Azbel-Kaner cyclotron resonance

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(Date: April 03, 2013)

A part of The development of the band theory of solids. L. Hoddeson et al, Out of Crystal maze (Oxford University Press, 1992).

The discovery of cyclotron resonance in metals opened up a new line of investigation of the Fermi surface of metals. Azbel remembers that when he first presented the idea at a seminar attended by Landau, the latter immediately raised a series of objections, and that only after two days of thrashing out the various possibilities was Landau convinced. Subsequently, Azbel and his younger Kharkov colleague E.A. Kaner showed how this technique could be used to determine Fermi surfaces. If the shape of the Fermi surface was known from other experimental methods, a study of the cyclotron resonance frequency could give the velocity of conduction electrons at every point on the Fermi surface, a quantity of fundamental importance in all problems involving electron transport,

I. Introduction

One of the important methods of observing the cyclotron resonance in a metal was proposed by Azbel and Kaner (1956). In the presence of a magnetic field $\boldsymbol{B}$, the electron undergoes a cyclotron motion. When the electron passes through the skin depth (anomalous) just near the surface, it is accelerated by the rf (radio frequency) electric field. The Azbel-Kaner cyclotron resonance (AKCR) occurs when the period of cyclotron motion is the same as that of the rf field (in phase). Only when the electron passes through the skin depth, the electron can be accelerated by the rf electric field. The method of AKCR is a powerful tool in determining the structure of the Fermi surface, as well as the de Haas-van Alphen effect.

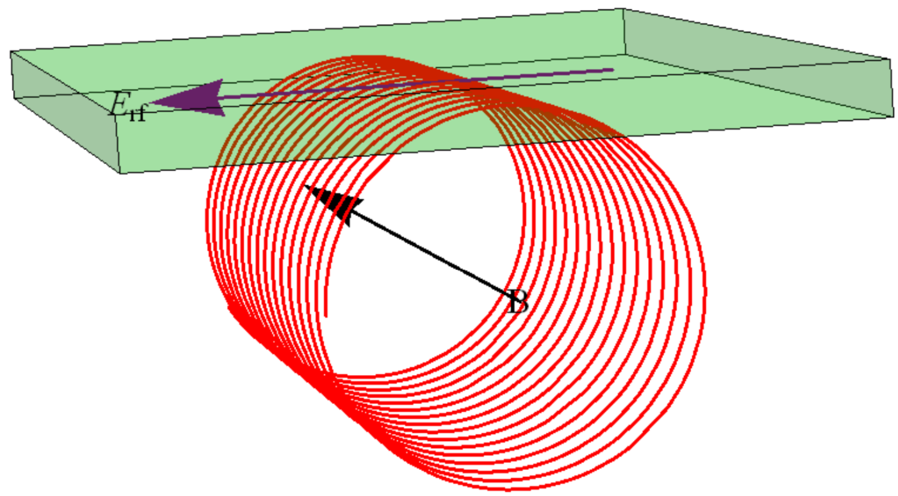

Fig.1 Azbel-Kaner type cyclotron resonance in metal.

2. Skin depth

If the frequency ω is not too high, such a field will penetrate into the metal a distance δ_c (the classical skin depth) given by

$$\delta_c = \sqrt{\frac{c^2}{2\pi\sigma\omega}} \;.$$

The derivation of this depth assumes that the field in the metal varies little over a mean free path; $\delta_c >> l$. When δ_c is comparable to l a much more complicated theory is required. First we evaluate the mean free path in Cu at 4 K. The conductivity and the relaxation time of Cu are given by

$$\sigma = 5.29\times10^{17}\times10^{5} = 5.29\times10^{22} \;\; (1/\mathrm{s})$$

and

$$\tau = \frac{\sigma m}{ne^2} = 2.47 \text{ x } 10^{-9} \text{ (s)}$$

where n is the number density in Cu and

$$n = 8.47 \text{ x } 10^{22}/\mathrm{cm}^3.$$

When v_F = 1.57 x 10^8 cm/s for Cu, then the mean free path l at 4 K is evaluated as

$$l = 1.57\times10^{8}\times2.47\times10^{-9} = 0.3 \text{ (cm)}.$$

On the other hand, the classical skin depth in Cu can be calculated as

$$\delta_c = \frac{65.6}{\sqrt{f_0(GHz)}} \text{ [Å]}$$

At 300 K, the conductivity of Cu is given by

$$\sigma = 5.29\times10^{17} \text{ s}^{-1}$$

Then the skin depth is calculated as

$$\delta_c = \frac{2.0745\times10^{-4}}{\sqrt{f_0(GHz)}} \text{ [cm]}$$

On the other hand, at 4.2 K, $\delta_c = 20.74$ Å for f = 10 GHz, which is much smaller than the electron mean free path. When $\delta_c << l$ one is in the extreme anomalous regime. The fact is that not all the electrons are participating in the absorption of the lectromagnetic wave. Only those that are running inside the skin depth for most of a mean free path l are capable of picking up much energy from the rf electric field. If the skin depth is δ', then only a fraction δ'/l of electrons are effective in the conductivity. Then the conductivity σ' can be expressed by

