## Thermoionic Emission of Electrons: Richardson effect Masatsugu Sei Suzuki Department of Physics, SUNY at Binghamton (Date: October 05, 2016)

## 1. Introduction

After the discovery of electron in 1897, the British physicist Owen Willans Richardson began work on the topic that he later called "thermionic emission". He received a Nobel Prize in Physics in 1928 "for his work on the thermionic phenomenon and especially for the discovery of the law named after him".

The minimum amount of energy needed for an electron to leave a surface is called the work function. The work function is characteristic of the material and for most metals is on the order of several eV. Thermionic currents can be increased by decreasing the work function. This oftendesired goal can be achieved by applying various oxide coatings to the wire. In 1901 Richardson published the results of his experiments: the current from a heated wire seemed to depend exponentially on the temperature of the wire with a mathematical form similar to the Arrhenius equation.

$$J = AT^2 \exp(-\frac{\phi}{k_B T})$$

where J is the emission current density, T is the temperature of the metal, W is the work function of the metal,  $k_{\rm B}$  is the Boltzmann constant, and  $A_{\rm G}$  is constant.

## 2. Richardson's law

We consider free electrons inside a metal. The kinetic energy of electron is given by

$$\varepsilon_p = \frac{1}{2m} (p_x^2 + p_y^2 + p_z^2)$$



Fig. The work function  $\phi$  of a metal represents the height of the surface barrier *over and above* the Fermi level.

Suppose that these electrons escape from the metal along the positive x direction. To this end, only the x components of the momentum should satisfy the inequality

$$\frac{p_x^2}{2m} \ge W \text{ or } p_x > \sqrt{2mW}$$

where  $W = \mu + \phi$ ,  $\mu$  is the chemical potential (the Fermi energy  $\varepsilon_F$ ) and  $\phi$  is the work function.



Fig. The number of electrons moving at the velocity  $v_x = \frac{p_x}{m}$  during a time period  $\Delta t$  passing through an area A. The volume V where electrons pass across the area A, is  $V = (v_x \Delta t)A$ .

The number of electrons leaving per unit area of the metal surface per unit time is given by

$$R = \frac{2}{(2\pi\hbar)^3} \int_{\sqrt{2mW}} dp_x \iint dp_y dp_z \frac{p_x}{m} \overline{n}(\boldsymbol{p})$$
$$= \frac{1}{4\pi^3 m\hbar^3} \int_{\sqrt{2mW}} p_x dp_x \iint dp_y dp_z \frac{1}{e^{\beta(\varepsilon_p - \mu)} + 1}$$

where  $\overline{n}(p)$  is the Fermi-Dirac distribution function

$$\overline{n}(\boldsymbol{p}) = \frac{1}{e^{\beta(\varepsilon_p - \mu)} + 1}$$

The integration over the variables  $p_y$  and  $p_z$  may be carried out by changing over to the corresponding polar co-ordinates  $(p', \varphi)$  with the result

$$\iint dp_{y} dp_{z} \frac{1}{e^{\beta(\varepsilon_{p}-\mu)} - 1} = 2\pi \int_{0}^{\infty} p' dp' \frac{1}{\exp[\beta(\frac{p_{x}^{2}}{2m} + \frac{p'^{2}}{2m} - \mu)] + 1}$$
$$= 2\pi \frac{m}{\beta} \ln[1 + \exp[-\beta(\frac{p_{x}^{2}}{2m} - \mu)]$$

using the Mathematica. Then we have

$$R = \frac{k_B T}{2\pi^2 \hbar^3} \int_{\sqrt{2mW}} p_x dp_x \ln[1 + \exp[-\beta(\frac{p_x^2}{2m} - \mu)]]$$

We use the notation

$$\varepsilon_x = \frac{p_x^2}{2m}, \qquad d\varepsilon_x = \frac{1}{m} p_x dp_x$$

Then we get the expression for R

$$R = \frac{mk_BT}{2\pi^2\hbar^3} \int_W^\infty d\varepsilon_x \ln[1 + \exp[-\beta(\varepsilon_x - \mu)]]$$
  
$$\approx \frac{mk_BT}{2\pi^2\hbar^3} \int_W^\infty d\varepsilon_x \exp[-\beta(\varepsilon_x - \mu)]]$$
  
$$= \frac{mk_B^2T^2}{2\pi^2\hbar^3} \exp[\beta(\mu - W)]$$
  
$$= \frac{4\pi mk_B^2T^2}{h^3} \exp[\beta(\mu - W)]$$

where we assume that

 $\ln(1+x) \approx x$ 

or

$$\ln[1 + \exp[-\beta(\varepsilon_x - \mu) \approx \exp[-\beta(\varepsilon_x - \mu)]]$$

The thermoionic current density is then given by

$$J = eR = \frac{4\pi m e k_{B}^{2}}{h^{3}} T^{2} \exp[\beta(\mu - W)]$$

where *e* is the charge of electron (-*e*) with e>0. We introduce the work function as

$$\phi = W - \mu \,.$$

Thus we have

$$J = \frac{4\pi m e k_B^2}{h^3} T^2 \exp(-\frac{\phi}{k_B T})$$
 (Richardson's law)

Note that

$$A = \frac{4\pi m e k_B^2}{h^3} = 1.20174 \text{ x } 10^6 \text{ A/(m^2 K^2)}$$
(SI units)





Fig. Thermionic current from tungsten (W) as a function of the temperature of the metal. The continuous line corresponds to r = 1.2 while the broken line corresponds to r = 0, r being the reflection coefficient of the surface.