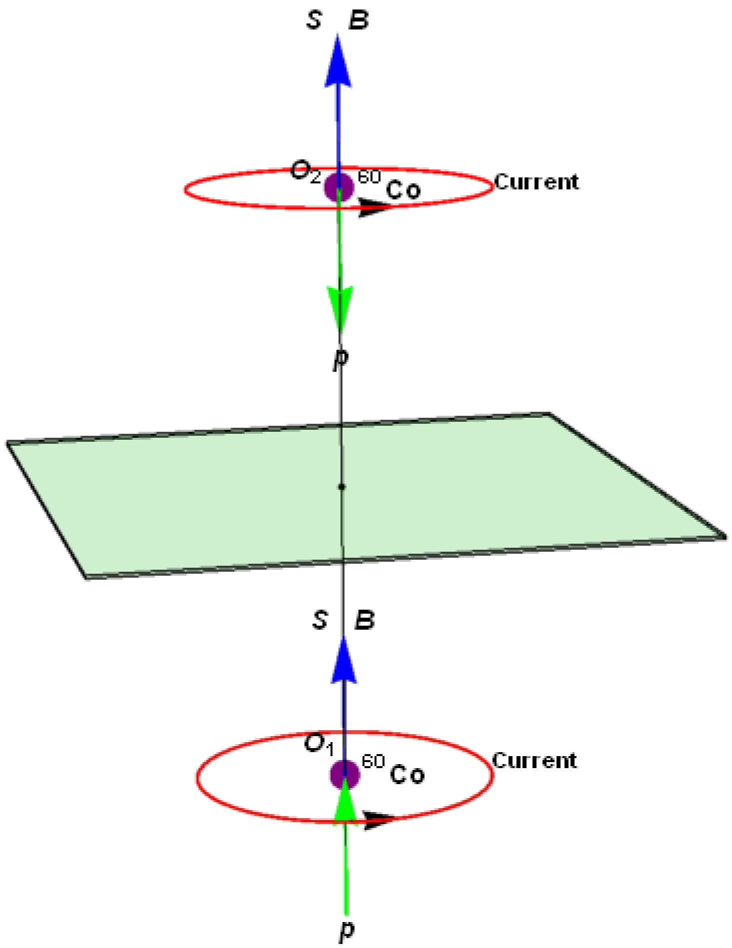

Fig. Actual experiment and mirror-reflected experiment under the parity conservation. O_1 (real). O_2 (image).

((**Real**))

Suppose that a crystal of ^{60}Co at very low temperature is located at the center of a solenoid, represented in the above figure. The direction of the current flowing in the solenoid is opposite to that of electrons moving in the solenoid. The magnetic field ***B*** (directed upward) is produced by this current. Then the spin direction of ^{60}Co is parallel to the direction of ***B***. The direction of electrons due to the β-decay is antiparallel the spin direction.

((**Image**))

We note that ***S*** and ***B*** are the axial vector. The momentum ***p*** of electron is the polar vector. Then the spin ***S*** and the magnetic field ***B*** in the image (O_2) are antiparallel to those in the real (O_1). The momentum ***p*** in the image is parallel to that in the real.

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8. Parity violation in the weak interaction

We consider an ensemble of ^{60}Co nuclei. This nucleus has spin 5 and is β-radioactive with lifetime 5.2 years. The given sample of ^{60}Co is cooled at very low temperature (≈0.01 K) to avoid thermal agitation and placed inside a solenoidal, electrically conducting coil. We start counting the electrons emitted in the various directions via the process of β-decay. With current in the coil the electrons are emitted isotropically from the source. This is obvious, since there is no preferred direction in the given sample, the spins of the ^{60}Co nuclei being oriented at random. When the current flows, however, it is found that more electrons are emitted in the direction antiparallel to the magnetic field ***B*** produced by the solenoid.

Which kind of angular distribution is expected if there is conservation of parity? Since the magnetic field ***B*** polarizes more ^{60}Co nuclei of the sample in the direction parallel to B, we can base our considerations on only one of these nuclei. We compare our experiment with its mirror image. That is to say, we use the second active point of view.

In Fig. S is the solenoid that produces the magnetic field. The direction of the motion of the electrons in S is shown. Note that the plane of S is perpendicular to the paper. From Fig. we understand immediately that if we suppose that the mirror images of the various particles involved in the process coincide with the corresponding particles, the hypothesis of parity conservation together with that of rotational invariance gives an isotropic angular distribution for the emitted electrons. That is, since the only vectors at our disposal are the ^{60}Co spin S and the electron momentum p (all the other vectors are averaged on), the probability has the form

$$W(\boldsymbol{p},\boldsymbol{\sigma}) = A_0(\boldsymbol{p}^2) \qquad \text{(parity conserved)}$$

The experiment shows, on the contrary, that the distribution is not isotropic – that is,

$$W(\boldsymbol{p},\boldsymbol{\sigma}) = A_0(\boldsymbol{p}^2) + A_1(\boldsymbol{p}^2)(\boldsymbol{S}\cdot\boldsymbol{p}) \qquad \text{(experimentally)}$$