$$\sigma' = \frac{3}{2}\beta\frac{\delta'}{l}\sigma .$$

where the number β is just a fudge factor, $\beta = \frac{8\pi}{3\sqrt{3}}$. The skin depth δ' is given by

$$\delta' = \sqrt{\frac{c^2}{2\pi\sigma'\omega}} = \sqrt{\frac{c^2}{2\pi\frac{3}{2}\beta\frac{\delta'}{l}\sigma\omega}}$$

which leads to the expressions of δ' and as

$$\delta' = \left(\frac{c^2}{3\pi\beta\frac{\sigma\omega}{l}}\right)^{1/3}.$$

and

$$\sigma' = \frac{3}{2}\beta\frac{\sigma}{l}\left(\frac{c^2}{3\pi\beta\frac{\sigma\omega}{l}}\right)^{1/3} = \left(\frac{9\beta^2}{8\pi}\right)^{1/3}\frac{1}{\omega^{1/3}}\left(\frac{c\sigma}{l}\right)^{2/3}$$

Thus the effective conductivity s' appears to behave as $\omega^{-1/3}$. Note that

$$\delta' = \left(\frac{2l\delta_c^{\,2}}{3\beta}\right)^{1/3}$$

3. Condition for AKCR

In the presence of an external magnetic field along the surface of Cu, the radius of the orbit is evaluated by

$$r_H = \frac{c\hbar}{eH}k_F = \frac{8.952}{H(Oe)} \text{ [cm]}$$

where k_F = 1.36 x 10^8 cm^{-1}. This implies that r_H is much larger than the anomalous skin depth. Since the field does not penetrate far into the metal, electrons can absorb energy only when they are within the skin depth of the surface. Because of the dimensions of the electron's real space orbit at the Fermi surface are comparable to the mean free path, the skin depth will also be small compared with the size of orbit.

Another important factor of the condition for the ACKR is that

$$T_E = \frac{2\pi}{\omega_c} << \tau,$$

or

$$\omega_c\tau >> 1.$$

where ω_c is the cyclotron angular frequency. The electron undergoes cyclotron roatation many times inside the metal within the relaxation time τ.

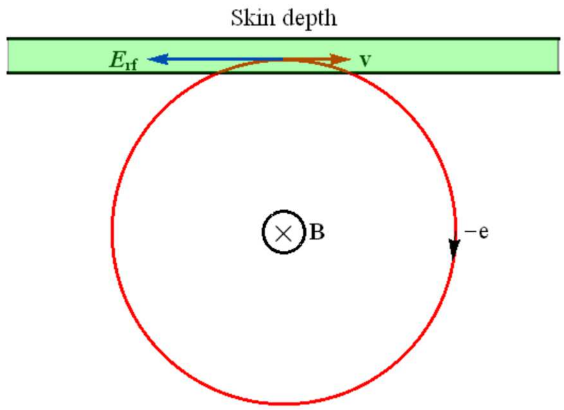

Fig.2 The AKCR geometry. The rf electric field may be perpendicular or parallel to B, but both E and B are parallel to the surface of the specimen. The penetration depth (skin depth) of the rf field is indicated by the green shading. An electron orbit is shown. near the top of each turn the electron enters the skin depth and experiences the rf electric field, gaining energy from the rf field.

((**Note**))

We note that in cyclotron resonance in semiconductors only the possibility $n = 1$ occurs because the rf field is assumed to penetrate the specimen uniformly.

