

**Density of states at the Fermi energy**  
**Masatsugu Sei Suzuki**  
**Department of Physics**  
**(Date: May 06, 12)**

**1. Density of states for the 3D system**

$$D(\varepsilon)d\varepsilon = 2 \frac{V}{(2\pi)^3} 4\pi k^2 dk = \frac{V}{\pi^2} k^2 dk .$$

The factor 2 comes from spin freedom. Using the dispersion relation

$$\varepsilon = \frac{\hbar^2}{2m} k^2 ,$$

we get

$$D(\varepsilon)d\varepsilon = \frac{V}{\pi^2} k^2 dk = \frac{V}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \sqrt{\varepsilon} d\varepsilon ,$$

or

$$D(\varepsilon) = \frac{V}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \sqrt{\varepsilon} = \alpha \sqrt{\varepsilon} ,$$

where

$$\alpha = \frac{V}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} .$$

Note that the number of free electrons is

$$N = \int_0^{\varepsilon_F} D(\varepsilon) d\varepsilon = \alpha \int_0^{\varepsilon_F} \sqrt{\varepsilon} d\varepsilon = \frac{2\alpha}{3} \varepsilon_F^{3/2}$$

Then we have

$$\frac{D(\varepsilon_F)}{N} = \frac{\alpha\sqrt{\varepsilon_F}}{\frac{2}{3}\alpha\varepsilon_F^{3/2}} = \frac{3}{2\varepsilon_F}$$

## 2. The total number of free electrons below the Fermi energy:

$$N = 2 \frac{V}{(2\pi)^3} \frac{4\pi}{3} k_F^3$$

or

$$n = \frac{N}{V} = \frac{1}{3\pi^2} k_F^3 \quad \text{or} \quad k_F = (3\pi^2 n)^{1/3}$$

Then the Fermi energy is given by

$$\varepsilon_F = \frac{\hbar^2}{2m} k_F^2 = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3}.$$

## 3. Electronic heat capacity coefficient

Suppose that the heat capacity of atoms (1 mol) with the volume  $V_m$  can be measured as a function of temperature. The heat capacity consists of lattice contribution and electronic contribution. The electronic heat capacity can be expressed by

$$C_{el} = \frac{1}{3} \pi^2 D(\varepsilon_F) k_B^2 T = \gamma T.$$

with

$$\gamma = \frac{1}{3} \pi^2 D(\varepsilon_F) k_B^2$$

and

$$D(\varepsilon_F) = \frac{V_m}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \sqrt{\varepsilon_F}$$

where  $V_m$  is the molar mass. Then we get

$$\begin{aligned}\gamma &= \frac{1}{3} \pi^2 \frac{D(\varepsilon_F)}{N_A} N_A k_B^2 \\ &= \frac{1}{3} \pi^2 D^A(\varepsilon_F) N_A k_B^2 \\ &= 2.35715 D^A(\varepsilon_F)\end{aligned}$$

where

$$D^A(\varepsilon_F) = \frac{D(\varepsilon_F)}{N_A} \quad (1/\text{eV}\cdot\text{atom}) \text{ or } (1/\text{eV}\cdot\text{at})$$

and

$$\frac{1}{3} \pi^2 N_A k_B^2 = 2.35715 \quad (\text{mJ eV/K})$$

((**Mathematica**))

```
Clear["Global`*"];
rule1 = {kB -> 1.3806504 * 10^-16, NA -> 6.02214179 * 10^23,
c -> 2.99792 * 10^10, h -> 1.054571628 * 10^-27,
me -> 9.10938215 * 10^-28, mp -> 1.672621637 * 10^-24,
mn -> 1.674927211 * 10^-24, qe -> 4.8032068 * 10^-10,
eV -> 1.602176487 * 10^-12, meV -> 1.602176487 * 10^-15,
keV -> 1.602176487 * 10^-9, MeV -> 1.602176487 * 10^-6,
A -> 10^-8, J -> 10^7};

1 / meV J * pi^2 NA kB^2 // . rule1
2.35715
```

