Cosmic neutrino background (CvB) Masatsugu Sei Suzuki Department of Physics, SUNY at Binghamton (Date: December 23, 2018)

The **cosmic neutrino background** (CvB)is the universe's background particle radiation composed of neutrinos. They are sometimes known as **relic neutrinos**. The CvB is a relic of the big bang; while the cosmic microwave background radiation (CMB) dates from when the universe was 379,000 years old, the CvB decoupled (separated) from matter when the universe was just one second old. It is estimated that today, the CvB has a temperature of roughly 1.95 K. As neutrinos rarely interact with matter, these neutrinos still exist today. They have a very low energy, around 10^{-4} to 10^{-6} eV. Even high energy neutrinos are notoriously difficult to detect, and the CvB has energies around 10^{-10} times smaller, so the CvB may not be directly observed in detail for many years, if at all. However, Big Bang cosmology makes many predictions about the CvB, and there is very strong indirect evidence that the CvB exists.

https://en.wikipedia.org/wiki/Cosmic neutrino background

1. Neutrino (*v*)

A **neutrino** (denoted by the Greek letter v) is a fermion (an elementary particle with halfinteger spin) that interacts only via the weak subatomic force and gravity. The mass of the neutrino is much smaller than that of the other known elementary particles. Although only differences of squares of the three mass values are known as of 2016, cosmological observations imply that the sum of the three masses must be less than one millionth that of the electron. The neutrino is so named because it is electrically neutral and because its rest mass is so small (*-ino*) that it was long thought to be zero.

Weak interactions create neutrinos in one of three leptonic flavors: electron neutrinos, muon neutrinos or tau neutrinos, in association with the corresponding charged lepton. A neutrino created with a specific flavor is in an associated specific quantum superposition of all three mass states. As a result, neutrinos oscillate between different flavors in flight.

For each neutrino, there also exists a corresponding <u>antiparticle</u>, called an <u>antineutrino</u>, which also has half-integer spin and no electric charge. They are distinguished from the neutrinos by having opposite signs of <u>lepton number</u> and <u>chirality</u>. To conserve total lepton number, in nuclear <u>beta</u> <u>decay</u>, electron neutrinos appear together with only positrons (anti-electrons) or electron-antineutrinos, and electron antineutrinos with electrons or electron neutrinos.

2. Chemical potential

A pair production (Huang): the grand canonical ensemble

The grand canonical ensemble includes systems with different particle numbers, with a mean value N determined by the chemical potential. This makes sense only if N is a conserved quantity, for otherwise the chemical potential would be zero, as in the case of photon. We consider a reaction

$$e^+ + e^- = \gamma$$
, $v + \overline{v} = 2\gamma$

The reaction establishes an average value for the conserved quantum number $N_- - N_+$. The grand partition function is given by

$$Z_{G} = \sum_{N_{-}} \sum_{N_{+}} z^{N_{-} - N_{+}} Z_{N_{-}} Z_{N_{+}}$$

where

$$z = e^{\beta \mu},$$

$$F_{N_{-}} = -k_{B}T \ln Z_{N_{-}} \quad \text{or} \quad Z_{N_{-}} = e^{-\beta F_{N_{-}}}$$

$$F_{N_{+}} = -k_{B}T \ln Z_{N_{+}} \quad \text{or} \quad Z_{N_{+}} = e^{-\beta F_{N_{+}}}$$

Then we have

$$z^{N_{-}-N_{+}}Z_{N_{-}}Z_{N_{+}} = \exp[\beta\mu(N_{-}-N_{+}) - \beta(F_{N_{-}}+F_{N_{+}})]$$

We put

$$f = \mu(N_{-} - N_{+}) - (F_{N_{-}} + F_{N_{+}})$$

We find the condition that

$$\frac{\partial f}{\partial N_{-}} = \mu - \left(\frac{\partial F_{N_{-}}}{\partial N_{-}}\right)_{T,V} = 0 \quad \text{or} \quad \mu - \mu_{-} = 0.$$
$$\frac{\partial f}{\partial N_{+}} = -\mu - \left(\frac{\partial F_{N_{+}}}{\partial N_{+}}\right)_{T,V} = 0 \quad \text{or} \quad \mu + \mu_{+} = 0.$$

where

$$\left(\frac{\partial F_{N_{-}}}{\partial N_{-}}\right)_{T,V} = \mu_{-}, \quad \text{and} \quad \left(\frac{\partial F_{N_{+}}}{\partial N_{+}}\right)_{T,V} = \mu_{+}.$$

Thus we have

$$\mu_{-} + \mu_{+} = 0$$
.