4. Principle of ACKR

If the electron experiences a rf electric field of the same phase every time it enters the skin depth, then it can resonantly absorb energy from the rf electric field. This will be the case if the applied field has completed an integral number of periods, T_E, each time the electron returns to the surface,

$$T = nT_E ,$$

where T is the period of the cyclotron motion and n is an integer. Since frequencies are inversely proportional to period, we get

$$\omega = n\omega_c ,$$

where

$$T = \frac{2\pi}{\omega_c}, \qquad T_E = \frac{2\pi}{\omega},$$

ω_c is the cyclotron frequency and is given by

$$\omega_c = \frac{eB}{m_c^* c} = \frac{eB}{\frac{\hbar^2 c}{2\pi}\frac{\partial A}{\partial \varepsilon}} = \frac{2\pi eB}{\hbar^2 c}\frac{1}{\frac{\partial A}{\partial \varepsilon}}$$

When ω is fixed and the magnetic field is varied, the resonance condition can be expressed by

$$\frac{1}{B_n} = \frac{2\pi e}{\hbar^2 c\omega}\frac{1}{\frac{\partial A}{\partial \varepsilon}} n,$$

or

$$\Delta\left(\frac{1}{B}\right) = \frac{2\pi e}{\hbar^2 c\omega}\frac{1}{\frac{\partial A}{\partial \varepsilon}},$$

Thus, if the absorption is plotted as a function of $1/B$, resonant peaks due to a given cyclotron period will be uniformly spaced.

The magnetic field ***B*** lies in the plane of the sample. The rf electric field also lies in the plane of the surface may be either parallel (longitudinal) or at right angles (transverse) to ***B***. If the relaxation time is sufficiently long, we may think of the carriers as spiraling about ***B***, dipping once each cycle in and out of the rf field localized in the skin depth. Resonant absorption of energy will occur if a carrier sees an electric field in the same phase every time the carrier is in the skin depth.

The penetration depth (skin depth) of the rf field is indicated by the shading. An electron orbit is shown. Near the top of each turn the electron enters the skin depth and experiences the rf electric field, gaining energy from the field.

Stationary values of $dA/d\varepsilon$ with respect to k_z define the section of the Fermi surface which contribute to the central and resolved portions of the cyclotron lines.

4. Cyclotron mass of electron

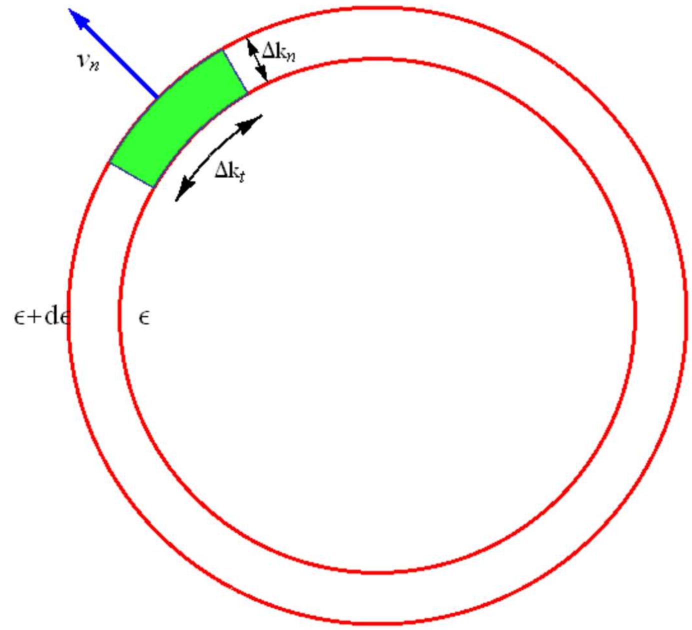

Fig.3 The area ΔA enclosed between two adjacent orbits on a given slice-plane is given by $\Delta A = \oint \Delta k_n dk_t$. The magnetic field ***B*** is applied in a direction (out of the page, the z direction). $A(\varepsilon, k_z)$ is the area of the orbit which is determined by the energy ε and k_z (the component of ***k*** parallel to ***B***.

From the above equation, we get

$$c\hbar \Delta k_t = e v_n B \Delta t ,$$

where k_t is measured around the orbit and v_n is the component of velocity $\boldsymbol{v}_k$ normal to ***B***. The component v_n is defined as

$$v_n = \frac{1}{\hbar}\frac{\partial \varepsilon}{\partial k_n},$$

where k_n is measured normal to the orbit. Here we define the period T_c, which is the time taken for the electron to travel once around the orbit,

$$\int dt = T_c = \oint \frac{c\hbar dk_t}{eBv_n} = \frac{c\hbar}{eB}\oint \frac{dk_t}{v_n} = \frac{c\hbar^2}{eB}\frac{\partial A(\varepsilon, k_z)}{\partial \varepsilon}.$$

The area ΔA enclosed between two adjacent orbits on a given slice-plane is given by

$$\Delta A = \oint \Delta k_n dk_t \; .$$

or

$$\frac{\partial A(\varepsilon, k_B)}{\partial \varepsilon} = \oint \frac{\partial k_n}{\partial \varepsilon} dk_t = \oint \frac{dk_t}{\frac{\partial \varepsilon}{\partial k_n}} = \frac{1}{\hbar} \oint \frac{dk_t}{v_n} \; .$$