#### 4. Expression of $D^A(\varepsilon_F)$

What is the expression for  $D^A(\varepsilon_F)$ ? It is expressed by

$$D^A(\varepsilon_F) = \frac{D(\varepsilon_F)}{N_A} \left( \frac{eV}{erg} \right) = \frac{V_m}{N_A} \frac{1}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \sqrt{\varepsilon_F} \left( \frac{eV}{erg} \right) = \Omega \frac{1}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \sqrt{\varepsilon_F} \left( \frac{eV}{erg} \right)$$

where

$$\Omega = \frac{V_m}{N_A} = \frac{M}{\rho N_A}$$

is the volume per atom.  $M$  is the molar mass and  $\rho$  is the density. Suppose that there are  $p$  electrons per atom. Then the number density of electrons is given by

$$n = \frac{p}{\Omega}$$

$$1 \text{ eV} = 1.602176487 \times 10^{-12} \text{ erg}$$

Then

$$D^A(\varepsilon_F) = \Omega \frac{1}{2\pi^2} \left( \frac{2m}{\hbar^2} \right)^{3/2} \sqrt{\varepsilon_F} \left( \frac{eV}{\text{erg}} \right) = \frac{3^{1/3}}{\pi^{4/3}} \frac{m}{\hbar^2} \Omega^{2/3} p^{3/2} \left( \frac{eV}{\text{erg}} \right) = 0.313454 \frac{m}{\hbar^2} \Omega^{2/3} p^{3/2} \left( \frac{eV}{\text{erg}} \right)$$

where

$$\varepsilon_F = \frac{\hbar^2}{2m} \left( p \frac{3\pi^2}{\Omega} \right)^{2/3}$$

In other words,  $D^A(\varepsilon_F)$  depends on the mass  $m$ , the atomic volume  $\Omega$ , and the number of electrons per atom.

## 5. Electronic heat capacity coefficient

$$\gamma = 2.35715 D^A(\varepsilon_F) = 0.73886 \frac{m}{\hbar^2} \Omega^{2/3} p^{3/2} \left( \frac{eV}{\text{erg}} \right)$$

When the mass is different from that of the free particle, we have

$$\gamma = 0.73886 \frac{m}{\hbar^2} \frac{m^*}{m} \Omega^{2/3} p^{3/2} \left( \frac{eV}{\text{erg}} \right) = 9.69637 \times 10^{14} \left( \frac{m^*}{m} \right) p^{3/2} \Omega^{2/3} \text{ (mJ/mol K}^2\text{)}$$

((Example-1))

Al:

Density:  $\rho = 2.6989 \text{ g/cm}^3$   
 Atomic weight = 26.9815386 g/mol  
 fcc;  $a = 4.05 \text{ \AA}$ .  $p = 3$  (trivalent)

$$\Omega = \frac{a^3}{4} = \frac{M}{\rho N_A}$$

The Fermi energy:

$$\varepsilon_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} = 11.6524 \text{ eV}$$

Since

$$p = 3$$

we get

$$n = 3 \times \frac{4}{a^3} = 1.80641 \times 10^{23} / \text{cm}^3$$

Then we have

$$\gamma = 9.69637 \times 10^{14} \left( \frac{m^*}{m} \right) p^{3/2} \Omega^{2/3}$$

For  $p = 3$  and  $\Omega = \frac{a^3}{4}$ ,

$$\gamma = 0.631169 \left( \frac{m^*}{m} \right) 3^{3/2} = 1.31289 \left( \frac{m^*}{m} \right) \quad (\text{mJ/mol K})$$

**((Example-2)) Pb**

Density:  $\rho = 11.342 \text{ g/cm}^3$   
 Atomic weight = 207.2 g/mol  
 fcc;  $a = 4.95 \text{ \AA}$ .  $p = 4$ .

$$\Omega = \frac{a^3}{4} = \frac{M}{\rho N_A}$$

The Fermi energy:

$$\varepsilon_F = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} = 9.44945 \text{ eV}$$

$$n = 4 \times \frac{4}{a^3} = 1.31918 \times 10^{23} / \text{cm}^3$$

$$\gamma = 9.69637 \times 10^{14} \left( \frac{m^*}{m} \right) p^{3/2} \Omega^{2/3}$$

