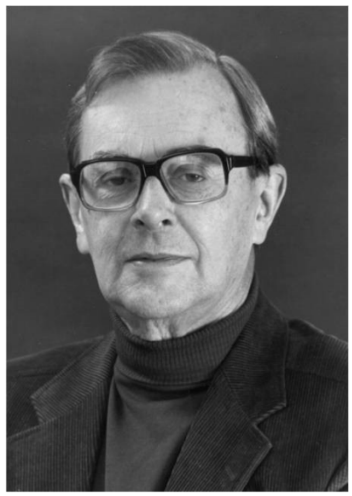
Shoenberg effect (magnetic interaction effect) in de Haas-van Alphen effect

Masatsugu Sei Suzuki
Physics Department, SUNY at Binghamton
(Date March 31, 2013)

Sir Alfred Brian Pippard, ScD, FRS (7 September 1920, Earl's Court, London – 21 September 2008, Cambridge), was a British physicist. He was Cavendish Professor of Physics from 1971 until 1984 and an Honorary Fellow of Clare Hall, University of Cambridge, of which he was the first President. He was educated at Clifton College.



http://en.wikipedia.org/wiki/Brian_Pippard

1. Introduction

The electrons in a metal move along the cyclotron orbits. The induced magnetic fields must therefore average out over the regions with dimensions of the order r_{H}

$$r_H = \frac{c\hbar k_F}{eH} \approx 10^{-3} \text{ cm}$$

in a field of 10 kOe. The mean distance between electrons is about 1 Å. Thus it is clear that the field acting on the conduction electron is the average field inside the metal. The average of the microscopic field is simply the magnetic induction $\boldsymbol{B}$. In the situation where the susceptibilti becomes large, the field $\boldsymbol{H}$ is not equal to B, and it will be related to $\boldsymbol{B}$ through the magnetization $M(B)$ as

$$\boldsymbol{H} = \boldsymbol{B} - 4\pi \boldsymbol{M}(\boldsymbol{B})$$

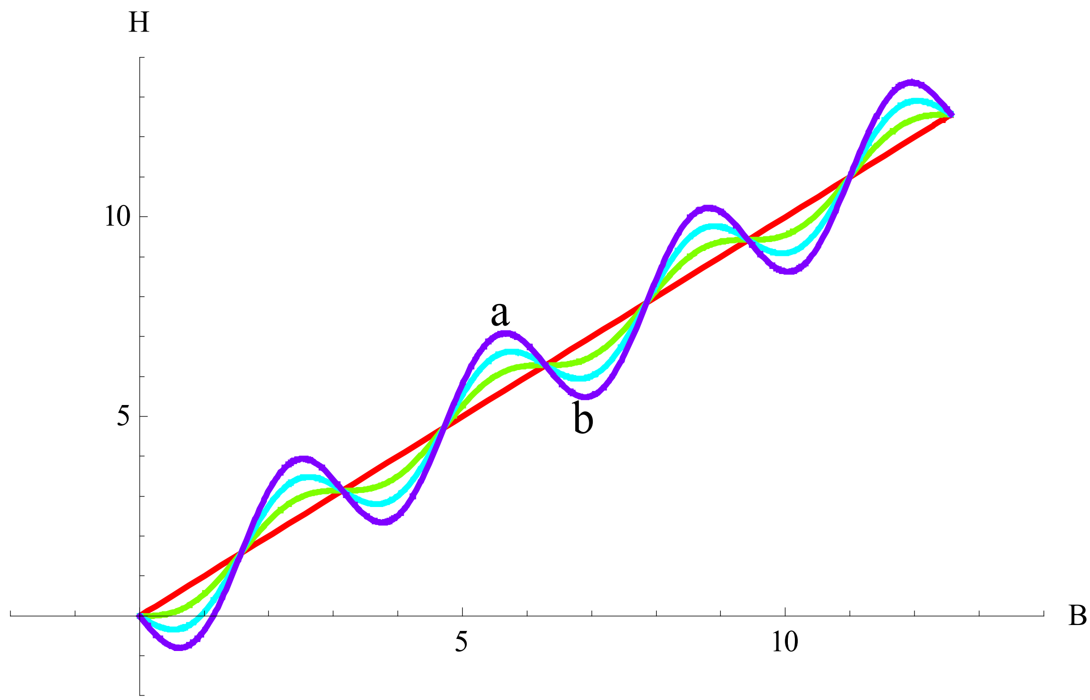
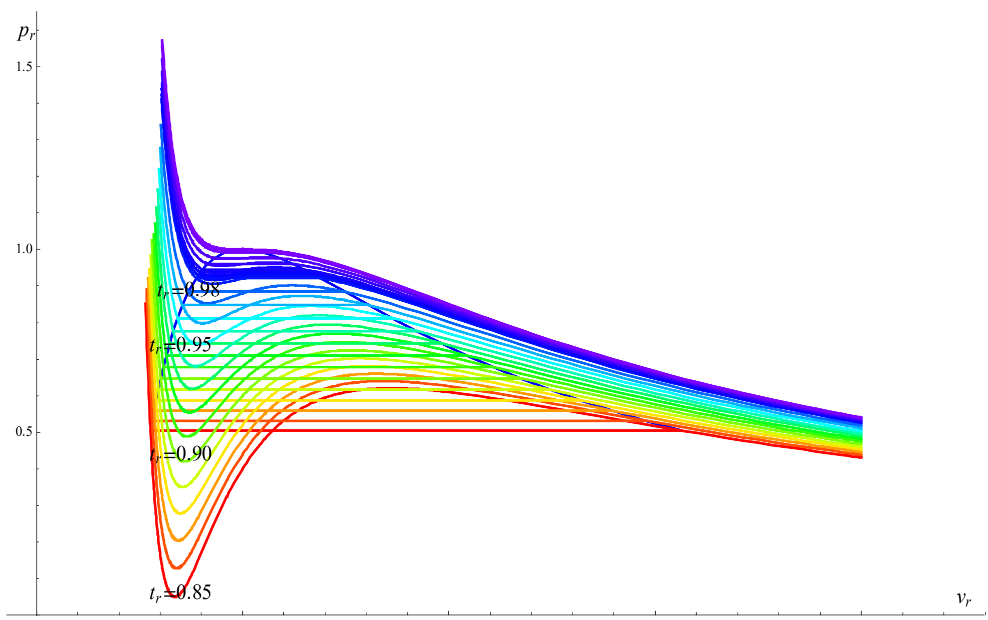


Fig. Relation between external magnetic field H and magnetic induction B arising as a result of dHvA effect. The portion a-b of the curve is unstable; $(\partial H / \partial B)_T < 0$.

When $|\boldsymbol{M}(\boldsymbol{B})| << |\boldsymbol{B}|$, $\boldsymbol{H} \approx \boldsymbol{B}$. However, if $|\boldsymbol{M}(\boldsymbol{B})| \approx |\boldsymbol{B}|$, the situation becomes different. For some values of *H*, there are three values of *B*. Such a situation suggests an instability similar to that occuring in the *P*-*V* phase diagram representing the van der Waals equation of state.



