Polarization of photon emitted from atom in excited state

Masatsugu Sei Suzuki
Department of Physics, SUNY at Binghamton
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In his book [R.P. Feynman, R.,B. Leighton, and M. Sands, *The Feynman Lectures in Physics*, 6th edition (Addison Wesley, Reading Massachusetts, 1977, Part.3 Chapter 18), Feynman discussed the angular momentum conservation for the radiation of light from an atom. The conservation of the angular momentum determines the polarization and angular distribution of the emitted light. When an atom is in an excited state of definite angular momentum (spin 1 with m = 1, the state denoted by $\left| j=1, m=1 \right\rangle$) and makes a transition to a state of angular momentum zero (spin zero, m = 0, the state denoted by $\left| j=0, m=0 \right\rangle$) at a lower energy, emitting a right-hand circularly (RHC) polarized photon, due to the electric dipole transition. When an atom is in a state $\left| j=1, m=-1 \right\rangle$ and a makes a transition to a state $\left| j=0, m=0 \right\rangle$, emitting a left-hand circularly (LHC) polarized photon, due to the electric dipole transition. From the angular momentum conservation, it is concluded that the RHC photon is in a spin state $\left| R \right\rangle = \left| j=1, m=1 \right\rangle$, while the LHC photon is in a spin state $\left| L \right\rangle = \left| j=1, m=-1 \right\rangle$. We note that there is no state corresponding to $\left| j=1, m=1 \right\rangle$ because of no rest mass for photon. This will be discussed in the APPENDIX (by Feynman).

Here we discuss the properties of RHC photon and LHC photon for the electric dipole transition, the light scattering and the annihilation of positronium, based on the discussion given by Feynman. We also note that similar topics are discussed by Bellac [M.L. Bellac, Quantum Physics (Cambridge Uniuversity Press, 2006)]. In order to understand the essential points of the discussion, we need to introduce the concept of the mirror-reflection operator $\hat{M}_y = \hat{Y}\hat{\pi}$.

1. Definition of RHC and LHC photons

(i) Right-hand circularly photon

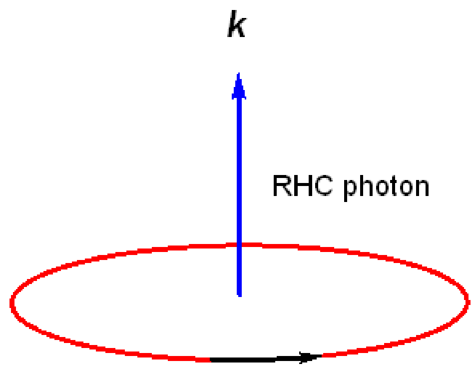


Fig. Right-hand circular photon (σ^{+} polarization). $J_z = \hbar$. The direction of ***J*** is along the rotation axis (parallel to ***k***). The sense of ***J*** is defined by the right-hand rule. The thumb of the right hand gives the sense of ***J***.

The state vector of RHC photon is expressed by

$$\left|R\right\rangle = \frac{1}{\sqrt{2}}(\left|x\right\rangle + i\left|y\right\rangle)$$

(ii) Left-hand circularly (LHC) photon

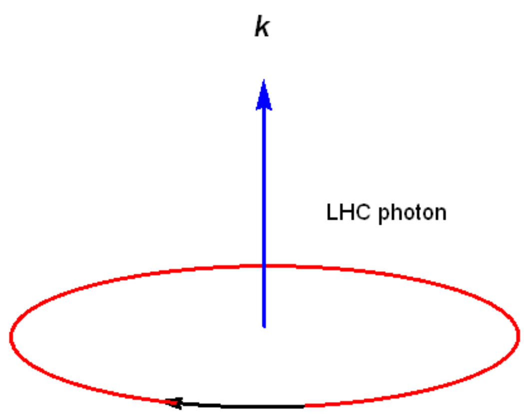


Fig. Left-hand circular photon (σ^{-} polarization). $L_z = -\hbar$. The sense of ***J*** is defined by the right-hand rule. The thumb of the right hand gives the sense of ***J***. The direction of ***J*** is anti-parallel to the direction of wavevector ***k***.

The state vector of LHC photon is expressed by

$$\left|L\right\rangle = \frac{1}{\sqrt{2}}(\left|x\right\rangle - i\left|y\right\rangle)\,.$$

2. Electric dipole radiation

(a) Emission of RHC photon (polarization σ^{-+})

Suppose that we have an atom which is in an excited state of the angular momentum with j = 1. Since the upper state of the atom is spin j = 1, there are three possibilities for its z-component of angular momentum. The value of m could be m = +1, 0, or -1. We will take m = +1 in this case. We suppose that the atom is sitting with its angular momentum along the +z axis. It will emit the right-hand circularly (RHC) polarized light upward along the +z axis according to the angular momentum conservation. So that atom ends up with j = 0, because of the selection rule for the electric dipole radiation ($\Delta j = \pm 1$). The RHC photon has an angular momentum of $\hbar$ about its direction of propagation. So after the photon is emitted, the atom is left with j = 0 (m = 0). Note that the rule for the electric dipole radiation ($\Delta m = 0, \pm 1$) is satisfied in this case since the transition of the state in atom occurs from $\left|j = 1, m = 1\right\rangle$ to $\left|j = 0, m = 0\right\rangle$.