For  $p = 4$  and  $\Omega = \frac{a^3}{4}$ ,

$$\gamma = 0.73886 \left( \frac{m^*}{m} \right) 4^{3/2} = 2.37585 \left( \frac{m^*}{m} \right) \quad (\text{mJ/mol K})$$

#### 4. Derivation of $D^A(\varepsilon_F)$ from $\gamma_{measured}$

$D^A(\varepsilon_F)$  can be experimentally determined from the relation

$$D^A(\varepsilon_F) = \frac{\gamma_{measured} (\text{mJ} / \text{mol} \cdot \text{K}^2)}{2.35715} \quad [1/(\text{eV at})].$$

	$\gamma$ (mJ/mol K <sup>2</sup> )	$D(\varepsilon_F)$ (1/eV at)
Al	1.35	0.573
Pb	2.98	1.264
Sn (white)	1.78	0.755

**Table 2 Experimental and free electron values of electronic heat capacity constant  $\gamma$  of metals**

(From compilations kindly furnished by N. Phillips and N. Pearlman. The thermal effective mass is defined by Eq. (38).)

Li		Be											B	C	N		
1.63	0.17																
0.749	0.500																
2.18	0.34																
Na		Mg											Al	Si	P		
1.38	1.3														1.35		
1.094	0.992														0.912		
1.26	1.3														1.48		
Observed $\gamma$ in $\text{mJ mol}^{-1} \text{K}^{-2}$ .																	
Calculated free electron $\gamma$ in $\text{mJ mol}^{-1} \text{K}^{-2}$ .																	
$m_{th}/m = (\text{observed } \gamma)/(\text{free electron } \gamma)$ .																	
K	Ca	Sc	Ti	V	Cr	Mn( $\gamma$ )	Fe	Co	Ni	Cu	Zn	Ga	Ge	As			
2.08	2.9	10.7	3.35	9.26	1.40	9.20	4.98	4.73	7.02	0.695	0.64	0.596		0.19			
1.668	1.511									0.505	0.753	1.025					
1.25	1.9									1.38	0.85	0.58					
Rb	Sr	Y	Zr	Nb	Mo	Tc	Ru	Rh	Pd	Ag	Cd	In	Sn ( $w$ )	Sb			
2.41	3.6	10.2	2.80	7.79	2.0	—	3.3	4.9	9.42	0.646	0.688	1.69	1.78	0.11			
1.911	1.790									0.645	0.948	1.233	1.410				
1.26	2.0									1.00	0.73	1.37	1.26				
Cs	Ba	La	Hf	Ta	W	Re	Os	Ir	Pt	Au	Hg( $\alpha$ )	Tl	Pb	Bi			
3.20	2.7	10.	2.16	5.9	1.3	2.3	2.4	3.1	6.8	0.729	1.79	1.47	2.98	0.008			
2.238	1.937									0.642	0.952	1.29	1.509				
1.43	1.4									1.14	1.88	1.14	1.97				

## APPENDIX

Myer Chapter 13 problems

1. Estimate the strength of the electron-phonon interaction,  $V$ , for Al and Pb, where

$$T_c = \Theta \exp\left[-\frac{1}{VN(\mathcal{E}_F)}\right]$$

and  $N(\mathcal{E}_F)$  is the density of states for a given spin at the Fermi energy.

((Solution))

$$T_c = \Theta \exp\left[-\frac{1}{VN(\mathcal{E}_F)}\right]$$

For Pb,  $T_c = 7.2$ ,  $\Theta = 105$  K.

For Al,  $T_c = 1.2$  K,  $\Theta = 428$  K.