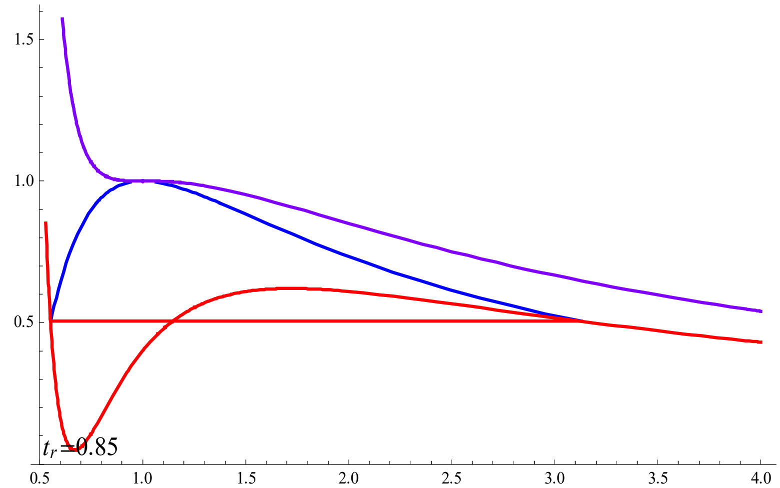


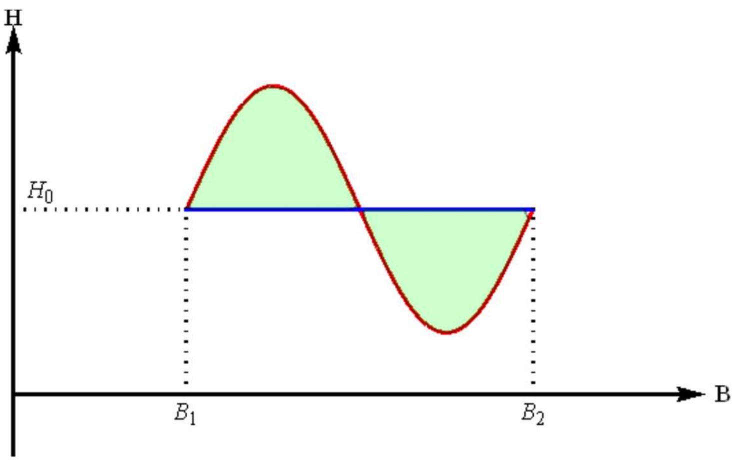
Fig. *P-V* phase diagram for the van der Waals gas with constant temperature (isotherm) exhibiting an unstable portion where $\partial P/\partial V < 0$.

In the case of the van der Waals equation of state, the portion of the P-V isothem where $(\partial P/\partial V)_T < 0$ is unstable thermodynamically. The condition for the appearance of discontinuous jump in B is the appearance of the interval with $(\partial H/\partial B)_T < 0$ and the *H-B* curve or, equivalently

$$\chi = \frac{\partial M(B)}{\partial B} > \frac{1}{4\pi}.$$

The shaded area showm in the figure below should be equal. Equivalently,

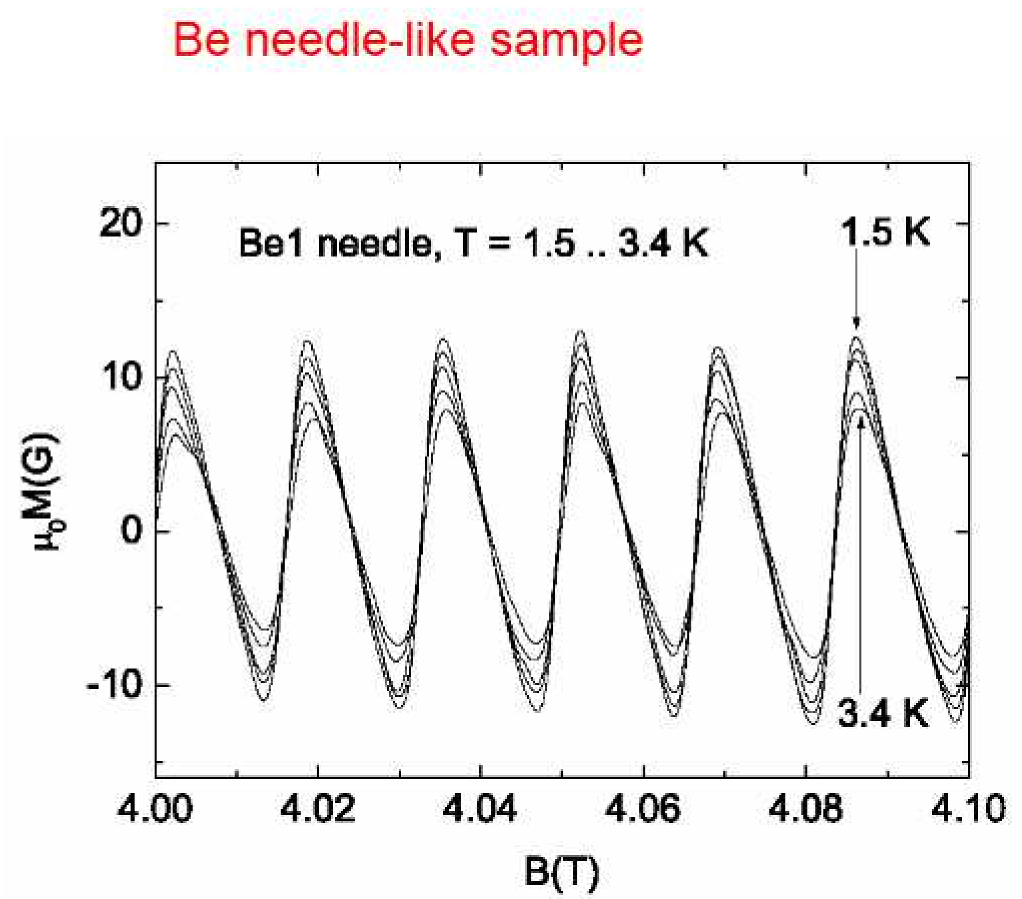
$$\int_{B_1}^{B_2} (H - H_0) dB = 0$$



2. Experimental results (Shoenberg effect)

The Shoenberg effect has been studied in the noble metals and particularly in Be where it is very strongly marked.

(1) Be needle-like sample



(2) Ag

A very interesting experimental result on the Shoenberg effect has been reported by Kramer et al. for Ag sample under strong Landau quantization in magnetic fields up to 10 T. They use a set of micro Hall probes for the detection of the local induction. Their results are reported in Phys. Rev. Lett. **95**, 267209 (2005).