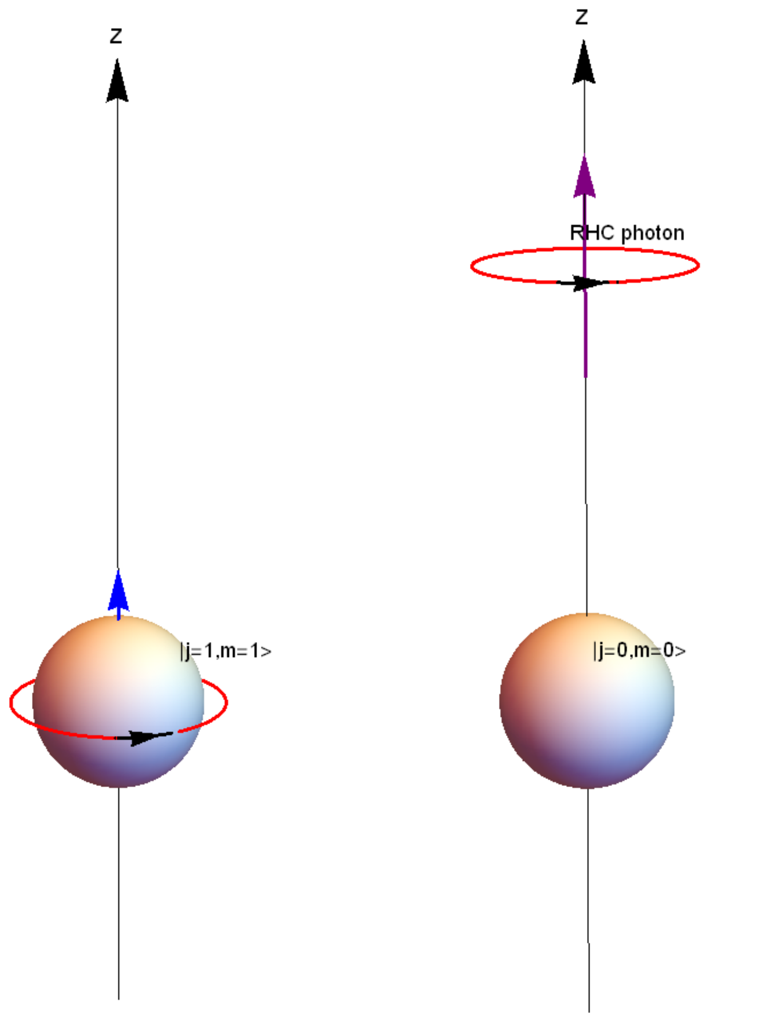


Fig. (a) An atom with $m = +1$ emits a RHC photon (with the angular momentum $+\hbar$) along the $+z$ axis. The final state of the atom is $\left| j = 0, m = 0 \right\rangle$ according to the rule for the electric dipole radiation ($\Delta j = \pm 1$ and $\Delta m = 0, \pm 1$). We assume that the probability amplitude is a for this transition.

The transition amplitude a is given by

$$a = \left\langle R, \theta = 0 \middle| \hat{T} \middle| j = 1, m = 1 \right\rangle$$

where $\hat{T}$ is the transition matrix, and θ is the angle between the photon emission direction. In this case, $\theta = 0$.

(b) Emission of LHC photon (polarization σ^-)

Suppose that we have an atom which is in an excited state of the angular momentum with j = 1. Since the upper state of the atom is spin j = 1, there are three possibilities for its z-component of angular momentum. The value of m could be m = +1, 0, or -1. We will take m = -1 in this case (down spin). We suppose that the atom is sitting with its angular momentum along the -z axis. It will emit the right-hand circularly (LHC) polarized light upward along the +z axis, where the angular momentum is conserved. So that atom ends up with j = 0, because of the selection rule for the electric dipole radiation ($\Delta j = \pm 1$). The LHC photon has an angular momentum of -$\hbar$ about its direction of propagation. So after the photon is emitted, the atom is left with j = 0 (m = 0). Note that the rule for the electric dipole radiation ($\Delta m = 0, \pm 1$) is satisfied in this case since the transition of the state in atom occurs from $\left| j = 1, m = -1 \right\rangle$ to $\left| j = 0, m = 0 \right\rangle$.

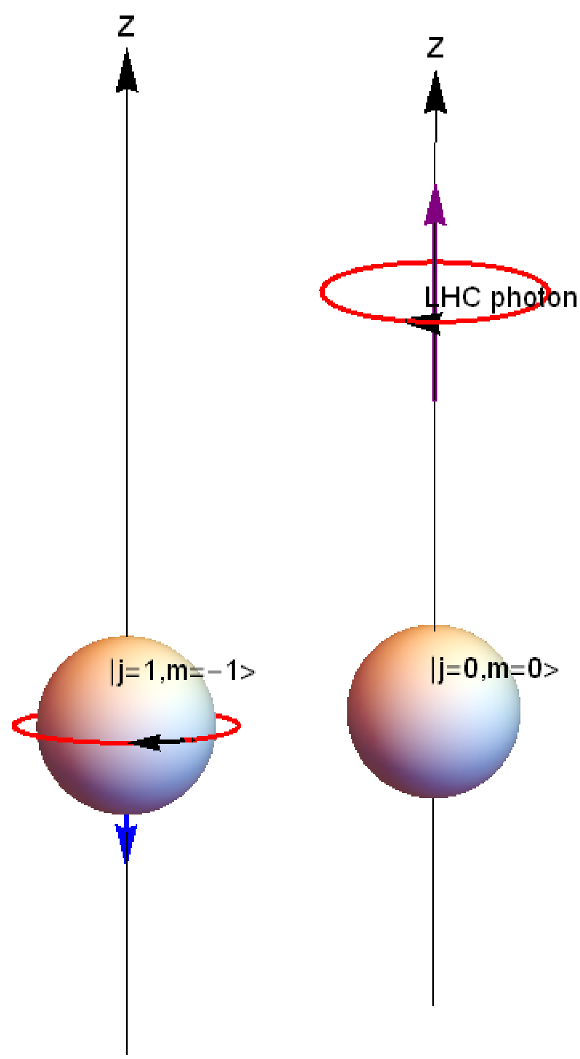


Fig. (a) An atom with m = -1 emits a LHC photon (with the angular momentum -$\hbar$) along the -z axis. The final state of the atom is $|j=0, m=0\rangle$ according to the rule for the electric dipole radiation ($\Delta j = \pm 1$ and $\Delta m = 0, \pm 1$). We assume that the probability amplitude is b for this transition.

The transition amplitude b is given by

$$b = \langle L, \theta = 0|\hat{T}|j=1, m=-1\rangle$$

where $\hat{T}$ is the transition matrix, and θ is the angle between the photon emission direction. In this case, $\theta = 0$.

3. Properties of the mirror-reflection operator $\hat{M}_y$

Here we show the properties of the mirror reflection operator $\hat{M}_y$ at y = 0 plane (the z-x plane). The detail of the discussion will be given elsewhere. The results are summarized as follows.