We assume that

$$N^A(\varepsilon_F) = \frac{1}{2} D^A(\varepsilon_F),$$

$V_A$  is in the units of [eV at].

$$VN(\varepsilon_F) = V^A N^A(\varepsilon_F) = \frac{1}{2} V^A D^A(\varepsilon_F) = -\frac{1}{\ln\left(\frac{T_c}{\Theta}\right)}.$$

or

$$V^A = -\frac{1}{2D^A(\varepsilon_F) \ln\left(\frac{T_c}{\Theta}\right)}.$$

Then we have

	$\gamma$ (mJ/.mol K <sup>2</sup> )	$D(\varepsilon_F)$ (1/eV at)	$V^A$ (eV.at)
Al	1.35	0.57	0.60
Pb	2.98	1.26	0.59

2.

Calculate the London penetration depth in Al at 0 K under the assumptions (a) that all the valence electrons are responsible for the superconducting property, and (b) that only electrons in an energy interval  $k_B\Theta/2$  in the neighborhood of the Fermi energy contribute to the supercurrent. To what fraction of the valence electron concentration does the latter case correspond?

((Solution))

Al fcc with the lattice constant  $a = 4.05$  Å.

Then the number density:

$$n_0 = 3 \times 4/a^3 = 18.06 \times 10^{22}/\text{cm}^3.$$

The Fermi energy is

$$\varepsilon_F = 11.6524 \text{ eV}.$$



The London penetration depth is

$$\lambda_L = \sqrt{\frac{mc^2}{4\pi m_0 e^2}} = 125.046 \text{ \AA}.$$

The energy interval is

$$\Delta\varepsilon = \frac{1}{2} k_B \Theta = 0.018441 \text{ eV}.$$

$$\gamma = 1.35 \quad (\text{mJ}/\text{mol K}^2)$$

$$D^A(\varepsilon_F) = 0.57 \quad (1/\text{eV at})$$

$$\Delta n = N_A D^A(\varepsilon_F) \Delta\varepsilon = 6.33014 \times 10^{20} / \text{cm}^3.$$

The revised penetration depth is

$$\lambda_L' = \sqrt{\frac{mc^2}{4\pi(\Delta n)e^2}} = 667.19 \text{ \AA}$$

$$\frac{\Delta n}{n} = 0.0352351$$

((**Mathematica**))

```

Clear["Global`*"];
rule1 = {kB → 1.3806504 × 10-16, NA → 6.02214179 × 1023,
c → 2.99792 × 1010, ħ → 1.054571628 × 10-27,
me → 9.10938215 × 10-28, qe → 4.8032068 × 10-10,
eV → 1.602176487 × 10-12, meV → 1.602176487 × 10-15,
keV → 1.602176487 × 10-9, MeV → 1.602176487 × 10-6,
Å → 10-8, n0 → 18.06 × 1022, Θ → 428, a1 → 4.05 Å,
DA1 → 0.57};

```

$$\lambda_L = \frac{1}{\text{\AA}} \sqrt{\frac{m_e c^2}{4 \pi n_0 q_e^2}} \quad // . \text{rule1}$$

125.046

$$\Delta E = \frac{1}{2} k_B \Theta / eV \quad // . \text{rule1}$$

0.0184411

$$n_1 = NA DA1 \Delta E / . \text{rule1}$$

$6.33014 \times 10^{21}$

$$\lambda_{L2} = \frac{1}{\text{\AA}} \sqrt{\frac{m_e c^2}{4 \pi n_1 q_e^2}} \quad // . \text{rule1}$$

667.916

$$\frac{n_1}{n_0} \quad // . \text{rule1} \quad // N$$

0.0350506

((Note)) The brief solution is shown in the Myer's book. Our result is slightly different.

$$\lambda_L = 127 \text{ \AA}. \quad \lambda_L' = 1250 \text{ \AA}. \quad \frac{\Delta n}{n} = 0.035\%.$$

---

### 3. The critical field is described as

$$H_c = H_{c0} \left(1 - \frac{T^2}{T_c^2}\right).$$

**Obtain an expression for  $H_{c0}$  in terms of  $T_c$  and  $(C_s - C_n)$  at  $T_c$ . Given that the latter quantities are 1.2 K and 1.6 mJ/K mol, estimate  $H_{c0}$  for Al.**