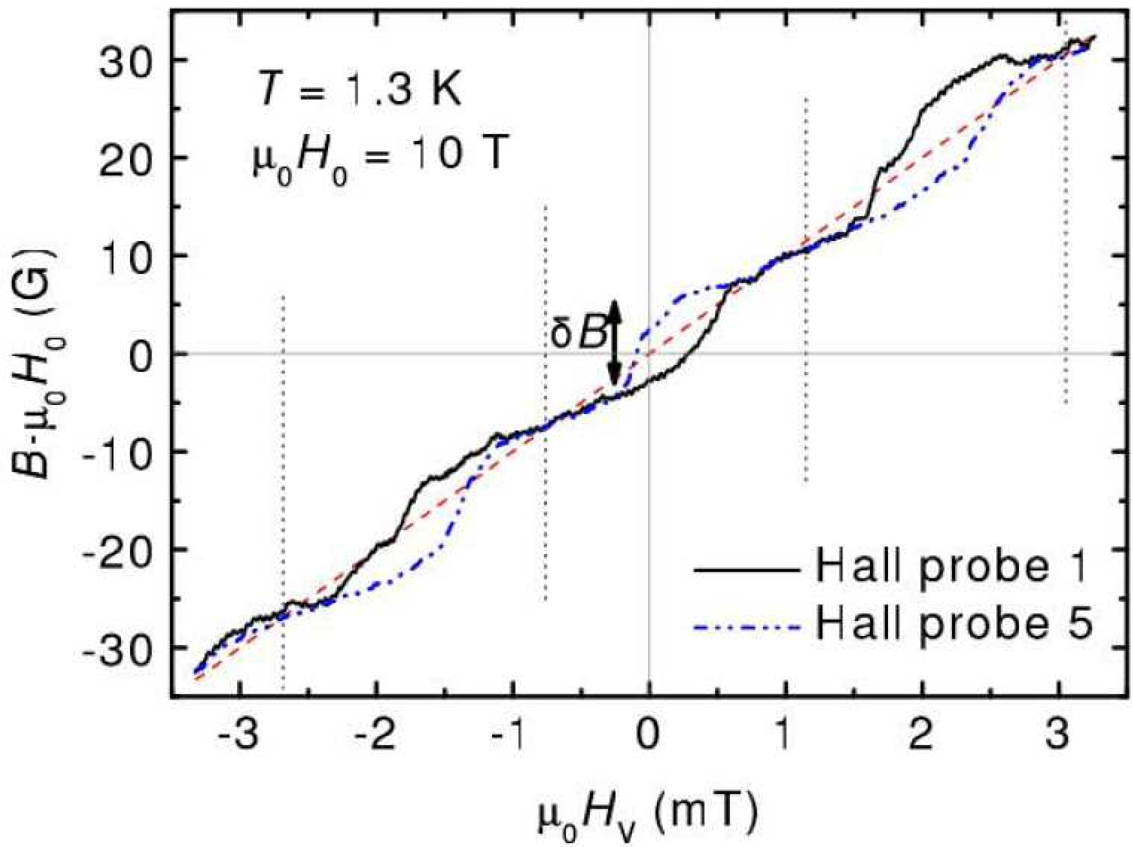


FIG. 2 (color online). $B(H)$ trace for silver single crystal with $H \parallel [100]$ showing the splitting δB of B_1 and B_5 of the L array for three dHvA periods separated by dotted lines. The dashed line is a guide to the eye for $B - \mu_0 H_0 = \mu_0 H_V$.

(Kramer et al. PRL. 95, 267209 (2005).)

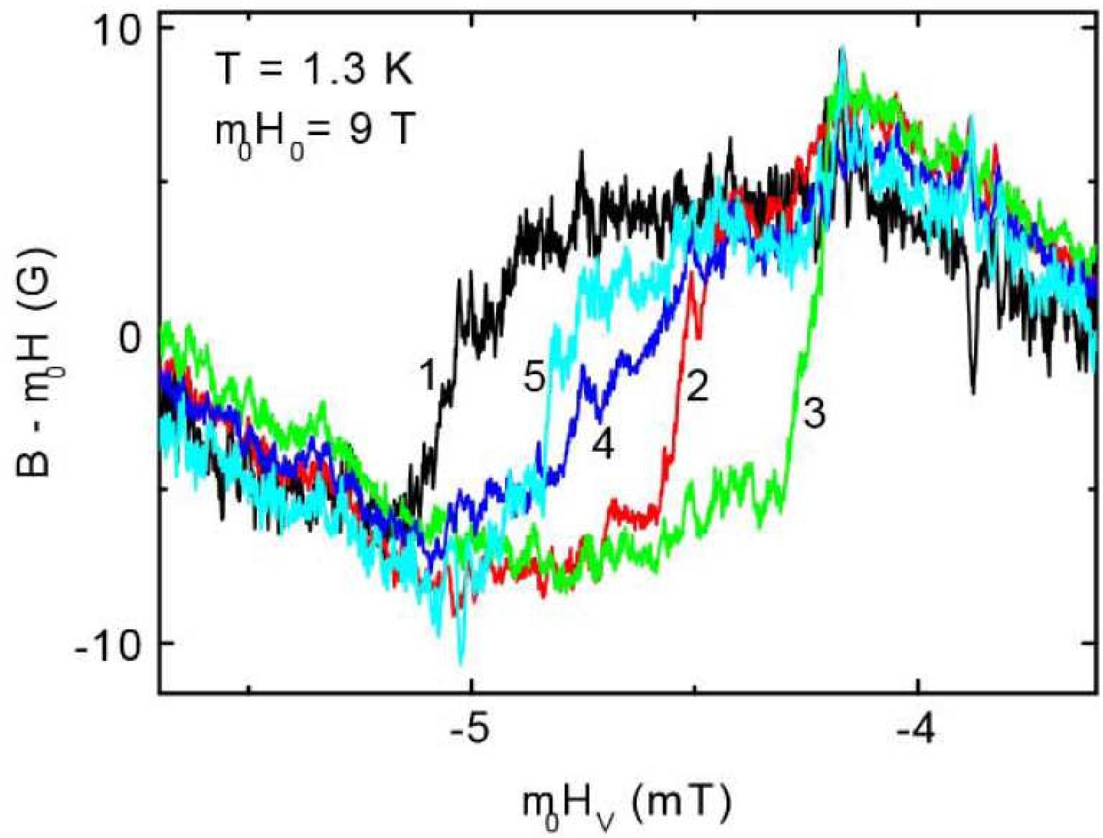


FIG. 4 (color online). Example of successive transitions for five T probes between diamagnetic and paramagnetic phase. The sweep rate was 0.5 mT/ min.

(Kramer et al. PRL 95, 267209 (2005))

---

3. Model for the Shoenberg effect (by Pippard)

Here we discuss the Shoenberg effect, which was discovered by Shoenberg in the course of experiments on the de Haas-van Alphen effect in the noble metals (1962). He observed a line shape from of de Haas-van Alphen oscillation from sinusoidal oscillation, which is dependent on the demagnatization factor. According to Pippard. there is one interaction which leads to observable effects, and which fortunately is a macroscopic in character and capable of analysis without probing too deeply into fundamentals. This is the magnetic interaction which becomes important when the de Haas-van Alphen effect is strong enough to make the internal field in the sample significantly different from the applied field H (Shoenberg effect). The internal field responsible for determining the energy levels inside the sample is the macroscopic B, and in the highly oscillatory de Haas-van Alphen effect it does not take much to make B differ from H by a substantial fraction of a cycle of the oscillation.