For the position operator, we get

$$\hat{M}_y^{-1}\hat{x}\hat{M}_y = \hat{x},$$

$$\hat{M}_y^{-1}\hat{y}\hat{M}_y = -\hat{y},$$

$$\hat{M}_y^{-1}\hat{z}\hat{M}_y = \hat{z}.$$

For the momentum operator we have

$$\hat{M}_y^{-1}\hat{p}_x\hat{M}_y = \hat{p}_x,$$

$$\hat{M}_y^{-1}\hat{p}_y\hat{M}_y = -\hat{p}_y,$$

$$\hat{M}_y^{-1}\hat{p}_z\hat{M}_y = \hat{p}_z.$$

For the orbital angular momentum, we have

$$\hat{M}_y^{-1}\hat{L}_x\hat{M}_y = -\hat{L}_x,$$

$$\hat{M}_y^{-1}\hat{L}_y\hat{M}_y = \hat{L}_y ,$$

$$\hat{M}_y^{-1}\hat{L}_z\hat{M}_y = -\hat{L}_z .$$

4. Mirror reflection operator for RHC and LHC photons

Under the mirror-reflection operator, the sense of rotation remain unchanged (which means that the direction of the angular momentum remain unchanged), while the direction of the wave vector changes from ***k*** to -***k***.

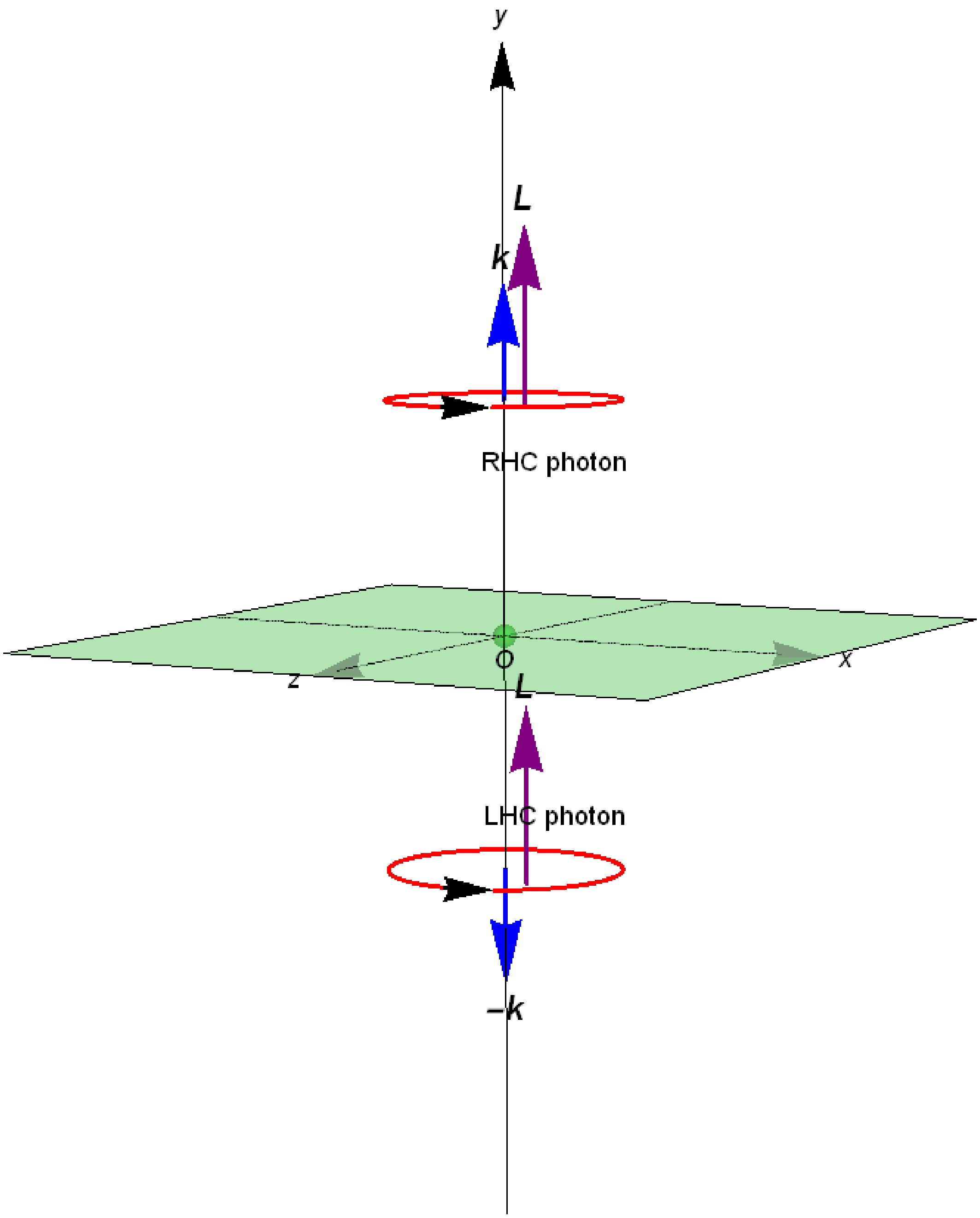


Fig. Role of the Mirror reflection operator (with respect to the $y = 0$ plane, or the y-z plane). After the mirror reflection, $\boldsymbol{k} \to -\boldsymbol{k}$, $\boldsymbol{L} \to \boldsymbol{L}$.

After the mirror-reflection, the RHC state changes into the LHC state, while the LHC state changes into the RHC state. That is to say, we have

$$\hat{M}_y|R\rangle = |L\rangle, \qquad \hat{M}_y|L\rangle = |R\rangle$$

The matrix of $\hat{M}_y$ is can be expressed by

$$\hat{M}_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \hat{\sigma}_x$$

under the basis of {$|R\rangle$, $|L\rangle$}. The eigenkets of $\hat{M}_y$ is given by

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}[|R\rangle + |L\rangle] = |x\rangle, \qquad \text{(eigenvalue +1)}$$

$$|\psi_2\rangle = \frac{1}{\sqrt{2}}[|R\rangle - |L\rangle] = i|y\rangle. \qquad \text{(eigenvalue -1).}$$

For convenience, we use $|\psi_2\rangle = |y\rangle$ instead of $i|y\rangle$. Then we have

$$\hat{M}_y|x\rangle = |x\rangle, \qquad \hat{M}_y|y\rangle = -|y\rangle.$$

5. Light scattering

We suppose that the same atoms are sitting in their ground state $|j=0,m=0\rangle$, and scatter an incoming beam of light from the -*z* direction. We consider the scattering of light as two steps. The photon is absorbed and then re-emitted.