**((Solution))**

At  $T = T_c = 1.2$  K,

$$(C_N - C_S)_V \Big|_{T=T_c} = -\frac{T_c}{4\pi} \left( \frac{dH_c(T)}{dT} \right)_{T=T_c}^2$$

Parabolic law:

$$H_c(T) = H_{c0} \left(1 - \frac{T^2}{T_c^2}\right).$$

$$\frac{dH_c(T)}{dT} = -H_{c0} \frac{2T}{T_c^2}$$

Then we have

$$(C_S - C_N)_V \Big|_{T=T_c} = \frac{T_c}{4\pi} H_{c0}^2 \frac{4}{T_c^2} = \frac{H_{c0}^2}{\pi T_c}$$

or

$$H_{c0}^2 = \pi T_c (C_S - C_N)_V \Big|_{T=T_c}$$

For  $T_c = 1.2$  K,  $(C_S - C_N)_V \Big|_{T=T_c} = 1.6$  mJ/mol K.

We note that the units of  $H_{c0}^2$  is mJ/mol. The unit of  $H_{c0}^2$  should be  $\text{Oe}^2 = \text{Gauss}^2 = \text{erg/cm}^3$ . Therefore we need to calculate the volume of 1 mol of Al. Using the molar mass  $M$  ( $= 26.98$  g/mol) and the density  $\rho$  ( $= 2.70$  g/cm<sup>3</sup>), we get the molar volume as

$$V_M = \frac{M}{\rho} = 9.99 \text{ cm}^3.$$

Then we get

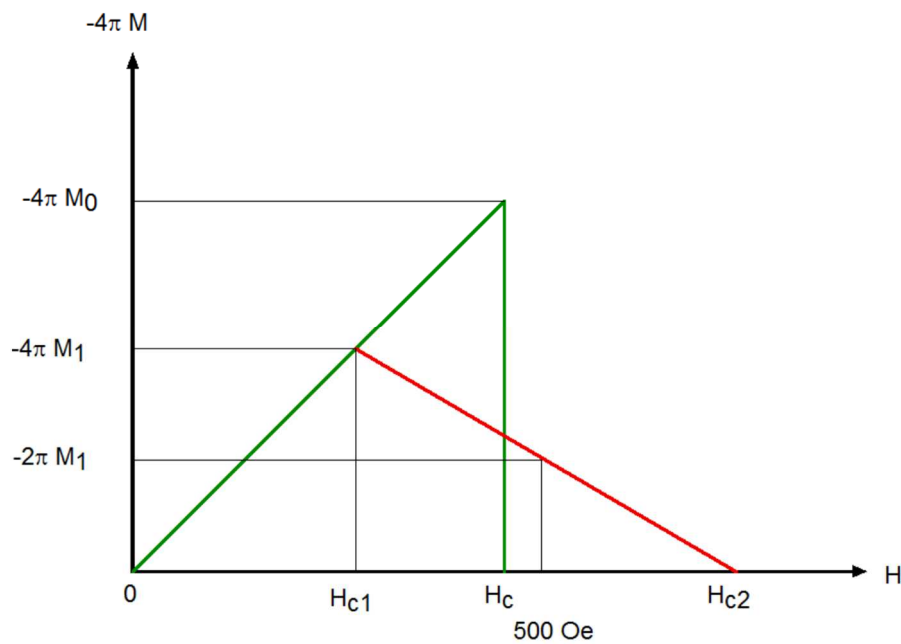
$$H_{c0}^2 = \pi \times 1.2 \times 1.6 \times 10^4 \text{ erg} / 9.99 \text{ cm}^3 = 0.604 \times 10^4 \text{ Oe}^2$$

or

$$H_{c0} = 78 \text{ Oe.}$$

4. It is found, for a certain superconducting alloy, that  $H_{c1}$  is 400 Oe and that the magnetization has fallen to half its (negative) value at  $H_{c1}$  when the applied field is 500 Oe. Assuming the Abrikosov model for the mixed state, find the distance between the centers of the flux vortices.