When $\mu = 0$,

 $\mu_{+} = \mu_{-} = 0$

3. ((**Problem and solution**))

D.V. Schroeder, An Introduction to Thermal Physics (Addison Wesley, 1999).

Problem 7-48

In addition to the cosmic background radiation of photons, the universe is thought to be permeated with a background radiation of neutrinos (ν) and antineutrinos ($\bar{\nu}$), currently at an effective temperature of 1.95 K. There are three species of neutrinos (electron neutrino, muon neutrino, and tau neutrino), each of which has an antiparticle, with only one allowed polarization state for each particle or antiparticle. For parts (a) through (c) below, assume that all three species are exactly massless.

(a) It is reasonable to assume that for each species, the concentration of neutrinos equals the concentration of neutrinos, so trhat their chemical potentials are equal; $\mu_{\nu} = \mu_{\overline{\nu}}$. Furthermore, neutrinos and antineutrinos can be produced and annihilated in pairs by the reaction

 $v + \overline{v} = 2\gamma$

where γ is photon. Assuming that this reaction is at equilibrium (as it would have been in the very early universe), prove that $\mu = 0$ for both the neutrinos and the antineutrinos.

- (b) If neutrinos are massless, they must be highly relativistic. They are also fermions: They obey the exclusion principle. Use these facts to derive a formula for the total energy density (energy per unit volume) of the neutrino-antineutrino background radiation. (Hint: there are very few differences between this "neutrino gas" and a photon gas. Antiparticles still have positive energy, so to include the antineutrinos all you need is a factor of 2. To account for the three species, just multiply by 3.) To evaluate the final integral, first change to a dimensionless variable and then use a computer or look it up in a mathematical table.
- (c) Derive a formula for the number of neutrinos per unit volume in the neutrino background radiation. Evaluate your result numerically for the present neutrino temperature of 1.95 K.

((Solution))

The condition for equilibrium is the same as the reaction equation, but with the name of each species replaced by its chemical potential. So for the reaction

 $\nu + \overline{\nu} \leftrightarrow 2\gamma$

the equilibrium condition would be

$$\mu_{v} + \mu_{\overline{v}} = 2\,\mu_{\gamma}$$

But the chemical potential for the photons is zero, while the chemical potentials of the neutrinos and ant-neutrinos are equal to each other, if they are equally abundant. Therefore we must have

$$\mu_{\nu} = \mu_{\overline{\nu}} = 0$$

For neutrino, we use the dispersion relation as

$$\varepsilon = cp = c\hbar k$$

There are three kinds of neutrinos; <u>electron neutrinos</u>, <u>muon neutrinos</u> or <u>tau neutrinos</u>. The internal energy is

$$U_{\nu} = 3(2s+1)\frac{V}{(2\pi)^3} \int_{0}^{\infty} 4\pi k^2 dk \frac{c\hbar k}{e^{\beta c\hbar k} + 1}$$

where s=1/2. We put $x = \beta c\hbar k$. Thus we get

$$U_{v} = 6 \frac{V}{8\pi^{3}} 4\pi \int_{0}^{\infty} \left(\frac{x}{\beta c \hbar}\right)^{2} \frac{dx}{\beta^{2} c \hbar} \frac{x}{e^{x} + 1}$$
$$= \frac{3V}{\pi^{2}} \frac{k_{B}^{4} T^{4}}{c^{3} \hbar^{3}} \int_{0}^{\infty} \frac{x^{3} dx}{e^{x} + 1}$$
$$= V \frac{7\pi^{2}}{40} \frac{k_{B}^{4} T^{4}}{c^{3} \hbar^{3}}$$

or

$$U_{v} = V \frac{7\pi^{2}}{40} \frac{k_{B}^{4}T^{4}}{c^{3}\hbar^{3}} = V \frac{21}{2c} \sigma_{SB}T^{4}$$

with

$$\sigma = \frac{4\sigma_{SB}}{c} = \frac{4}{c} \frac{\pi^2 k_B^4}{60\hbar^3 c^2} = \frac{\pi^2 k_B^4}{15\hbar^3 c^3},$$