(i) After the absorption of the RHC photon by atom, the state of the atom will change from $|j=1,m=1\rangle$ to $|j=0,m=0\rangle$. The atom can then emit a RHC photon in the direction θ(=0). We call the transition amplitude for this process *a*.

$$a = \langle R,\theta=0|\hat{T}|j=1,m=1\rangle,$$

where $\hat{T}$ is the transition operator.

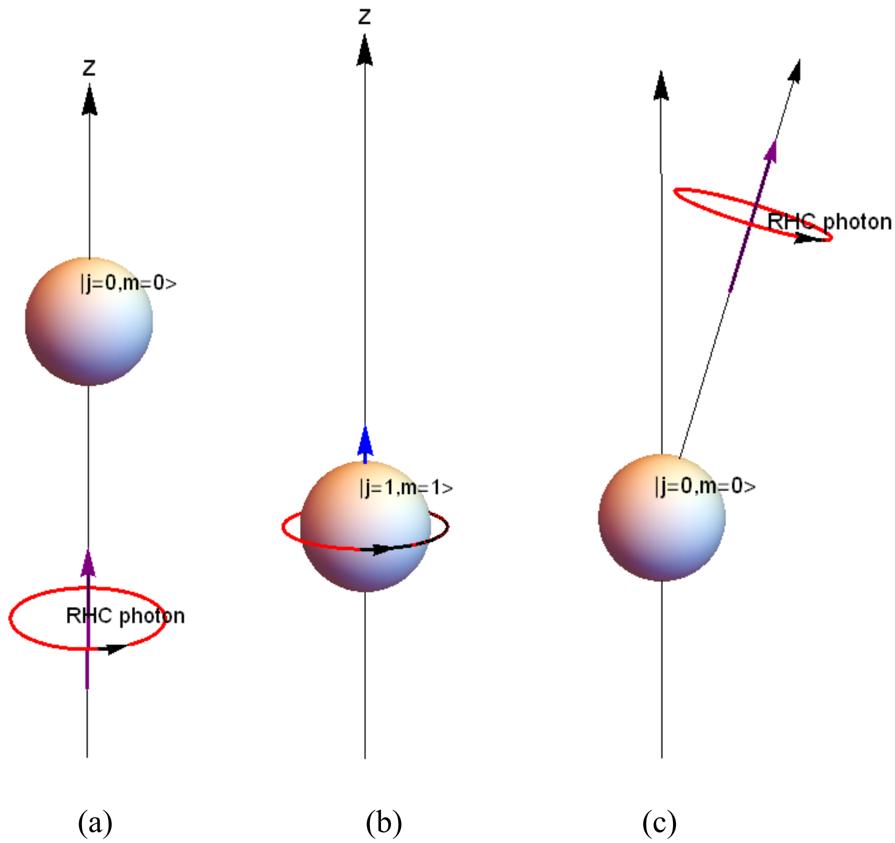
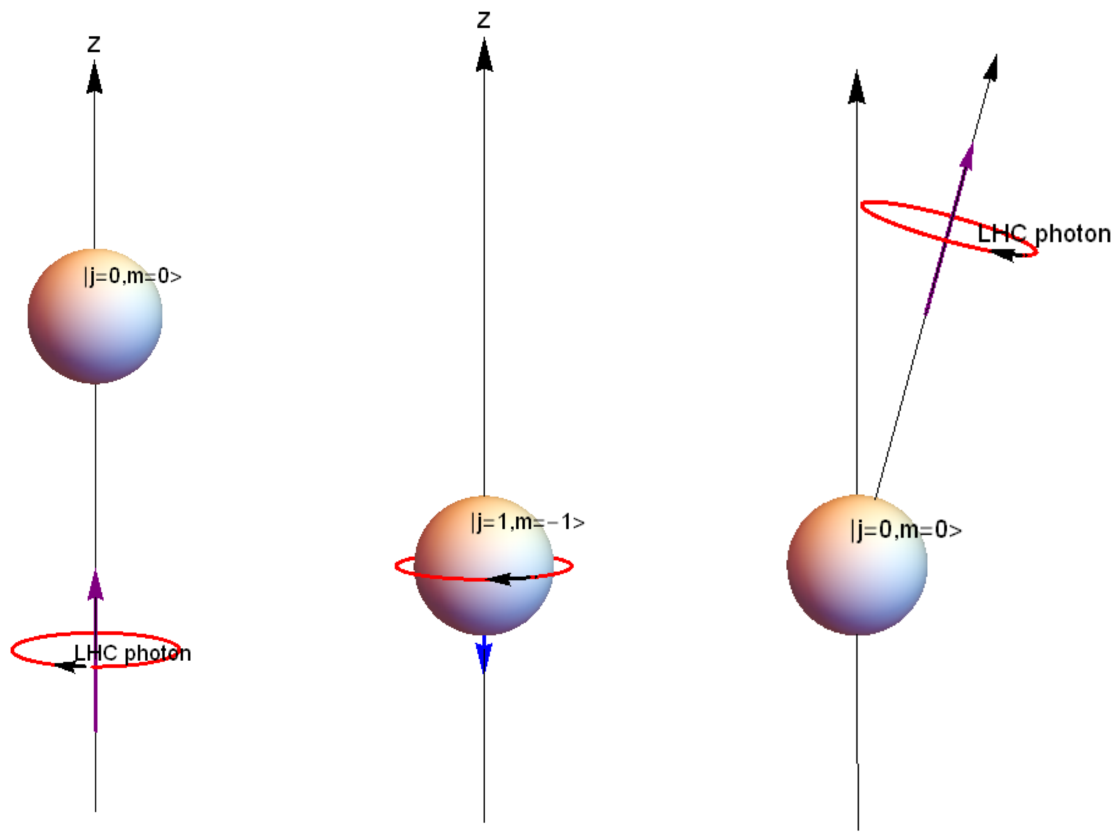