((Solution))



At  $H_{c1} = 400 \text{ Oe.}$   $-4\pi M_1 = 400 \text{ Gauss.}$

At  $H = 500 \text{ Oe.}$   $B = H + 4\pi M = H + 2\pi M_1 = 500 - 200 = 300 \text{ Gauss.}$

The most stable arrangement of vortices is triangular.

$$\Phi_0 = \frac{\sqrt{3}}{2} a^2 B,$$

with

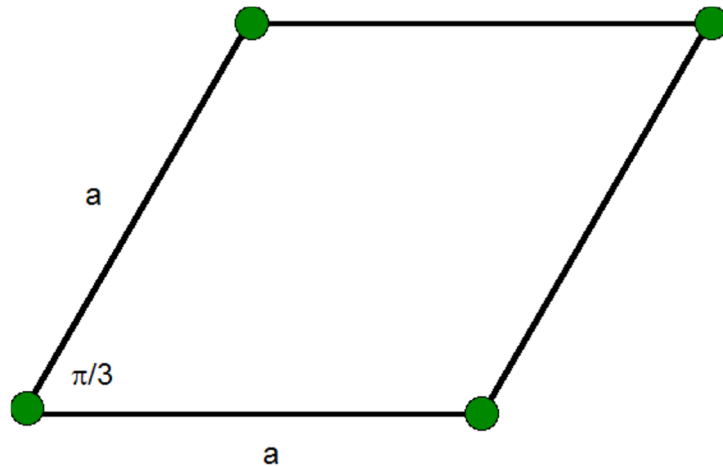
$$\Phi_0 = 2.0678 \times 10^{-7} \text{ Gauss cm}^2$$

Then we get

$$a^2 = \frac{2\Phi_0}{\sqrt{3}B} = \frac{2 \times 2.0678 \times 10^{-7}}{\sqrt{3} \times 300} = 7.96 \times 10^{-10} \text{ cm}^2$$

or

$$a = 2820 \text{ \AA}.$$



**Fig.** Quantum fluxoids forming triangular lattice.

- 
5. A thick specimen of type I superconductor is known to have a critical field of 500 Oe. It is found that a film of  $5 \times 10^{-5}$  cm thickness has a critical field of 550 Oe. What value would you expect for the critical field of a sample of only  $10^{-6}$  cm thickness? You may assume that the penetration of field into the superconductor is as given by the London theory and that the penetration depth is independent of magnetic field and you may neglect demagnetization effects.

**((Solution))**

Type-I superconductor

$$H_c = 500 \text{ Oe.}$$

From Problem ((10-2)) Kittel 8th edition,

$$H_c' = \sqrt{12} \frac{\lambda}{\delta} H_c$$

where  $\delta$  is the thickness and  $\lambda$  is the penetration depth.

Suppose that

$$H_c' = \alpha \frac{H_c}{\delta}$$

where  $\alpha$  is constant.

$$\text{For } \delta = 5 \times 10^{-5} \text{ cm,} \quad H_c' = 550 \text{ Oe.}$$

Then we get

$$\alpha = \frac{H_c'}{H_c} \delta = \frac{550}{500} (5 \times 10^{-5}) = 5.5 \times 10^{-5}.$$

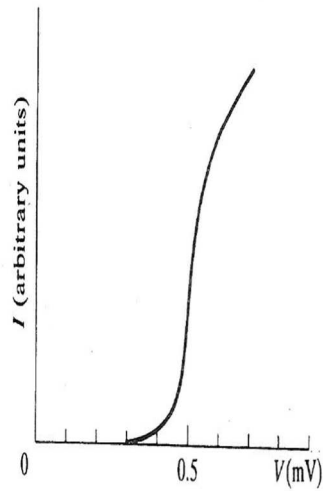
Using the relation

$$H_c' = (5.5 \times 10^{-5}) \frac{H_c}{\delta},$$

we can estimate

$$H_c' = (5.5 \times 10^{-5}) \frac{500}{10^{-6}} = 27.5 \text{ kOe}$$

- 
6. In a tunneling experiment using a junction of Sn and Al at 2 K, the  $I$  vs  $V$  characteristic as shown in Fig.1 was obtained. What can be deduced from this value?