$$\sigma_{SB}=\frac{\pi^2 k_B^4}{60\hbar^3 c^2},$$

and

$$\int_{0}^{\infty} \frac{x^{3} dx}{e^{x} + 1} = \frac{7\pi^{4}}{120}.$$

The number of neutrinos is

$$N_{v} = 3(2s+1)\frac{V}{(2\pi)^{3}}\int_{0}^{\infty} 4\pi k^{2} dk \frac{1}{e^{\beta c\hbar k} + 1}$$

where s=1/2. We put $x = \beta c\hbar k$. Thus we get

$$N_{\nu} = V \frac{3k_B^{3}T^{3}}{\pi^2 c^3 \hbar^3} \int_{0}^{\infty} dx \frac{x^2}{e^x + 1}$$
$$= V \frac{3(1.80309)}{\pi^2} \left(\frac{k_B T}{c\hbar}\right)^3$$

where

$$\int_{0}^{\infty} dx \frac{x^2}{e^x + 1} = 1.80309$$

The number density of neutrino at T = 1.95 K (cosmic neutrino background)

$$n_{\nu} = \frac{N_{\nu}}{V} = \frac{3}{\pi^2} (1.80309) \left(\frac{k_B T}{c\hbar}\right)^3 = 3.38457 \text{ x } 10^8/\text{m}^3.$$

The entropy is

$$S_{\nu} = \frac{4U_{\nu}}{3T} = V \frac{42\sigma_{SB}}{3c}T^3$$

4. Photon (*y*)

We note that the internal energy of photon is

$$U_{\gamma} = V\sigma T^{4} = V \frac{\pi^{2}k_{B}^{4}T^{4}}{15\hbar^{3}c^{3}} = V \frac{4\sigma_{SB}T^{4}}{c}.$$

The entropy:

$$S_{\gamma} = \frac{4U_{\gamma}}{3T} = V\frac{4}{3}\sigma T^{4} = V\frac{16\sigma_{SB}}{3c}T^{3}$$

The number of photons is

$$N_{\gamma} = \frac{2V}{(2\pi)^3} \int 4\pi k^2 dk \overline{n}_k = \frac{V}{\pi^2} \int \frac{k^2 dk}{e^{\beta \varepsilon_k} - 1}$$

The zero-momentum state is ignored in the continuum approximation used. The energy dispersion for photon;

$$\varepsilon = \hbar c k$$

Since $d\varepsilon = \hbar c dk$, we have

$$N_{\gamma} = \frac{V}{\pi^2} \frac{1}{(\hbar c)^3} \int \frac{\varepsilon^2 d\varepsilon}{e^{\beta \varepsilon} - 1}$$

We put

$$x = \beta \varepsilon$$

Then we get

$$N_{\gamma} = \frac{V}{\pi^2} \frac{(k_B T)^3}{(\hbar c)^3} \int_0^{\infty} \frac{x^2 dx}{e^x - 1}$$

We note that

$$\int_{0}^{\infty} \frac{x^2 dx}{e^x - 1} = 2\zeta_3(z = 1) = 2.40412, \qquad \qquad \zeta_3(z = 1) = 1.20206$$

Then we have

$$N_{\gamma} = \frac{V}{\pi^2} \frac{(k_B T)^3}{(\hbar c)^3} 2.40416$$

The number density of photon at T = 2.72 K (cosmic microwave background)

$$n_{\gamma} = \frac{N_{\gamma}}{V}$$

= $(\frac{k_B T}{\hbar c})^3 0.2435926$
= $4.08255 \times 10^8 / \text{m}^3$

5. Electron-positron pair

For positron and electron, we use the dispersion relation as

$$\varepsilon = cp = c\hbar k$$

The internal energy is

$$U_{p-e} = 2(2s+1)\frac{V}{(2\pi)^3} \int_0^\infty 4\pi k^2 dk \frac{c\hbar k}{e^{\beta c\hbar k} + 1}$$

where s=1/2. We put $x = \beta c\hbar k$. Thus we get

$$U_{p-e} = 4 \frac{V}{8\pi^3} 4\pi \int_0^\infty \left(\frac{x}{\beta c\hbar}\right)^2 \frac{dx}{\beta^2 c\hbar} \frac{x}{e^x + 1}$$
$$= \frac{2V}{\pi^2} \frac{k_B^4 T^4}{c^3 \hbar^3} \int_0^\infty \frac{x^3 dx}{e^x + 1}$$
$$= V \frac{7\pi^2}{60} \frac{k_B^4 T^4}{c^3 \hbar^3}$$

or

$$U_{p-e} = V \frac{7\pi^2}{60} \frac{k_B^4 T^4}{c^3 \hbar^3} = V \frac{7}{c} \sigma_{SB} T^4$$