Fig. The scattering of light by an atom seen as a two-step process

(ii) After the absorption of the LHC photon by atom, the state of the atom will change from $\left| j=0, m=0 \right>$ to $\left| j=1, m=-1 \right\rangle$. The atom can then emit a LHC photon in the direction θ (= 0) We call the transition amplitude for this process as *b*.

$$b = \left\langle L, \theta = 0 \middle| \hat{T} \middle| j=1, m=-1 \right\rangle .$$



(a) (b) (c)

Fig. The scattering of light by an atom seen as a two-step process

The matrix element can be calculated as

$$
\begin{aligned}
a &= \langle R, \theta = 0 | \hat{T} | j = 1, m = 1 \rangle \\
&= \langle R, \theta = 0 | \hat{M}_y^{-1} (\hat{M}_y \hat{T} \hat{M}_y^{-1}) \hat{M}_y | j = 1, m = 1 \rangle \\
&= \langle L, \theta = 0 | \hat{M}_y \hat{T} \hat{M}_y^{-1} | j = 1, m = -1 \rangle \\
&= \eta_T \langle L, \theta = 0 | \hat{T} | j = 1, m = -1 \rangle \\
&= \eta_T b
\end{aligned}
$$

where

$$b = \langle L, \theta = 0 | \hat{T} | j = 1, m = -1 \rangle$$

$$\hat{M}_y | R, \theta = 0 \rangle = - | L, \theta = 0 \rangle, \qquad \hat{M}_y | L, \theta = 0 \rangle = - | R, \theta = 0 \rangle$$

$$\hat{M}_y \left| j=1, m=1 \right\rangle = -\left| j=1, m=-1 \right\rangle,$$

$$\hat{M}_y \hat{T} \hat{M}_y^{-1} = \eta_T \hat{T}$$

So there are two possible cases.

$$\eta_T = 1 \rightarrow b = a,$$

$$\eta_T = -1 \rightarrow b = -a$$

((**Note**)) Conventional notation

(i) RHC and LHC photon

$$\hat{M}_y \left| R \right\rangle = -\left| L \right\rangle \qquad \hat{M}_y \left| L \right\rangle = -\left| R \right\rangle$$

If we choose

$$\left| \psi_{+p} \right\rangle = \frac{1}{\sqrt{2}} (\left| R \right\rangle - \left| L \right\rangle), \qquad \left| \psi_{-p} \right\rangle = \frac{1}{\sqrt{2}} (\left| R \right\rangle + \left| L \right\rangle)$$

we have

$$\hat{M}_y \left| \psi_{+p} \right\rangle = \left| \psi_{+p} \right\rangle, \qquad \text{(even parity)}$$

$$\hat{M}_y \left| \psi_{-p} \right\rangle = -\left| \psi_{-p} \right\rangle \qquad \text{(odd parity)}$$

(ii) $\left| j=1, m=1 \right\rangle$ and $\left| j=1, m=-1 \right\rangle$

$$\hat{M}_y \left| 1,1 \right\rangle = -\left| 1,-1 \right\rangle \qquad \hat{M}_y \left| 1,-1 \right\rangle = -\left| 1,1 \right\rangle$$

If we choose

$$\left| \psi_{-A} \right\rangle = \frac{1}{\sqrt{2}} (\left| 1,1 \right\rangle + \left| 1,-1 \right\rangle), \qquad \left| \psi_{+A} \right\rangle = \frac{1}{\sqrt{2}} (\left| 1,1 \right\rangle - \left| 1-1 \right\rangle)$$

we have

$$\hat{M}_y|\psi_{+A}\rangle = |\psi_{+A}\rangle, \qquad \text{(even parity)}$$

$$\hat{M}_y|\psi_{-A}\rangle = -|\psi_{-A}\rangle \qquad \text{(odd parity)}$$

We now consider the case where the photon emission angle is $\theta \neq 0$, where θ is the rotation angle from the z axis about the y-axis. We have

$$|R,\theta\rangle = \hat{R}_y(\theta)|R,\theta = 0\rangle,$$

$$|L,\theta\rangle = \hat{R}_y(\theta)|L,\theta = 0\rangle.$$

The transition amplitude

$$\begin{aligned}\langle R,\theta|\hat{T}|j=1,m=1\rangle &= \langle R,\theta=0|\hat{R}_y^{\ +}(\theta)\hat{T}|j=1,m=1\rangle \\ &= \langle R,\theta=0|\hat{T}\hat{R}_y^{\ +}(\theta)|1,1\rangle \\ &= \langle R,\theta=0|\hat{T}|1,1\rangle\langle 1,1|\hat{R}_y^{\ +}(\theta)|1,1\rangle \\ &= \frac{a}{2}(1+\cos\theta)\end{aligned}$$

$$\begin{aligned}\langle L,\theta|\hat{T}|j=1,m=-1\rangle &= \langle L,\theta=0|\hat{R}_y^{\ +}(\theta)\hat{T}|j=1,m=-1\rangle \\ &= \langle L,\theta=0|\hat{T}\hat{R}_y^{\ +}(\theta)|1,1\rangle \\ &= \langle L,\theta=0|\hat{T}|1,-1\rangle\langle 1,-1|\hat{R}_y^{\ +}(\theta)|1,-1\rangle \\ &= \frac{b}{2}(1+\cos\theta)\end{aligned}$$

$$\begin{aligned}\langle R,\theta|\hat{T}|j=1,m=-1\rangle &= \langle R,\theta=0|\hat{R}_y^{\ +}(\theta)\hat{T}|j=1,m=-1\rangle \\ &= \langle R,\theta=0|\hat{T}\hat{R}_y^{\ +}(\theta)|1,-1\rangle \\ &= \langle R,\theta=0|\hat{T}|1,1\rangle\langle 1,1|\hat{R}_y^{\ +}(\theta)|1,-1\rangle \\ &= \frac{a}{2}(1-\cos\theta)\end{aligned}$$

$$
\begin{aligned}
\langle L,\theta|\hat{T}|\, j=1, m=1\rangle &= \langle L,\theta=0|\hat{R}_y{}^{+}(\theta)\hat{T}|\, j=1, m=1\rangle \\
&= \langle L,\theta=0|\hat{T}\hat{R}_y{}^{+}(\theta)|1,1\rangle \\
&= \langle L,\theta=0|\hat{T}|1,-1\rangle\langle 1,-1|\hat{R}_y{}^{+}(\theta)|1,1\rangle \\
&= \frac{b}{2}(1-\cos\theta)
\end{aligned}
$$

