Experimental determination of g: Rűchhardt's method and Rinkel's modification

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Here we discuss the Rűchhardt's method of measuring the ratio g., and Rinkel's modification.

Blundell and Blundell
Thermal Physics
Problem 12-6

(12.6) In Rüchhardt's method of measuring γ, illustrated in Fig. 12.2, a ball of mass m is placed snugly inside a tube (cross-sectional area A) connected to a container of gas (volume V). The pressure p of the gas inside the container is slightly greater than atmospheric pressure p_0 because of the downward force of the ball, so that

$$p = p_0 + \frac{mg}{A}. \tag{12.38}$$

Show that if the ball is given a slight downwards displacement, it will undergo simple harmonic motion with period τ given by

$$\tau = 2\pi\sqrt{\frac{mV}{\gamma p A^2}}. \tag{12.39}$$

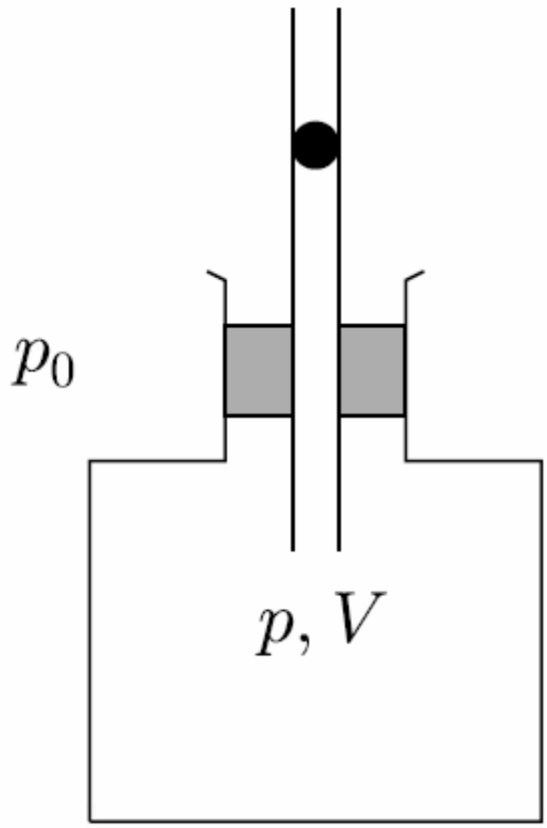


Fig. 12.2 Rüchhardt's apparatus for measuring γ. A ball of mass m oscillates up and down inside a tube.

[You may neglect friction. As the oscillations are fairly rapid, the changes in p and V which occur can be treated as occurring adiabatically.]
In Rinkel's 1929 modification of this experiment, the ball is held in position in the neck where the gas pressure p in the container is exactly equal to air pressure, and then let drop, the distance L that it falls before it starts to go up again is measured. Show that this distance is given by

$$mgL = \frac{\gamma P A^2 L^2}{2V}. \tag{12.40}$$

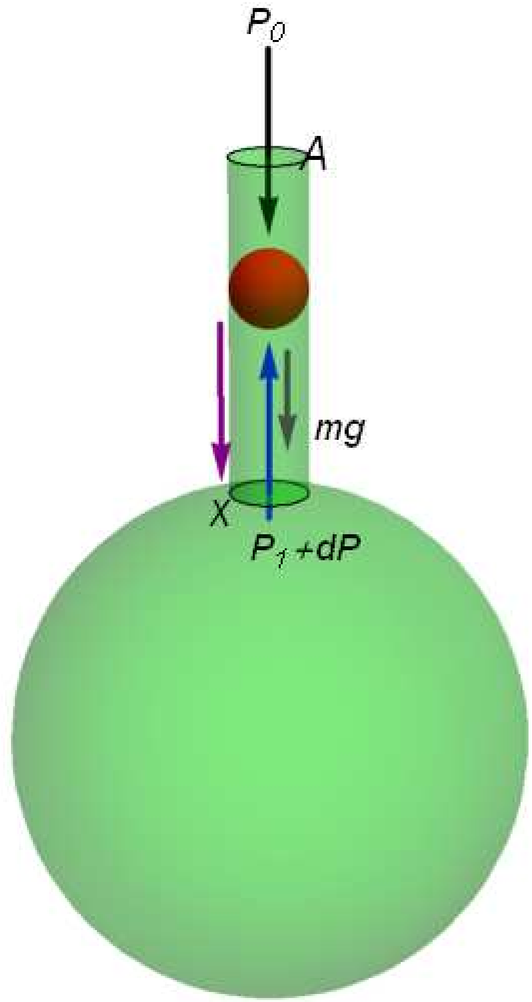
(a) Rűchhardt's method

Newton's second law

$$m\ddot{x} = mg + A[P_0 - (P_1 + dP)] = -AdP$$

where the positive x direction is downward

$$P_1 = P_0 + \frac{mg}{A}$$



In the adiabatic process,

$$\ln P + \gamma \ln V = const$$

or

$$\frac{dP}{P} + \gamma \frac{dV}{V} = 0$$

leading to

$$dP = -\gamma \frac{P_1}{V} dV = -\gamma \frac{P_1}{V}(-Ax) = \frac{\gamma P_1 A}{V} x$$