**((Solution))**

$$T_c = 3.72 \text{ K} \quad \text{for Sn}$$

$$T_c = 1.14 \text{ K} \quad \text{for Al}$$



At  $T = 2 \text{ K}$ , Sn is in the S-state, but Al is in the N-state. Then Sn-Al junction is a junction of S-I-N, where I is an insulator. In this case,  $V = 0.5 \text{ meV}$  corresponds to  $\Delta/e$ , where  $2\Delta$  is the energy gap of Sn.

$$2\Delta = 0.5 \times 2 = 1.0 \text{ meV}.$$

This value is close to the accepted value  $1.15 \text{ meV}$ .

$$2\Delta(0\text{K}) = 3.5277k_B T_c = 1.12478 \text{ meV} \quad (\text{BCS theory}).$$

**((Note))**

$$1 \text{ meV} = 1.602176487 \times 10^{-15} \text{ erg}$$

7. An electric current is passing through a junction that consists of Pb and Al separated by a very thin insulating layer. The plot of tunnel current against applied voltage at  $T = 0.5$  K is shown schematically in Fig.2(a), the maximum and minimum being at voltages  $V_1 = 11.8 \times 10^{-4}$  V and  $V_2 = 15.2 \times 10^{-4}$  V, respectively. Explain the shape of the curve and derive values for the energy gaps of superconducting Pb and Al. At what temperature the maximum and minimum to disappear to yield a curve as shown in Fig.2(b)? You may assume that the energy gap of a superconductor does not change appreciably in the temperature range from 0 K to  $(1/2)T_c$ .

Fig.2(a)

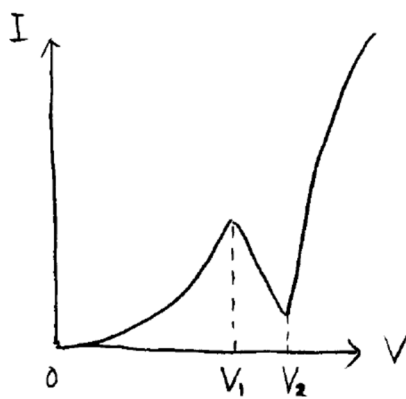
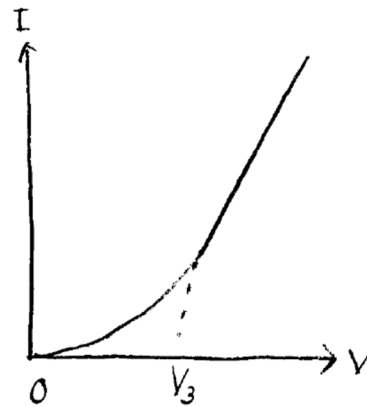


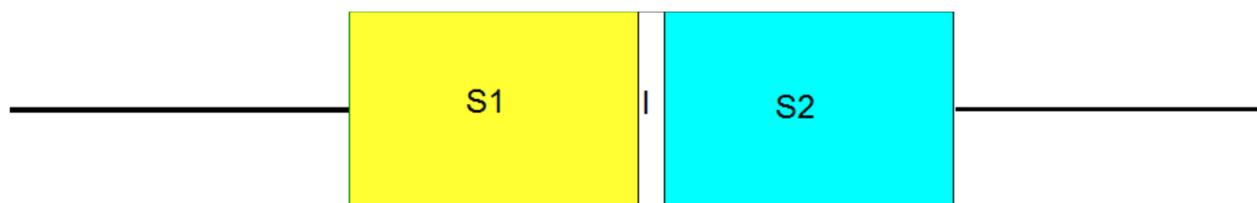
Fig.2(b)



((Solution))

Pb:  $T_c = 7.20$  K

Al:  $T_c = 1.20$  K



At  $T = 0.5$  K, Pb and Al are in the S-state.  $2\Delta_1$  and  $2\Delta_2$  are the energy gap of superconductor S1 (Pb) and the superconductor S2 (Al).