The oscillatory magnetization due to the dHvA can be described by

$$M = M_0 \sin\left(\frac{2\pi F}{H} - \phi\right),$$

where ϕ is a phase factor and F is the dHvA frequency, and H is the external magnetic field. Suppose that H is changed around $H = H_0$;

$$H = H_0 + h, \qquad B = H_0 + b$$

where $||h|$ is very small compared to H_0.Then we have

$$M = A_0 \sin\left(\frac{2\pi F}{H_0^2} h\right)$$

$$I = I_0 \sin\left(\frac{2\pi F}{H_0^2} h\right) = I_0 \sin(\lambda H)$$

where

$$\lambda = \frac{2\pi F}{H_0^2}$$

The magnetic induction (effective magnetic field) B is expressed by

$$B = H + 4\pi M = H_0 + h + 4\pi M = H_0 + b$$

where

$$b = h + 4\pi M$$

We now simply write the magnetization M as

$$M = M_0 \sin(\lambda b)$$

where h is replaced by b.

Pippard has considered the problem of magnetic interaction and has proved that Shoenberg's conjecture is true, i.e., the dHvA oscillations are simple oscillating function of the magnetic induction *B*. Over limited ranges of field, the magnetization can be approximated by

$$M = M_0 \sin(\lambda b)$$

where the magnetic induction *B* is related to an external magnetic field *H* as

$$b = h + 4\pi(1 - D)M$$

where *D* is the demagnetization factor. From the above equations, we have

$$\begin{aligned} y &= 4\pi\lambda M \\ &= 4\pi M_0 \sin(\lambda b)] \\ &= a \sin[x + (1 - D)y] \end{aligned}$$

where

$$y = 4\pi\lambda M\,, \qquad x = \lambda h\,, \qquad a = 4\pi\lambda M_0\,,$$

Note that λb is expressed by

$$\begin{aligned} \lambda b &= \lambda[h + 4\pi(1 - D)M] \\ &= x + (1 - D)y \end{aligned}$$

The criterion for the magnetic interaction to be significant is simply

$$a>2.$$

In order to solve the nonlinear equations, we introduce

$$\theta = x + (1 - D)y\,.$$

Then we get

$$y = a \sin\theta$$

$$x = \theta - a(1 - D)\sin\theta\,.$$

In other words, x and y are expressed in terms of θ, a and D. The Gibbs free energy is obtained as

$$G = -\int M dh = -\frac{1}{4\pi\lambda^2}\int y dx ,$$

or

$$g = 4\pi\lambda^2 G = -\int y dx .$$

This can be rewritten as

$$\begin{aligned} g &= -\int y dx = -\int a\sin\theta[1 - a(1-D)\cos\theta]d\theta \\ &= -\int [a\sin\theta - a^2(1-D)\sin\theta\cos\theta]d\theta \\ &= -\int [a\sin\theta - \frac{a^2}{2}(1-D)\sin 2\theta]d\theta \\ &= a\cos\theta - \frac{a^2}{4}(1-D)\cos(2\theta) + g_0 \end{aligned}$$

in terms of θ, where g_0 is a constant. The local maximum and local minimum of the value of x for the curve of x vs θ, are obtained as

$$\frac{dx}{d\theta} = 1 - a(1-D)\cos\theta .$$

When $\frac{dx}{d\theta}$=0, we have

$$\frac{1}{a(1-D)} = \cos\theta , \qquad \text{or} \qquad \theta = \arccos[\frac{1}{a(1-D)}] .$$

This problem is very similar to the phase transition of van der Waals system such as (liquid-gas transition).

4. Numerical calculation

Here we solve the problem by using the Mathematica in the following ways. We show typical results of y vs x, g vs x, and b vs h) where the magnetization factor D and $a(= 4\pi\lambda M_0)$

are appropriately chosen as parameters. We notice that the dHvA oscillation (y vs x) becomes distorted when a approaches unity from small a (<1). For a>1, y is a multi-valued function of x.

(i) y vs x (magnetization vs external magnetic field) and g vs x (free energy vs an external magnetic field)

ParametricPlot of { $x = \theta - a(1-D)\sin\theta$, $y = a\sin\theta$ }

ParametricPlot of {x, g}

(ii) b vs h (magnetic induction vs external magnetic field) relation

ParametricPlot of {x, θ}, where $\lambda b = \theta$, $\lambda h = x$

(1) $a = 2, D = 0.$

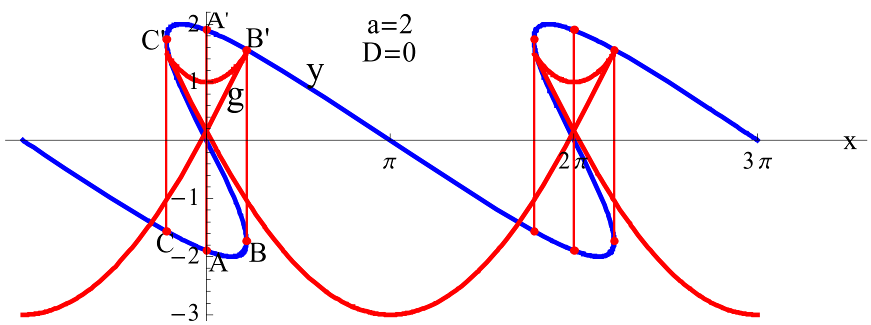


Fig. y vs x (denoted by blue line). g vs x (denoted by red line).