The period T is also defined as

$$T = \frac{2\pi}{\omega_c} ,$$

using the cyclotron (angular) frequency ω_c, defined by

$$\omega_c = \frac{eB}{m_c^* c} \; .$$

The cyclotron mass m_c^* is thus defined as

$$m_c^* = \frac{\hbar}{2\pi} \oint \frac{dk_t}{v_n} = \frac{\hbar^2}{2\pi} \frac{\partial A(\varepsilon, k_z)}{\partial \varepsilon}$$

For the electron-like Fermi surface, $m_c^*>0$ and for the hole-like Fermi surface, $m_c^*<0$.

5. Experimental results of AKCR in metals

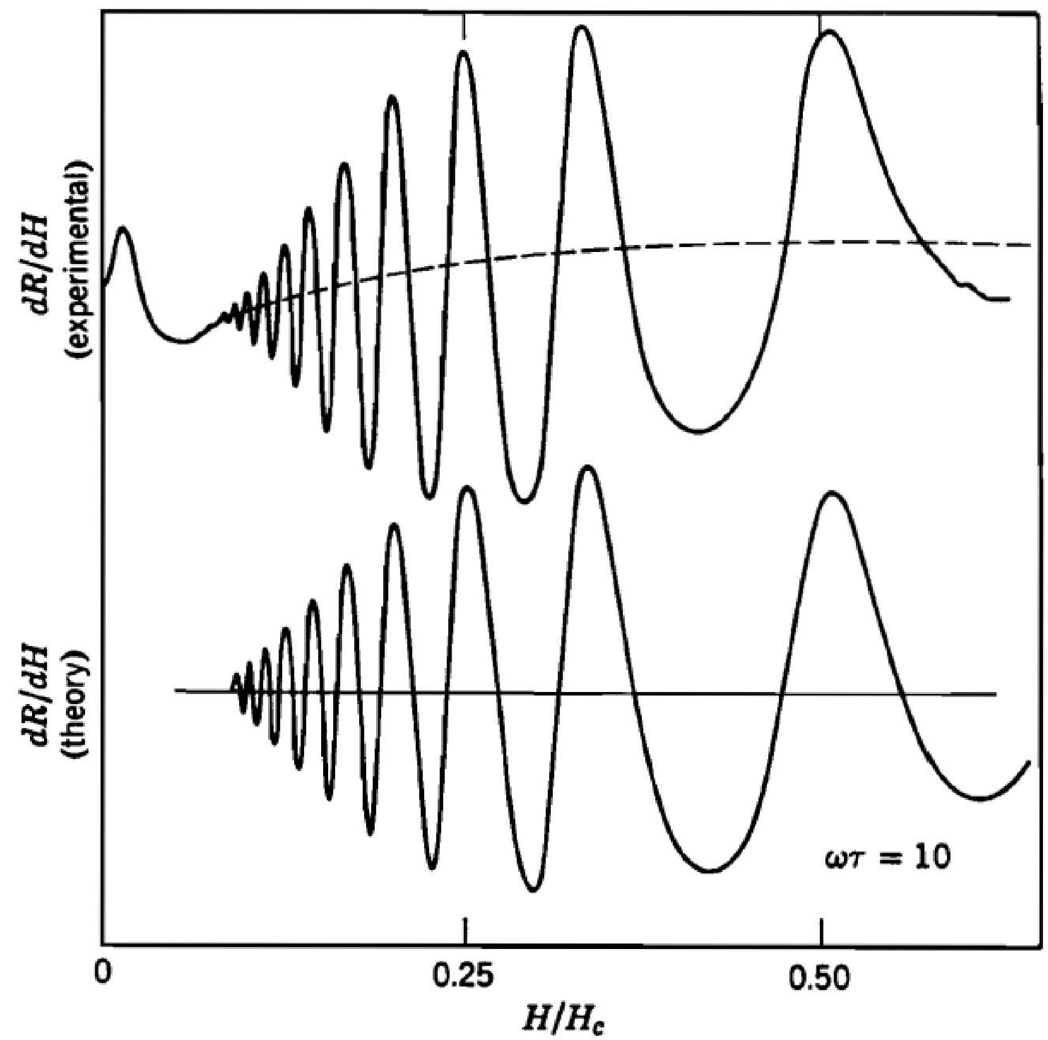

Fig.4 AKCR in Cu. Comparison of calculation of the magnetic fiel dependence of the derivative of the surface resistivity with experimental results at 24GHz (After Kip, Langenberg, and Moore). (Kittel, ISSP,2005)

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