where

$$
\begin{aligned}
\langle 1,1|\hat{R}_y{}^{+}(\theta)|1,1\rangle &= \langle 1,1|\hat{R}_y(\theta)|1,1\rangle^{*} = \frac{1}{2}(1+\cos\theta) \\
\langle 1,1|\hat{R}_y{}^{+}(\theta)|1,-1\rangle &= \langle 1,-1|\hat{R}_y(\theta)|1,1\rangle^{*} = \frac{1}{2}(1-\cos\theta) \\
\langle 1,-1|\hat{R}_y{}^{+}(\theta)|1,1\rangle &= \langle 1,1|\hat{R}_y(\theta)|1,-1\rangle^{*} = \frac{1}{2}(1-\cos\theta) \\
\langle 1,-1|\hat{R}_y{}^{+}(\theta)|1,-1\rangle &= \langle 1,-1|\hat{R}_y(\theta)|1,-1\rangle^{*} = \frac{1}{2}(1+\cos\theta)
\end{aligned}
$$

((**Note**))
For $\phi = 0$, the rotation operator is given by

$$
\hat{R} = D^{(1)}(\theta,\phi) = \begin{pmatrix} \dfrac{1+\cos\theta}{2} & -\dfrac{\sin\theta}{\sqrt{2}} & \dfrac{1-\cos\theta}{2} \\ \dfrac{\sin\theta}{\sqrt{2}} & \cos\theta & -\dfrac{\sin\theta}{\sqrt{2}} \\ \dfrac{1-\cos\theta}{2} & \dfrac{\sin\theta}{\sqrt{2}} & \dfrac{1+\cos\theta}{2} \end{pmatrix}
$$

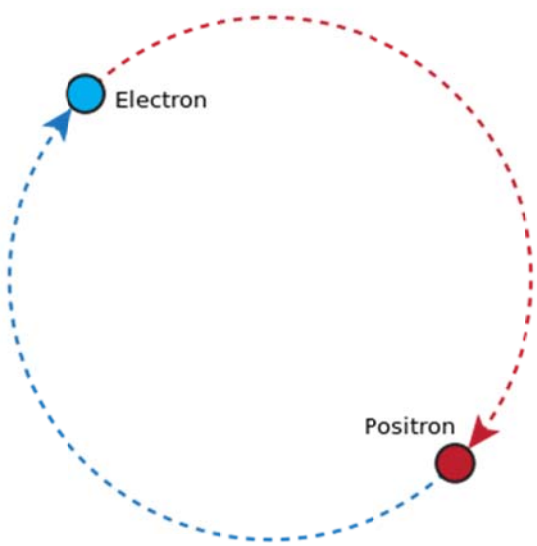
and

$$
|1,1\rangle_n = \hat{R}|1,1\rangle = \begin{pmatrix} \dfrac{1+\cos\theta}{2} \\ \dfrac{\sin\theta}{\sqrt{2}} \\ \dfrac{1-\cos\theta}{2} \end{pmatrix}
$$

$$|1,0\rangle_n = \hat{R}|1,0\rangle = \begin{pmatrix} -\dfrac{\sin\theta}{\sqrt{2}} \\ \cos\theta \\ \dfrac{\sin\theta}{\sqrt{2}} \end{pmatrix}$$

$$|1,-1\rangle_n = \hat{R}|1,-1\rangle = \begin{pmatrix} \dfrac{1-\cos\theta}{2} \\ -\dfrac{\sin\theta}{\sqrt{2}} \\ \dfrac{1+\cos\theta}{2} \end{pmatrix}$$

5. Annihilation of positronium: entanglement

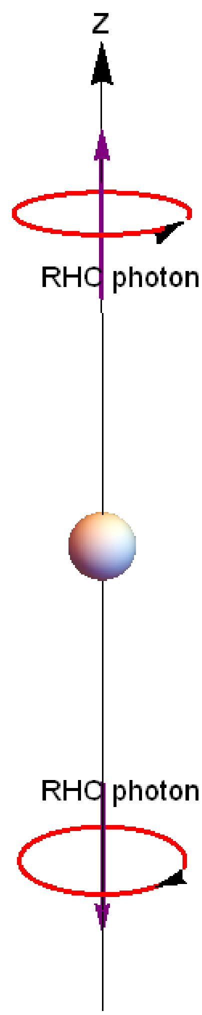


The positronium is an "atom" made up of an electron and a positron. It is a bound state of an e+ and e-. It is like a hydrogen atom. The ground state is split into a hyperfine structure by the interaction of the magnetic moments. The spins of the electron and positron are each S = 1/2, and they can either parallel or antiparallel to any given axis. In the ground state there is no other angular momentum due to orbital motion. So there are four states, S = 1 (three symmetric states, and S = 0 (single anti-symmetric state).

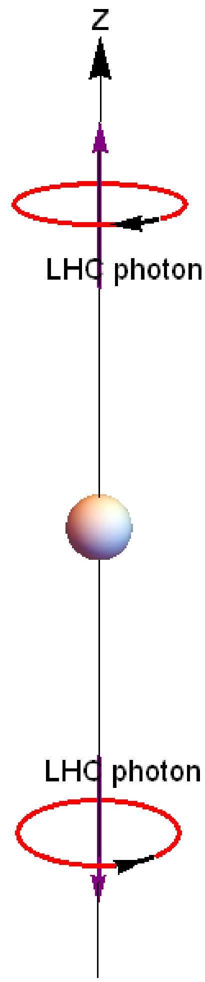
$$D_{1/2} \times D_{1/2} = D_1 + D_0$$

We consider only the case of S = 0. The positronium cannot last forever. The positron is the antiparticle of the electron. They can annihilate each other. The two particles disappear completely- converting their rest energy into radiation, which appears as g-rays (photon). In the disintegration, two particles with a finite rest mass go into two or more objects which have zero rest mass.