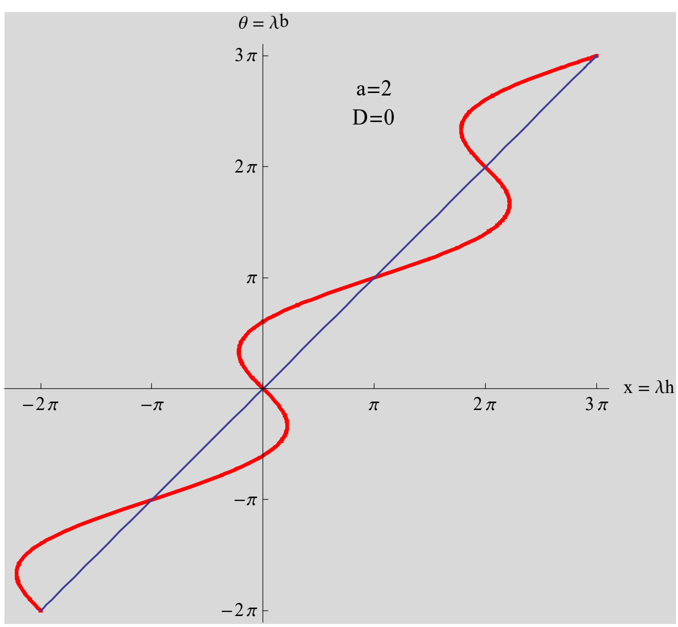
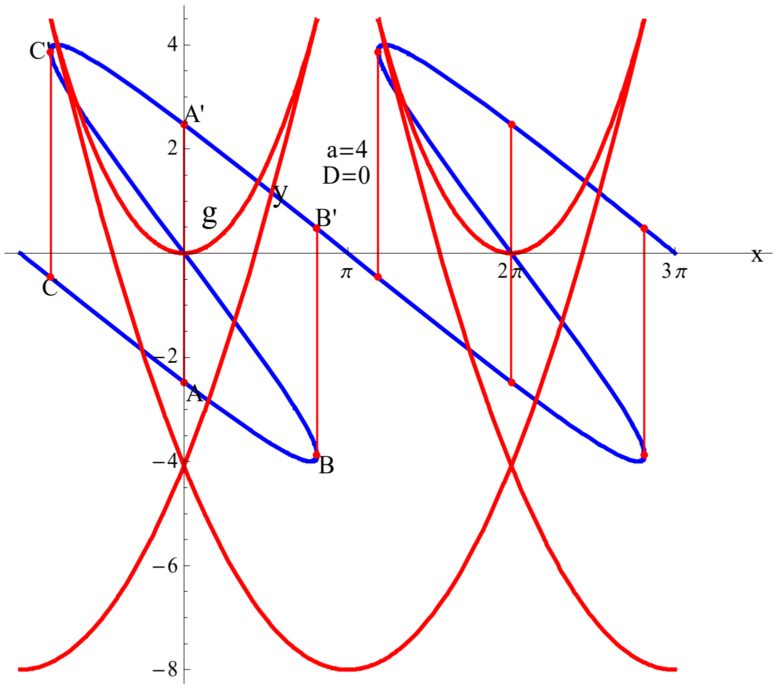
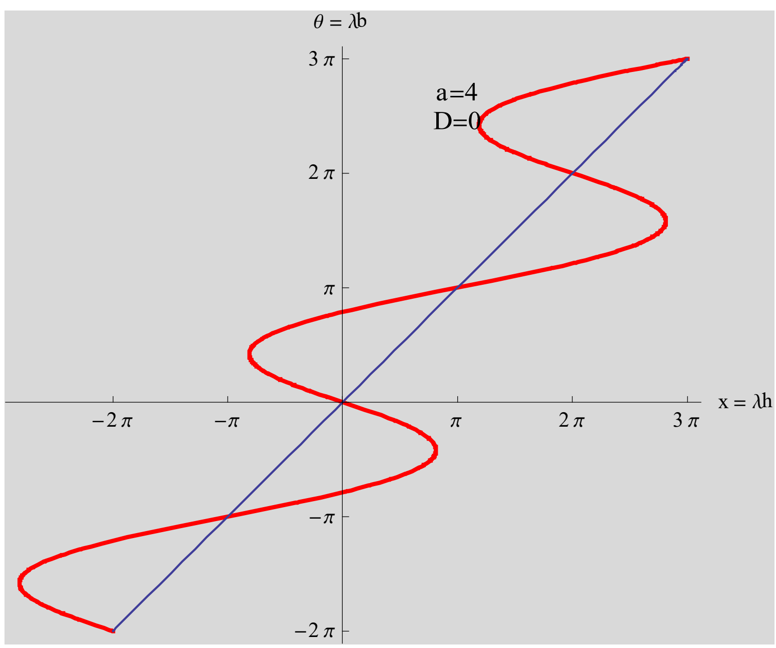


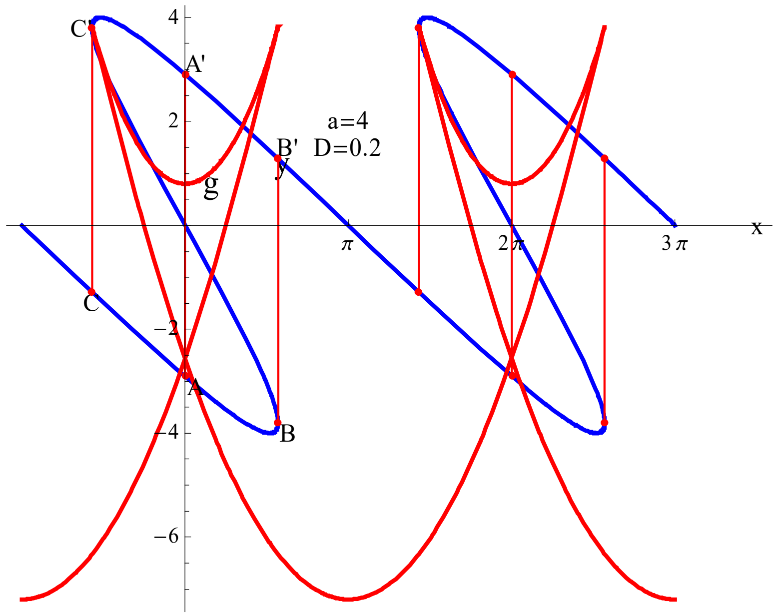
Fig. Relation between λb vs λh.