In particular, the β-rays from the oriented ^{60}Co nuclei are preferentially emitted in the direction opposite to that of the ^{60}Co spin. This means that the mirror image of our experiment does not represent a possible experiment. The principle of parity conservation is violated. This pioneering experiment was performed by Wu et al. in December, 1956 and provided the first conclusive evidence. Thus the whole system is symmetrical with respect to this plane and no effects asymmetrical with respect to it could be observed. ^{60}Co decays and produces β rays (electrons), and the faulty reasoning described would lead us to expect that the electron count would be identical above and below the plane shown in the drawing. This experiment could have been done many years before, but it was not tried because the result seemed obvious. When it was done, however, the electron count observed was different on either side of the plane. This result was stated in terms of the property of parity, rather than that of symmetry with respect to a reflection plane as we have done. The system appears to be symmetrical with respect to inversion at the origin, a property which is called parity. The result of this experiment was thus that parity may not be conserved, a find that was to be one of the most important ideas of the 1960's.

Parity violation in the weak interaction was confirmed in the correlation of electron momentum with nuclear spin S in the decay of spin polarized nuclei and muons. If the parity is conserved, mirror processes with $\boldsymbol{p} \to -\boldsymbol{p}$ and $\boldsymbol{S} \to \boldsymbol{S}$ should occur with equal probability while the angle between these vectors for electron velocity is consistent with

$$I(\theta) = 1 - v\cos\theta$$

which favors emission anti-parallel to the nuclear spin.

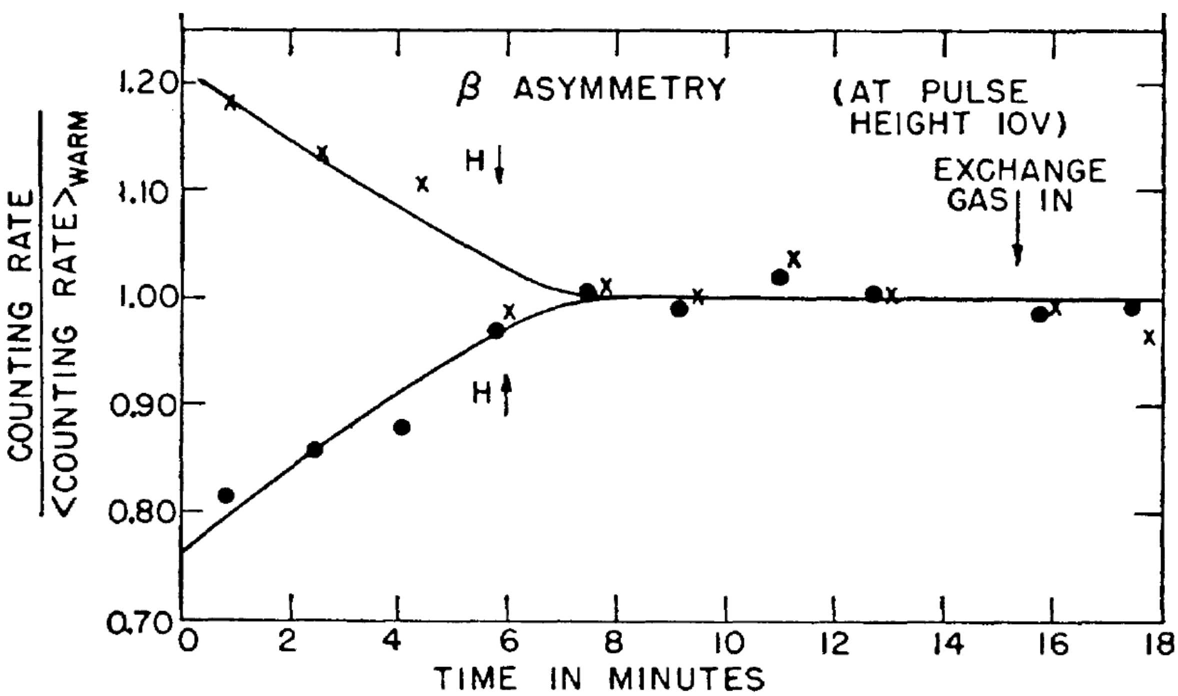

Fig. Experimental result (Wu et al.). As time goes on, the sample warms up and Co nuclei depolarize.

9. Conclusion

In the Wu experiment, the expectation value of the ^{60}Co spin ($\langle \boldsymbol{S} \rangle$ has a fixed orientation. The decay electrons are emitted preferentially in the direction opposite to that of the spin angular momentum: If $\boldsymbol{p}$ is the electron momentum, $\langle \boldsymbol{S} \cdot \boldsymbol{p} \rangle < 0$. However, $\langle \boldsymbol{S} \cdot \boldsymbol{p} \rangle$, the expectation value of the scalar product of a polar vector and an axial vector, is a pseudoscalar which changes sign under the parity operation. The mirror image of the experiment does not appear to be physically possible. In the mirror image the rotations are reversed, and the electrons are emitted preferentially in the direction of $\boldsymbol{S}$.

$$\hat{\pi}(\hat{\boldsymbol{S}} \cdot \hat{\boldsymbol{p}})\hat{\pi} = (\hat{\pi}\hat{\boldsymbol{S}}\hat{\pi}) \cdot (\hat{\pi}\hat{\boldsymbol{p}}\hat{\pi}) = -\hat{\boldsymbol{S}} \cdot \hat{\boldsymbol{p}} .$$

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