After the disintegration, there are two photons going out with equal and opposite, because the total momentum after the disintegration must be zero. If the photon going upward is the RHC photon, then angular momentum will be conserved if the downward going photon ia also RHC photon. Each will carry $+\hbar$, with respects to its momentum direction. It means $+\hbar$ and $-\hbar$ about the z axis. The total angular momentum is zero, and the angular momentum after the disintegration will be the same as before.



Similarly, if the photon going upward is the LHC photon, then angular momentum will be conserved if the downward going photon ia also LHC photon. Each will carry $+\hbar$, with respects to its momentum direction. It means $+\hbar$ and $-\hbar$ about the z axis. The total angular momentum is zero, and the angular momentum after the disintegration will be the same as before.



So the final state can be expressed by

$$|F\rangle = |R_1R_2\rangle - |L_1L_2\rangle$$

Then an inversion changes the R's into L's and gives the state

$$P|F\rangle = |L_1L_2\rangle - |R_1R_2\rangle = -|F\rangle$$

So the final state $|F\rangle$ has negative parity, which is the same as the initial spin zero state of the positronium. This is the only final state that conserves both angular momentum and parity.

$$\langle x_1 y_2 | F \rangle = i$$

$$\langle x_1 x_2 | F \rangle = 0$$

6. Quantum entanglement

The first experiment on entanglement was done in 1949 at Columbia University by Chien-Shiung Wu [a professor of physics at Columbia who was also known as “Madame Wu”]—and she didn’t study entanglement. She studied positronium, an artificial element comprised of an electron and a positron that exists for only a fraction of a second before the electron and positron annihilate each other. When they do, the two high-energy photons (gamma rays, which are one type of radioactivity) that come out of this annihilation are entangled. Madame Wu and her colleague I. Shaknov didn’t know that the photons were entangled when they did the experiment, and that wasn’t the purpose of their study. But years later, in the 1970s, scientists looked at the experiment’s results, and they realized that this was probably the first example of entanglement. In the 1970s and 1980s, two groups, one in the United States and one in France, proved that entanglement is a real phenomenon by using visible light, rather than the high-energy photons of the 1949 experiments.(Inside Angels and Demons, D. Burstein and A. de Keijzer (Vanguard Press, 2004, 2009).

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M.L. Bellac, Quantum Physics (Cambridge Uniuversity Press, 2006).

APPENDIX I Operator $\hat{Y}$

The photon has a vector nature and spin 1, which permits $|R\rangle$ and $|L\rangle$, to be identified as the states $|j,m\rangle$,

$$|R\rangle = |j=1, m=1\rangle = |1,1\rangle, \qquad |L\rangle = |j=1, m=-1\rangle = |1,-1\rangle,$$

where the angular momentum quantization axis (Oz) is taken to lie along the propagation direction. The value of m is called the photon helicity; m = +1 corresponds to positive helicity and m = -1 to negative helicity. We now check that the above definition corresponds to a standard angular momentum basis.

We use the rotation operator $R_y(\pi) = \hat{Y} = \exp(-\frac{i}{\hbar}\hat{J}_y\pi)$ related to the rotation around the y axis by the angle π.

$$\hat{Y}|1{,}1\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = |1{,}-1\rangle, \quad \hat{Y}|1{,}-1\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = |1{,}1\rangle$$

We use the operator $\hat{Y}$, which changes the direction of the phonon propagation while leaving O*y* unchanged. Action of $\hat{Y}$ on the linear polarization states yields

$$\hat{Y}|x\rangle = -|x\rangle, \quad \hat{Y}|y\rangle = |y\rangle .$$

Then we get

$$\hat{Y}|R\rangle = \hat{Y}(\frac{|x\rangle + i|y\rangle}{\sqrt{2}}) = -|L\rangle$$

$$\hat{Y}|L\rangle = \hat{Y}(\frac{|x\rangle - i|y\rangle}{\sqrt{2}}) = |R\rangle .$$

APPENDIX-II

Classical explanation why the RHC photon has an angular momentum of $\hbar$ (Feynman).

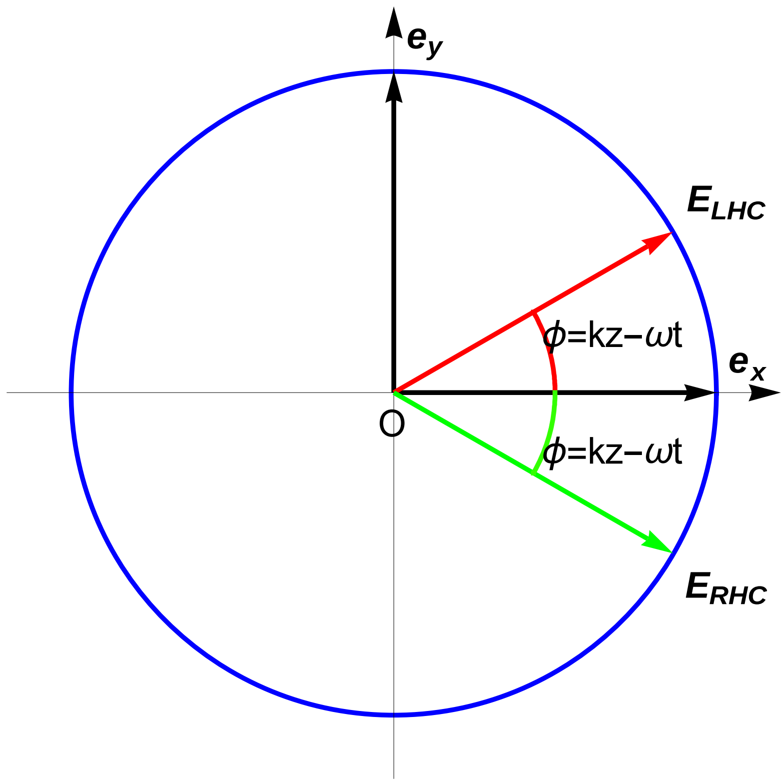

Fig. Direction of the electric field for the RHC (right-hand circularly) photon and LHC (left-hand circularly) photon (*x*-*y* plane which is perpendicular to the propagation direction *z* axis). The phase angle is given by $\phi = kz - \omega t$.

APPENDIX-III Feynman's explanation I; why RHC and LHC photon have spin angular momentum?