with

$$\int_{0}^{\infty} \frac{x^3 dx}{e^x + 1} = \frac{7\pi^4}{120}$$

The entropy:

$$S_{p-e} = \frac{4U_{p-e}}{3T} = V \frac{28\sigma_{SB}}{3c} T^{3}$$

The number density:

$$N_{p-e} = 2(2s+1)\frac{V}{(2\pi)^3}\int_0^\infty 4\pi k^2 dk \frac{1}{e^{\beta c \hbar k} + 1}$$

where s=1/2. We put $x = \beta c\hbar k$. Thus, we get

$$N_{p-e} = V \frac{k_B^3 T^3}{2\pi^2 c^3 \hbar^3} \int_0^\infty dx \frac{x^2}{e^x + 1}$$
$$= V \frac{1.80309}{2\pi^2} \left(\frac{k_B T}{c\hbar}\right)^3$$

where

$$\int_{0}^{\infty} dx \frac{x^2}{e^x + 1} = 1.80309$$

The total entropy of positron-electron, neutrino, and photon,

$$S_{tot} = V \frac{\sigma_{SB}}{c} T^{3} (\frac{28}{3} + \frac{16}{3} + \frac{42}{3}) = V \frac{\sigma_{SB}}{c} T^{3} \frac{86}{3}$$
$$S_{e-p} + S_{\gamma} = V \frac{\sigma_{SB}}{c} T^{3} (\frac{28}{3} + \frac{16}{3}) = V \frac{\sigma_{SB}}{c} T^{3} \frac{44}{3}$$
$$S_{e-p} = V \frac{\sigma_{SB}}{c} T^{3} \frac{28}{3}$$
$$S_{\nu} = V \frac{\sigma_{SB}}{c} T^{3} (\frac{42}{3})$$

$$S_{\gamma} = V \frac{\sigma_{SB}}{c} T^{3} \frac{16}{3}$$

$$\frac{S_{\nu}(T_{\nu})}{S_{\gamma}(T_{\gamma})} = \frac{V \frac{\sigma_{SB}}{c} (\frac{42}{3}) T_{\nu}^{3}}{V \frac{\sigma_{SB}}{c} (\frac{16}{3}) T_{\gamma}^{3}}$$

$$= \frac{43}{21} \frac{T_{\nu}^{3}}{T_{\gamma}^{3}}$$

$$= 0.75$$

$$\frac{S_{total}}{S_{\nu}} = \frac{V \frac{\sigma_{SB}T^{3}}{c} (\frac{86}{3})}{V \frac{\sigma_{SB}T^{3}}{c} (\frac{42}{3})}$$

$$= \frac{43}{21}$$

$$\frac{S_{total}}{S_{\gamma}} = \frac{V \frac{\sigma_{SB}T^{3}}{c} (\frac{86}{3})}{V \frac{\sigma_{SB}T^{3}}{c} (\frac{16}{3})}$$

$$= \frac{21}{4}$$

6. Cosmic neutrino background (CvB)

Given the temperature of the CMB, the temperature of the CvB can be estimated. Before neutrinos decoupled from the rest of matter, the universe primarily consisted of neutrinos, electrons, positrons, and photons, all in thermal equilibrium with each other. Once the temperature dropped to approximately 2.5 MeV, the neutrinos decoupled from the rest of matter. Despite this decoupling, neutrinos and photons remained at the same temperature as the universe expanded. However, when the temperature dropped below the mass of the electron, most electrons and positrons annihilated, transferring their heat and entropy to photons, and thus increasing the temperature of the photons. So the ratio of the temperature of the photons before and after the electron-positron annihilation is the same as the ratio of the temperature of the neutrinos and the photons today. To find this ratio, we assume that the entropy of the universe was approximately conserved by the electron-positron annihilation. Then using

where S is the entropy, g is the effective degrees of freedom and T is the temperature, we find that

where T_{ν} denotes the temperature before the electron-positron annihilation and T_{γ} denotes after the electron-positron annihilation:

16/3 for photons, since they are massless bosons 28/3 for electrons and positrons, since they are fermions.

Given the current value of $T_{\gamma} = 2.725$ K, it follows that $T_{\nu} = 1.945$ K.

Neutrino	\rightarrow	Neutrino \rightarrow	Neutrino
Electron		Electron	
Positron	\rightarrow	Positron	
Photon		Photon	Photon
Thotom	_	Thotom	
	T_{ν}		T_{γ}

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