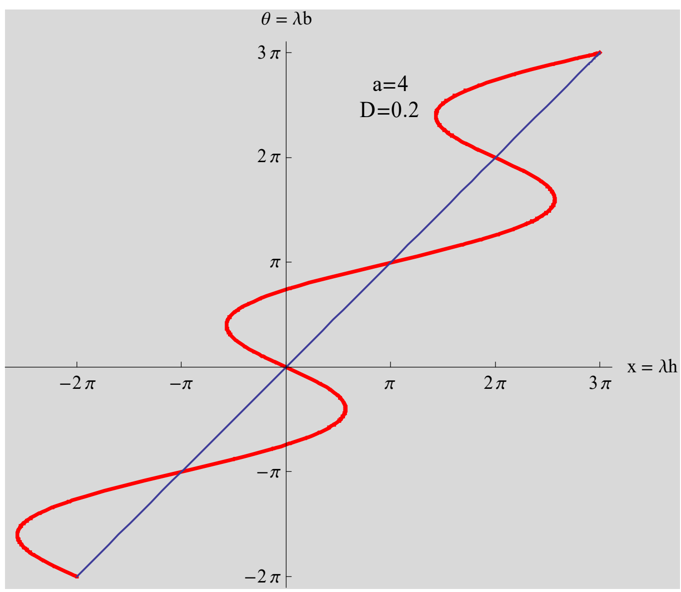
(2) $a = 4.\ D = 0.$





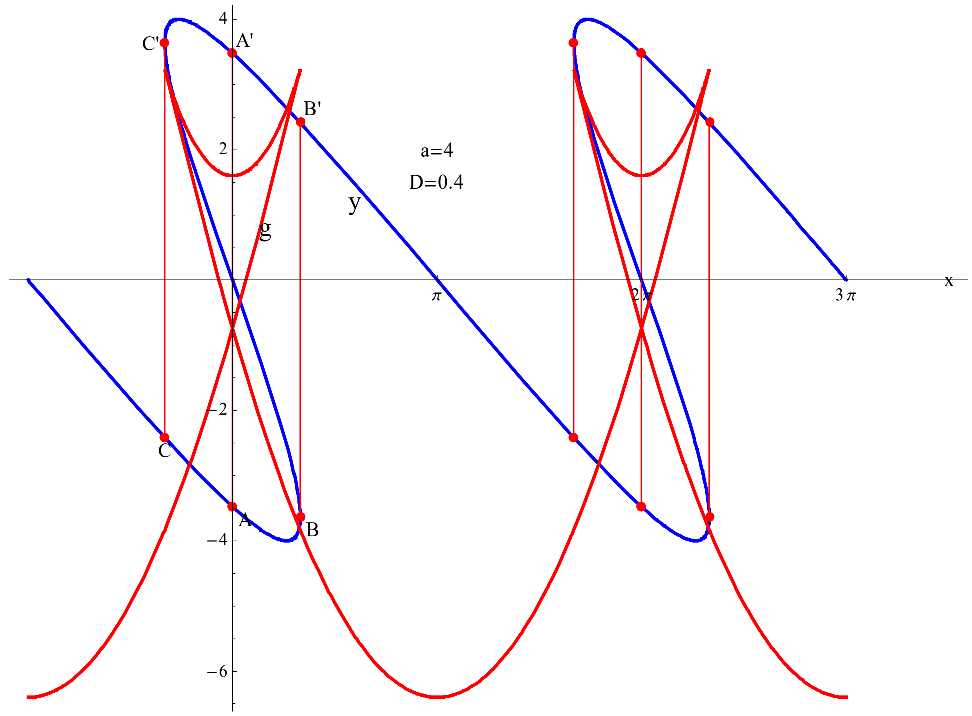
(3) $a = 4.\ D = 0.2.$

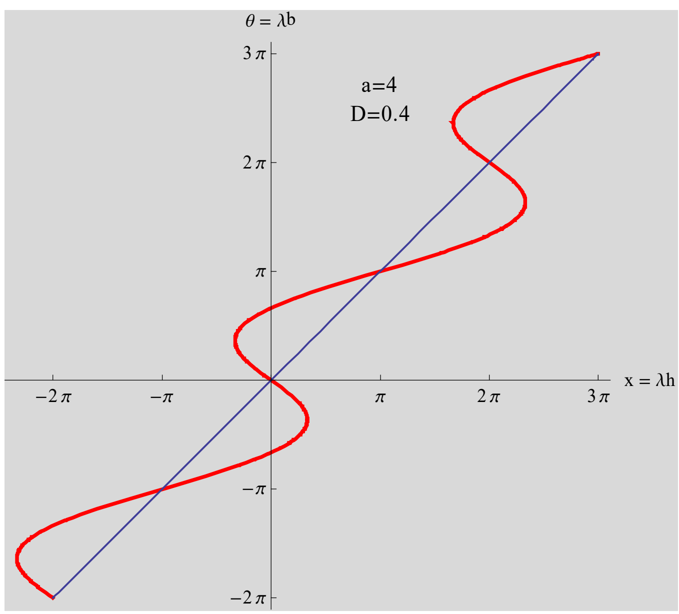




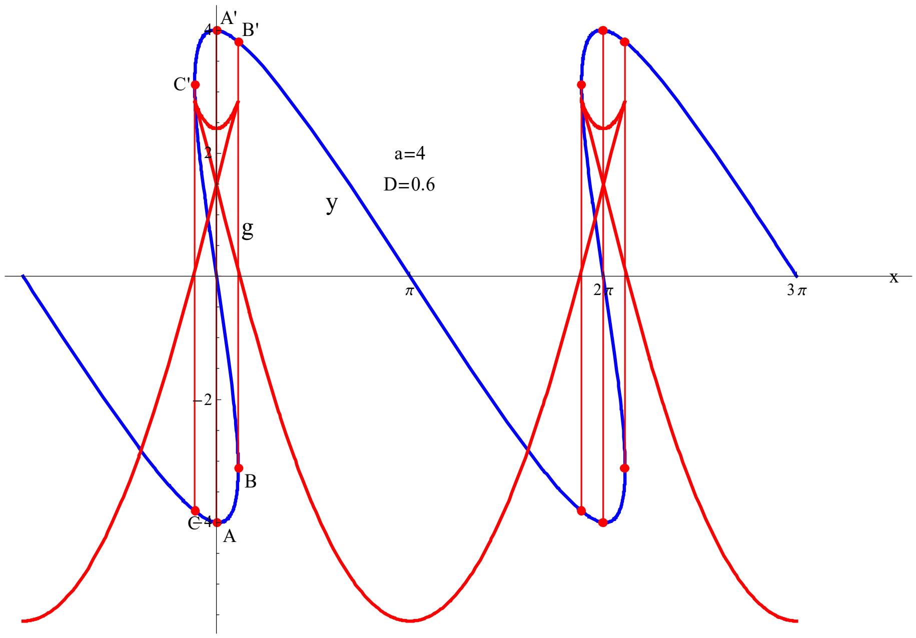
---

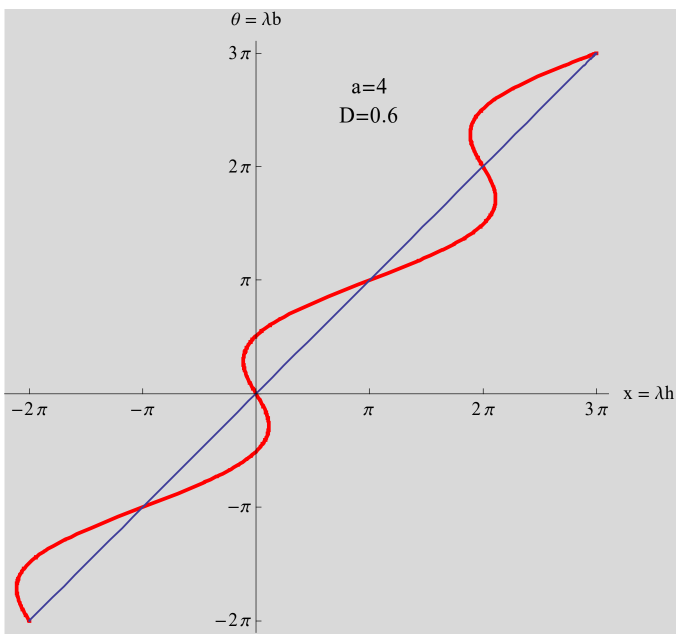
(4) $a = 4.\ D = 0.4.$


4
2
−2
−4
−6
C'
A'
B'
C
A
B
a=4
D=0.4
y
g
x
π
2π
3π


θ = λb
3π
2π
π
−π
−2π
a=4
D=0.4
x = λh
−2π
−π
π
2π
3π

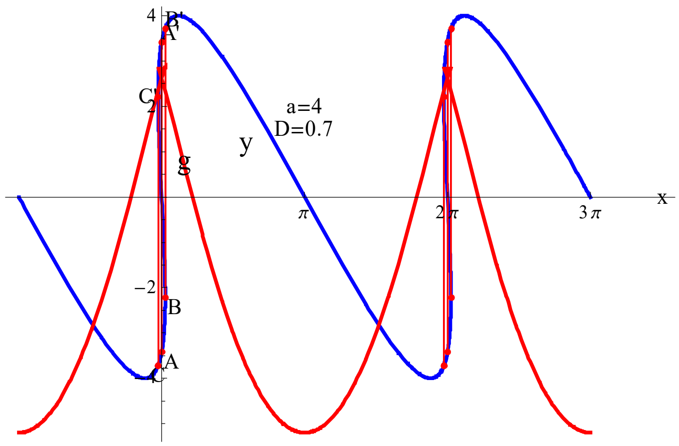
(5) $a = 4.\ D = 0.6$

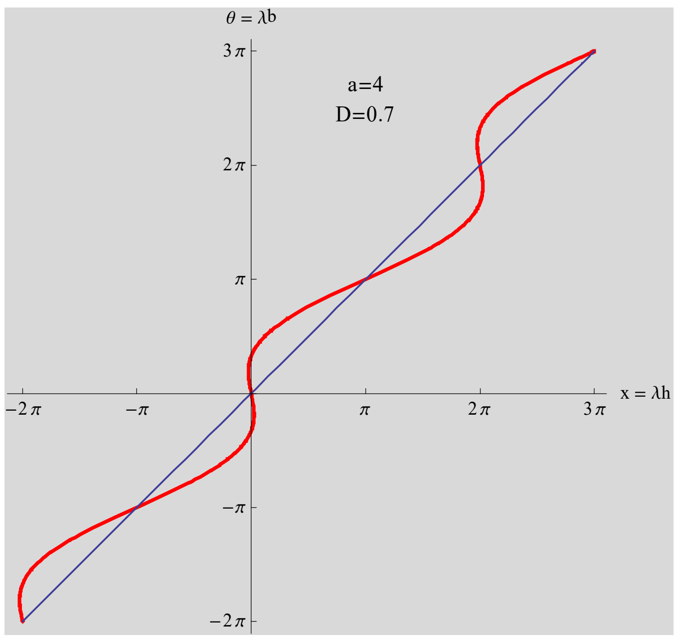

A'
B'
C'
a=4
D=0.6
y
g
x
π
2π
3π
B
C
A
4
2
−2
−4


θ = λb
3π
2π
π
−π