Thus we get an equation of the simple harmonics,

$$m\ddot{x} = -\frac{\gamma P_1 A^2}{V} x = -k_1 x$$

where

$$k_1 = \frac{\gamma P_1 A^2}{V}$$

The angular frequency of the simple harmonic is

$$\omega_1 = \sqrt{\frac{k_1}{m}} = \sqrt{\frac{\gamma P_1 A^2}{mV}}$$

The period T is given by

$$T = \frac{2\pi}{\omega_1} = 2\pi\sqrt{\frac{mV}{\gamma P_1 A^2}}$$

(b) Rinkel's modification

Newton's second law

$$m\ddot{x} = mg + A[P_0 - (P_0 + dP)] = mg - AdP$$

where the positive x direction is downward and

$$dP = -\gamma \frac{P_0}{V} dV = -\gamma \frac{P_0}{V}(-Ax) = \frac{\gamma P_0 A}{V} x$$

where we put $P = P_0$. Then we get the differential equation

$$m\ddot{x} = mg - \frac{\gamma P_0 A^2}{V} x = mg - k_0 x$$

In equilibrium;

$$mg = k_0 x_0 . \qquad (1)$$

where

$$k_0 = \frac{\gamma P_0 A^2}{V}$$

We put $y = x - x_0$

$$m\ddot{y} = -k_0 y$$

with

$$\omega_0^{\ 2} = \frac{k_0}{m} = \frac{\gamma P_0 A^2}{mV}$$

The solution is given by

$$y = x - x_0 = A\cos(\omega_0 t) + B\sin(\omega_0 t) ,$$

or

$$x = x_0 + A\cos(\omega_0 t) + B\sin(\omega_0 t)$$

$$\dot{y} = \dot{x} = -\omega A\sin(\omega_0 t) + B\omega_0 \cos(\omega_0 t)$$

From the initial condition that at t = 0, $x = 0$, and $\dot{x} = 0$. Then we have $B = 0$ and $A = -x_0$ (the amplitude)

$$x = x_0[1 - \cos(\omega_0 t)] ,$$

The system undergoes a motion of simple harmonics. The total amplitude of oscillation is $L = 2x_0$. So we have

$$L = 2x_0 = 2\frac{mg}{k_0} = \frac{2mg}{\frac{\gamma P_0 A^2}{V}} = \frac{2mgV}{\gamma P_0 A^2}$$

or

$$\boxed{\frac{\gamma P_0 A^2 L^2}{2V} = mgL}$$

((**Note**)) **Approach from the Energy conservation**

We now return to the original equation,

$$m\ddot{x} = mg - k_0 x$$

Multiplying $\dot{x}$ on both sides,

$$m\ddot{x}\dot{x} = mg\dot{x} - k_0 x\dot{x}$$

or

$$\frac{d}{dt}\left(\frac{m}{2}\dot{x}^2 - mgx + \frac{1}{2}k_0 x^2\right) = 0$$

This leads to the energy conservation law,

$$\frac{m}{2}\dot{x}^2 - mgx + \frac{1}{2}k_0 x^2 = E \ \text{(constant)}$$

Since $\dot{x} = 0$ and $x = 0$, $\dot{x} = 0$ and $x = x_1$, we have

$$E = 0 = -mgx_1 + \frac{k_0}{2}{x_1}^2$$

leading to

$$2mg = kx_1$$

or

$$\boxed{x_1 = \frac{2mg}{k} = 2x_0 = L}$$

leading to the relation

$$\boxed{\frac{\gamma P_0 A^2 L^2}{2V} = mgL}$$

where m = mass (kg), g = gravitational acceleration (m/s^2), V = volume of tube (m^3), P_0 = 1 atm (Pa), A = cross sectional area (m^2), and L = distance where mass has initially dropped (m).

((**Blundell**))

I think that the second part of the solution is wrong.

12.6 The change is adiabatic, so that

$$\frac{dp}{p} = -\gamma \frac{dV}{V}. \quad (1)$$

If the ball moves up a distance x, then $dV = A dx$ and the extra force on the ball is $A dp = m\ddot{x}$ and so

$$m\ddot{x} + kx = 0,$$

where

$$k = \frac{A^2 p \gamma}{V}$$

and hence simple harmonic oscillation results with

$$\omega^2 = \frac{A^2 p \gamma}{mV},$$

and the period $\tau = 2\pi/\omega$ results.

In Rinkel's modification, one equates gravitational PE with "spring" energy, so that

$$mgL = \frac{1}{2}k(L/2)^2 = \frac{\gamma p A^2 L^2}{8V}.$$

(Note that in this case the amplitude of the oscillation is L, which is from $-L/2$ to $L/2$, so the stored "spring" energy is $\frac{1}{2}k(L/2)^2$.)