Definition of the uncertainty:

The Heisenberg's uncertainty of principle is frequently discussed in the quantum mechanics. The uncertainty Δx is sometimes called as the standard deviation. The fluctuation $(\Delta x)^2$ is sometimes called as the variance or the square of the uncertainty.

Uncertainty	$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$
Standard deviation	$\Delta x = \sqrt{\langle x^2 \rangle - \langle x \rangle^2}$
Fluctuation	$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2$
The square of the uncertainty	$(\Delta x)^2$
Variance	$(\Delta x)^2$
Mean (or expectation)	$\langle x \rangle$

((**Gaussian**)) **Normal distribution**

A random variable with a Gaussian distribution is said to be **normally distributed** and is called a **normal deviate**.

$$f(x;\mu,\sigma) = \frac{1}{\sqrt{2\pi}\sigma}\exp[-\frac{(x-\mu)^2}{2\sigma^2}]$$

$$\langle x^2 \rangle = \int_{-\infty}^{\infty} x^2 f(x;\mu,\sigma)dx = \sigma^2 + \mu^2, \quad \langle x \rangle = \int_{-\infty}^{\infty} xf(x;\mu,\sigma)dx = \mu$$

$$(\Delta x)^2 = \langle x^2 \rangle - \langle x \rangle^2 = \sigma^2$$

The parameter μ in this definition is the *mean* or *expectation* of the distribution (and also its median and mode). The parameter σ is the standard deviation. σ^2 is called as the variance.

The Compton wavelength

$$\lambda_c = \frac{h}{m_e c} = \frac{2\pi\hbar}{m_e c}$$

$$\bar{\lambda}_c = \frac{\lambda_c}{2\pi} = \frac{\hbar}{m_e c} = 3.8616 \text{ x } 10^{-3} \text{ Å}.$$

Bohr radius

$$a_B = \frac{\hbar^2}{m_e e^2} = \frac{\hbar^2}{m_e \alpha \hbar c} = \frac{\hbar}{m_e \alpha c} = \frac{\lambda_c}{\alpha} = 5.29177 \text{ x } 10^{-1} \text{ Å}$$

The fine structure constant is defined by

$$\alpha = \frac{e^2}{\hbar c} = 7.2973525376 \text{ x } 10^{-3}.$$

$$\frac{1}{\alpha} = 137.036$$

((**Mathematica**))

```
Clear["Global`*"];
rule1 = {c → 2.99792 × 10^10, ℏ → 1.054571628 10^-27,
   me → 9.10938215 10^-28, qe → 4.8032068 × 10^-10,
   eV → 1.602176487 × 10^-12, Å → 10^-8, α → 7.2973525376 × 10^-3};

Lc = ℏ/(me c Å) /. rule1; Lc // ScientificForm

3.8616 × 10^-3

LB = ℏ^2/(me qe^2 Å) /. rule1; LB // ScientificForm

5.29177 × 10^-1

α LB /. rule1

0.00386159
```