−2π
a=4
D=0.6
x = λh
−2π
−π
π
2π
3π

---

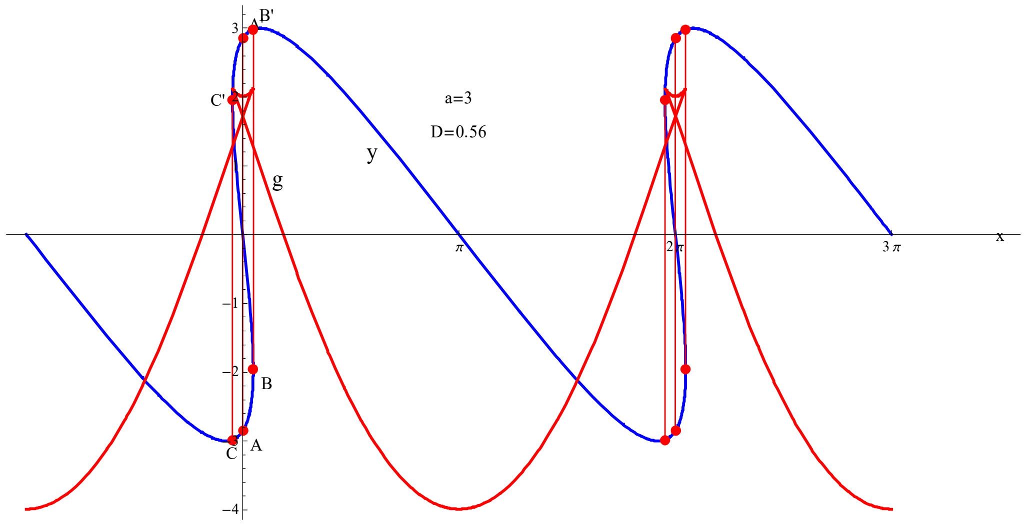
(6) $a = 4.\ D = 0.7$

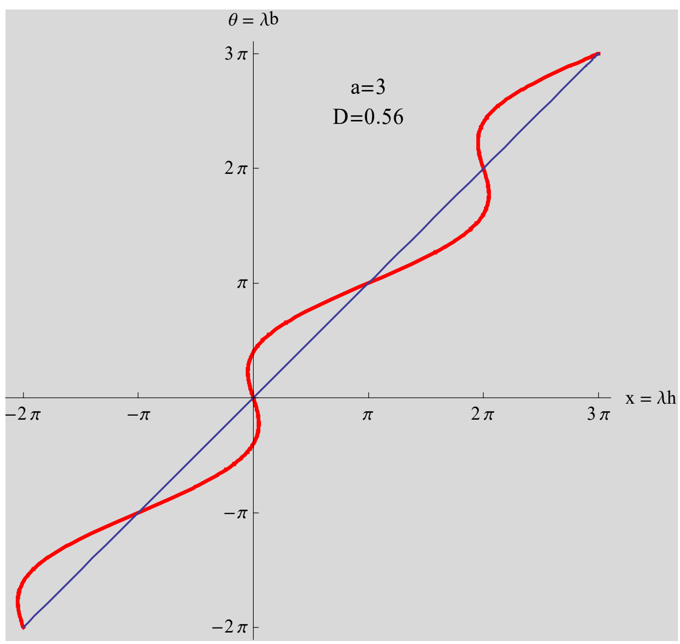




---

(7) $a = 3.\ D = 0.56$





REFERENCES

D. Shoenberg, Phil. Trans. Roy. Soc. (London) A**255**, 85 (1962).

A.B. Pippard, Proc. Roy. Soc. (London) A**272**, 192 (1963).

A.B. Pippard, Theory of Metals, I Electron, edited by J.M. Ziman, p.154 (Cambridge University Press, 1969)

D. Shoenberg, *Magnetic oscillations in metals*, Cambridge University Press Cambridge, 1984).

D.H. Condon; Phys. Rev. **145**, 526 (1966).