$$V_2 = \frac{\Delta_1 + \Delta_2}{e}, \quad V_1 = \frac{\Delta_1 - \Delta_2}{e} \quad (\Delta_1 > \Delta_2)$$

Then we get

$$\Delta_1 + \Delta_2 = eV_2 = 1.52 \text{ meV}, \quad \Delta_1 - \Delta_2 = eV_1 = 1.18 \text{ meV},$$

or

$$\Delta_1 = 1.35 \text{ meV for Pb}, \quad \Delta_2 = 0.17 \text{ meV for Al},$$

For  $1.2\text{K} \leq T < 7.2 \text{ K}$ ,

Pb is in the S-state, while Al is in the N-state.

$$V_3 = \frac{\Delta_1}{e} = 1.35 \text{ mV}.$$

- 8. Estimate the penetration depth and coherence length for pure Sn, where**
- |                             |   |
|-----------------------------|---|
| <b>critical temperature</b> | $T_c = 3.7 \text{ K}$ ,                 |
| <b>density</b>              | $\rho = 7.3 \text{ g/cm}^3$ ,           |
| <b>atomic molar mass</b>    | $M = 118.7 \text{ g/mol}$               |
| <b>effective mass</b>       | $m^* = 1.9 m$ ( $m$ ; mass of electron) |

**((Solution))**

Suppose that each atom has one electron. The electron concentration  $n$  is

$$n = \frac{N_A}{V} = \frac{M}{V} \frac{N_A}{M} = \rho \frac{N_A}{M} = 3.70359 \times 10^{22} / \text{cm}^3.$$

The London penetration depth is evaluates as

$$\lambda_L = \sqrt{\frac{m^* c^2}{4\pi n e^2}} = 380.622 \text{ \AA}.$$

The Fermi velocity is given by

$$v_F = \frac{\hbar}{m^*} k_F = \frac{\hbar}{m^*} (3\pi^2 n)^{1/3} = 6.28321 \times 10^7 \text{ cm/s}$$

Then the coherent length

$$\xi_0 = \frac{\hbar v_F}{\pi \Delta} = \frac{2\hbar v_F}{\pi(2\Delta)} = \frac{2\hbar v_F}{3.5277\pi k_B T_c} = 2344.1 \text{ \AA}$$

with

$$2\Delta = 3.5277 k_B T_c = 1.12318 \text{ meV} \quad (\text{BCS theory}).$$

((**Mathematica**))

```
Clear["Global`*"];
```

```
rule1 = {kB → 1.3806504 × 10-16, NA → 6.02214179 × 1023,  
c → 2.99792 × 1010, ħ → 1.054571628 × 10-27,  
me → 9.10938215 × 10-28, qe → 4.8032068 × 10-10,  
eV → 1.602176487 × 10-12, meV → 1.602176487 × 10-15,  
keV → 1.602176487 × 10-9, MeV → 1.602176487 × 10-6,  
Tc → 3.70, M → 118.7, Å → 10-8, ρ → 7.3,  
m1 → 1.9 me};
```

$$n1 = \frac{\rho NA}{M} /. rule1$$

3.70359 × 10<sup>22</sup>

$$\lambda L = \frac{1}{\text{Å}} \sqrt{\frac{m1 c^2}{4 \pi n1 qe^2}} //. rule1$$

380.622

$$vF = \frac{\hbar}{m1} (3 \pi^2 n1)^{1/3} //. rule1$$

6.28321 × 10<sup>7</sup>

$$\xi 0 = \frac{1}{\text{Å}} \frac{2 \hbar vF}{3.5227 \pi kB Tc} //. rule1$$

2344.1

$$\frac{3.5227 kB Tc}{meV} //. rule1$$

1.12318

---

## REFERENCES

- H.P. Myers, Introductory Solid State Physics (Taylor & Francis, London, 1990).  
C. Kittel, Introduction to Solid State Physics, 8th edition (John Wiley & Sons, New York, 2005)