Universal constants: Lorenz number, Wiedermann Franz law for conduction electrons

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Thermal conductivity of electron is expressed by

$$\kappa_e = \frac{1}{3} c_V v l$$

where c_V is the heat capacity of electron per unit volume.

$$c_V = \frac{C}{V} = \frac{1}{3}\pi^2 \frac{D(\varepsilon_F)}{V} k_B{}^2 T = \frac{1}{3}\pi^2 \frac{3N}{V 2\varepsilon_F} k_B{}^2 T = \frac{\pi^2 n k_B{}^2}{2\varepsilon_F} T$$

The heat capacity of electron is

$$C = \frac{1}{3}\pi^2 D(\varepsilon_F) k_B{}^2 T$$

The density of states is

$$D(\varepsilon_F) = \frac{3N}{2\varepsilon_F}$$

Then the thermal conductivity can be rewritten as

$$\kappa_e = \frac{1}{3} c_V v l = \frac{1}{9}\pi^2 \frac{D(\varepsilon_F)}{V} k_B{}^2 v l T = \frac{1}{9}\pi^2 \frac{3n}{2\varepsilon_F} k_B{}^2 v l T$$

or

$$\kappa_e = \frac{1}{6}\pi^2 \frac{n k_B{}^2}{\varepsilon_F} v^2 \tau T$$

Using the relation,

$$\varepsilon_F = \frac{1}{2} m v_F{}^2$$

the thermal conductivity for electron us

$$\kappa_e = \frac{1}{3}\pi^2 \frac{n k_B{}^2}{m v_F{}^2} v_F{}^2 \tau T = \frac{\pi^2 n \tau k_B{}^2}{3m} T$$

2. Lorenz number *L*

The conductivity is given by

$$\sigma = \frac{n e^2 \tau}{m}$$

The Lorenz number L is defined by

$$L = \frac{\kappa}{\sigma T} = \frac{\pi^2 n \tau k_B{}^2}{3m} \frac{m}{n e^2 \tau} = \frac{\pi^2}{3} \left(\frac{k_B}{e}\right)^2$$

which is equal to 2.44301×10^{-8} WΩ/K^2 in SI units and 2.71821×10^{-13} erg K^{-2} cm^{-1} (=esu/K)

((**Experimental results**))

Lorenz number $\times 10^8$ WΩ/K^2

	T = 273 K	T = 373 K
Ag	2.31	2.37
Au	2.35	2.40

Cd	2.42	2.43
Cu	2.23	2.33
Ir	2.49	2.49
Mo	2.61	2.79
Pb	2.47	2.56
Pt	2.51	2.60
Sn	2.52	2.49
W	3.04	3.20
Zn	2.31	2.33

The simple relation between thermal and electrical conductivity applies component by component to electrical and thermal conductivity so long as the relation time approximation is valid at 300 K. The comparison of theory and experiment is considerably less successful at 20 K, because at low temperatures the relaxation time approximation is less reliable. (M.P. Madar).

4. Widerman-Franz law and Maxwell-Boltzmann statistics

Drude used the Maxwell-Boltzmann statistics to interpret the Wiedermann-Franz law. The heat capacity in the Maxwell-Boltzmann statistics is

$$c_V = \frac{C}{V} = \frac{3}{2} n k_B$$

where we use the relation;

$$C = \frac{3}{2} N k_B$$

The thermal conductivity is

$$\kappa_e = \frac{1}{3} c_V v l = \frac{1}{2} n k_B \langle v^2 \rangle \tau = \frac{3n}{2m} \tau {k_B}^2 T$$

where

$$\frac{1}{2} m \langle v^2 \rangle = \frac{3}{2} k_B T$$

This relation leads to a simple relation between κ and σ;

$$L_{classic} = \frac{\kappa}{\sigma T} = \frac{3}{2}\left(\frac{k_B}{e}\right)^2$$

which is equal to 1.11388 x 10^{-8} WΩ/K^2.

3. Pauli susceptibility

The Pauli susceptibility

$$\chi_P = {\mu_B}^2 D(\varepsilon_F) = \frac{3N{\mu_B}^2}{2\varepsilon_F}$$

or

$$\frac{\chi_P}{V} = \frac{3n{\mu_B}^2}{2\varepsilon_F}$$

The ratio:

$$\frac{C}{T\chi_{Pauli}} = \frac{\pi^2 {k_B}^2}{3{\mu_B}^2}$$

which is equal to 7.2914. Note that this ration can be evaluated directly as

$$\frac{C}{T\chi_{Pauli}} = \frac{\frac{1}{3}\pi^2 {k_B}^2 T D(\varepsilon_F)}{{\mu_B}^2 D(\varepsilon_F) T} = \frac{\pi^2 {k_B}^2}{3{\mu_B}^2}$$

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