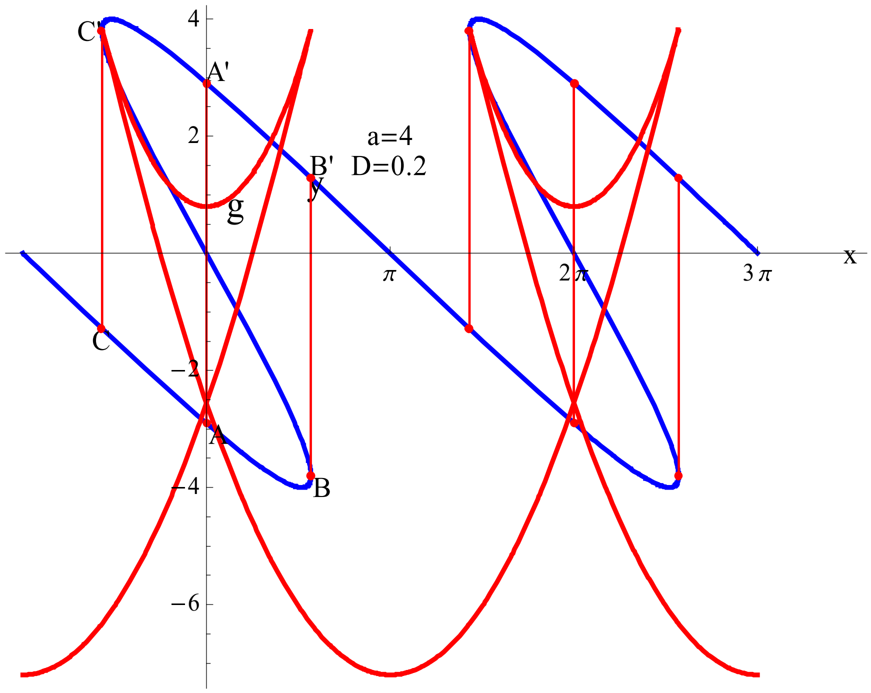
R.B.G. Kramer, Ph.D. Thesis, "Magnetic Oscillations in Metals Condon Domain," Université Jopseph Fourier – Grenoble I en Physique (2005).

R.B.G. Kramers, V.S. Egorov, V.A. Gasparov, A.G.M. Jansen, and W. Joss, Phys. Rev. Lett. **95**, 267209 (2005).

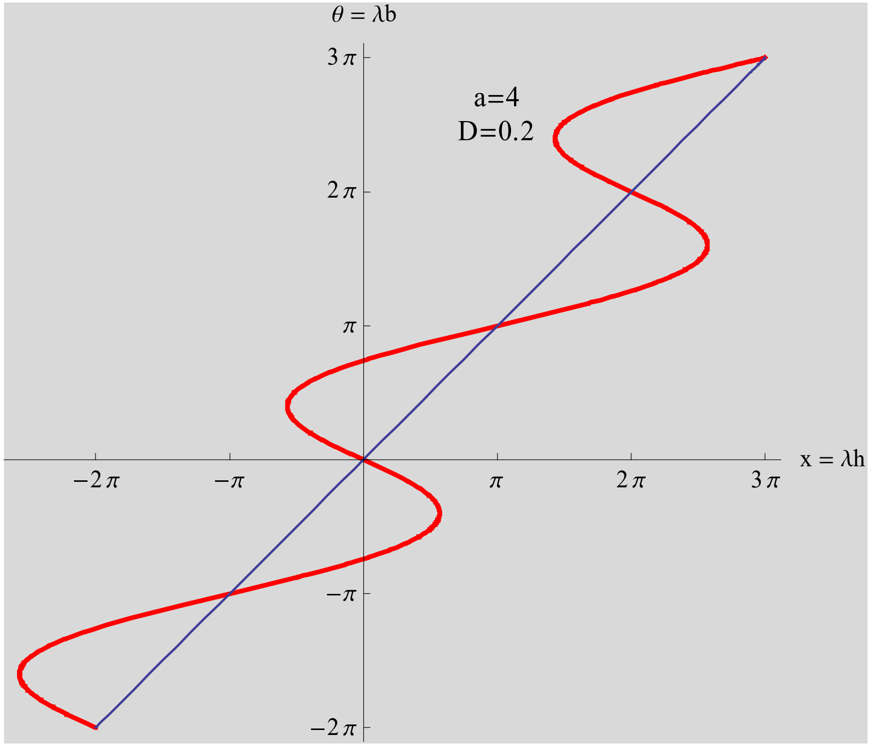
APPENDIX

Mathematica program

```
Clear["Global`*"]; a = 4; D1 = 0.2; x1 = θ - a (1 - D1) Sin[θ];
y1 = a Sin[θ]; g1 = a Cos[θ] - a^2/4 (1 - D1) Cos[2 θ]; θ1 = ArcCos[1/((1 - D1) a)];
x11 = x1 /. θ → θ1 // N; y11 = y1 /. θ → θ1 // N;
eq1 = FindRoot[x1 == 0, {θ, π / 20, π}]; y22 = y1 /. eq1[[1]];
eq2 = FindRoot[x1 == -x11, {θ, π / 20, π}]; y21 = y1 /. eq2[[1]] ;
f0 =
 Graphics[{PointSize[0.02], Red,
   Table[{PointSize[0.01], Red, Point[{x11 + 2 n π, y11}],
     Point[{-x11 + 2 n π, y21}], Point[{-x11 + 2 n π, -y11}],
     Point[{x11 + 2 n π, -y21}], Point[{2 n π, -y22}],
     Point[{2 n π, y22}], Line[{{x11 + 2 n π, -y21}, {x11 + 2 n π, y11}}],
     Line[{{-x11 + 2 n π, -y11}, {-x11 + 2 n π, y21}}],
     Line[{{2 n π, -y22}, {2 n π, y22}}]}, {n, 0, 1, 1}] }] ;
f1 = ParametricPlot[{x1, y1}, {θ, - π, 3 π},
  Ticks → {Range[0, 4 π, π]}, PlotStyle → {Blue, Thick}];
f2 = ParametricPlot[{x1, g1}, {θ, - π, 3 π},
  Ticks → {Range[- π, 3 π, π]}, PlotStyle → {Red, Thick}];
f3 = Graphics[{Text[Style["g", Black, 15], {0.5, 0.8}],
   Text[Style["y", Black, 15], {0.6 π, 1.2}],
   Text[Style["x", Black, 12], {3.5 π, 0}],
   Text[Style["C", Black, 12], {x11, -y21 - 0.2}],
   Text[Style["C'", Black, 12], {x11 - 0.2, y11}],
   Text[Style["A", Black, 12], {0.2, -y22 - 0.2}],
   Text[Style["A'", Black, 12], {0.2, y22 + 0.2}],
   Text[Style["B", Black, 12], {-x11 + 0.2, -y11 - 0.2}],
   Text[Style["B'", Black, 12], {-x11 + 0.2, y21 + 0.2}],
   Text[Style["a=" <> ToString[a], Black, 12], {π, 2.0}],
   Text[Style["D=" <> ToString[D1], Black, 12], {π, 1.5}]}];
Show[f1, f2, f3, f0, PlotRange → All]
```




```
f4 = ParametricPlot[{x1, θ}, {θ, -2 π, 3 π},
  Ticks → {Range[- 2 π, 3 π, π], Range[- 2 π, 3 π, π]},
  PlotStyle → {Red, Thick}, Background → LightGray,
  AxesLabel → {"x = λh", "θ = λb"}]; f5 = Plot[θ, {θ, -2 π, 3 π}];
f6 = Graphics[{Text[Style["a=" <> ToString[a], Black, 12], {π, 8.5}],
   Text[Style["D=" <> ToString[D1], Black, 12], {π, 7.7}]}];
Show[f4, f5, f6]
```



θ = λb
3π
2π
π
−π
−2π
a=4
D=0.2
x = λh
−2π
−π
π
2π
3π