How can we explain that the photons of light and left that are right circularly polarized carry spin angular momentum of $\hbar$ and $-\hbar$ along the z axis. This fact is explained classically by Feynman as follows.

"If we have a beam of light containing a large number of photons all circularly polarized the same way – as we would have in a classical beam – It will carry angular momentum. If the total energy carried by the beam in a certain time is W, then there are

$$N = \frac{W}{\hbar\omega}, \tag{1}$$

photons. Each one carries the angular momentum $\hbar$, so there is a total angular momentum of

$$J_z = N\hbar = \frac{W}{\omega}. \tag{2}$$

Can we prove classically that light which is right circularly polarized carries an energy and angular momentum in proportional to W/ω? That should be a classical proposition if everything is right. Here we have a case where we can go from the quantum thing to the classical thing. We should see if the classical physics checks. It will give us an idea whether we have a right to call m the angular momentum. Remember what right circularly polarized light is, classically. It is described by an electric field with an oscillating x-component and an oscillating y-component 90° out of phase so that the resultant electric field vector $\boldsymbol{\varepsilon}$ goes in a circle-as drawn in Fig.1. Now suppose that such light shines on a wall which is going to

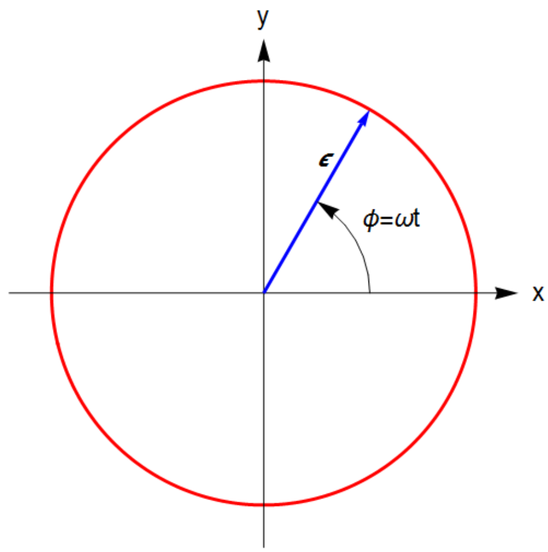

Fig.1 The electric field ε in a right-hand circularly polarized light wave. Note that the electric field rotates counter-clock wise since the phase factor is given by $(\omega t - kz)$, instead of $(kz - \omega t)$.

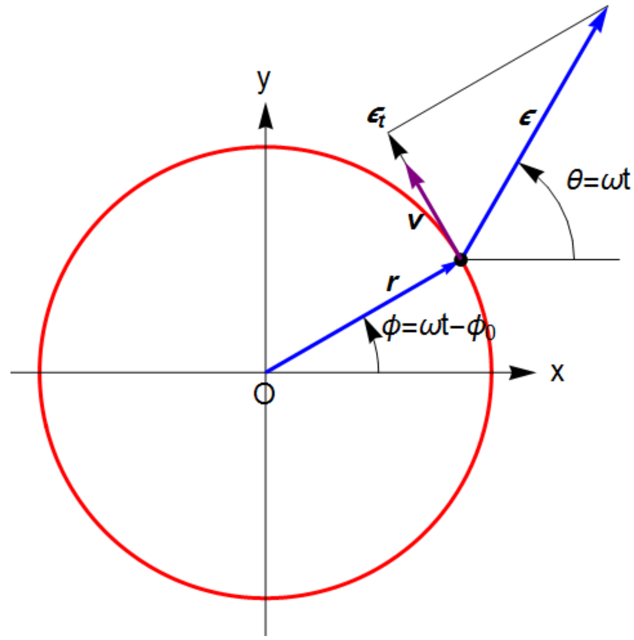

Fig.2 The motion of electron being driven by the right-hand circularly polarized light

absorb it - or at least some of it - and consider an atom in the wall according to the classical physics. We have often described the motion of the electron in the atom as a harmonics oscillator which can be driven into oscillation by an external electric field. We will suppose that the atom is isotropic, so that it can oscillate equally well in the *x*- or *y*-directions. Then in the circularly polarized light, the x-displacement and the y-displacement are the same, but one is 90° behind the other. The net result is that the electron moves in a circle, as shown in Fig.2. The electron is displaced at some displacement $\boldsymbol{r}$ from its equilibrium position at the origin and goes around with some phase lag with respect to the vector $\boldsymbol{\varepsilon}$. The relation between $\boldsymbol{\varepsilon}$ and $\boldsymbol{r}$ might be as shown in Fig.2. As time goes on, the electric field rotates and the displacement rotates with the same frequency, so their relative orientation stays the same. Now let us look at the work being done on this electron. The rate that energy is being put into this electron is $\boldsymbol{v}$, its velocity, times the component of $q\varepsilon$ parallel to the velocity:

$$\frac{dW}{dt} = q\varepsilon_t v\ .$$

But look, there is angular momentum being poured into this electron, because there is always a torque about the origin. The torque is $q\varepsilon_t r$, which must be equal to the rate of change of angular momentum dJ_z/dt:

$$\frac{dJ_z}{dt} = q\varepsilon_t r \ .$$

Remembering that $v = \omega r$, we have that

$$\frac{dJ_z}{dW} = \frac{1}{\omega}$$

Therefore, if we integrate the total angular momentum which is absorbed, it is proportional to the total energy – the constant of proportionality being $1/\omega$, which agrees with Eq.(2). Light does carry angular momentum – 1 unit (times $\hbar$) if it is right circularly polarized along the z axis -1 unit along the z axis if it is left circularly polarized."

APPENDIX-IV Feynman's explanation II: why there is no spin state $\left| j = 1, m = 0 \right\rangle$ for photon?

"The light does not have three states, but only two – although a photon is still on object of spin one. How is this consistent with our earlier proofs – based on what happens under rotation in space – that is for spin one particles three states are necessary? For a particle at rest, rotations can be made about any axis without changing the momentum state. Particles with rest mass (like photons and neutrinos) cannot be at rest; only rotation about the axis along the direction of motion do not change the momentum state. Arguments abou rotations around one axis only are insufficient to prove that three states are required, given that one of them varies as $e^{i\varphi}$ under rotation by the angle ϕ.