Note on stretched exponential relaxation in spin glass phase

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We consider a spin glass system which is cooled from high temperature to a temperture T below a spin freezing temperature T_{SG} [so called the ZFC (zero-field cooled) cooling protocol] and kept at T for a wait time t_w. After a magnetic field H is applied at $t = 0$, the ZFC susceptibility χ_{ZFC} is measured as a function of the time t. It is known that $\chi_{ZFC}(t)$ exhibits a stretched exponential relaxation with a relaxation time τ. Correspondingly the relaxation rate $S(t)$ shows a peak at a characteristic time t_{cr}. We find some relation between t_{cr} and τ, which may be useful in analyzing the aging dynamics of the spin glass system.

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Here we present a simple review on the stretched exponential relaxation of χ_{ZFC} of SG phase after the ZFC aging protocol. Theoretically[1,2] and experimentally[3–16] it has been accepted that the time variation of $\chi_{ZFC}(t)$ may be described by a product of a power-law form and a stretched exponential function

$$\chi_{ZFC}(t) = M_{ZFC}(t)/H = \chi_0 - At^{-m}\exp[-(t/\tau)^{1-n}], \tag{1}$$

where the exponent m may be positive and is very close to zero, n is between 0 and 1, τ is a characteristic relaxation time, and χ_0 and A are constants. In general, these parameters are dependent on t_w. This form of $\chi_{ZFC}(t)$ incorporates both the nonequilibrium aging effect through the stretched exponential factor $[\exp[-(t/\tau)^{1-n}]$ in the crossover region ($t \approx t_w$ and $t > t_w$) between the quasi equilibrium state and nonequilibrium state, and an equilibrium relaxation response at $t \ll t_w$ through a pure power-law relaxation (t^{-m}). Note that Ogielski[1] fits his data by a stretched exponential multiplied by a power function. For $0.6 < T/T_{SG} < 1$, Ogielski[1] fits it by a power law with a different temperature dependence of exponent m. When $t \ll \tau$, $\chi_{ZFC}(t)$ is well described by a power law form given by At^{-m}. However, in the regime of $t \approx \tau$, the stretched exponential relaxation is a very good approximation in spite of finite m that is very small.

For all temperatures, $\chi_{ZFC}(t)$ increases with increasing t and the relaxation rate $S(t)$, which is defined by

$$S(t) = \mathrm{d}\chi_{ZFC}(t)/\mathrm{d}\ln t = t\mathrm{d}\chi_{ZFC}(t)/\mathrm{d}t, \tag{2}$$

exhibits a maximum at t_{cr} that is close to t_w. Using Eq.(1) for $\chi_{ZFC}(t)$, the relaxation rate $S(t)$ can be derived as

$$S(t) = -(A\tau^{-m})\exp[-(t/\tau)^{1-n}] (t/\tau)^{-(m+n)}[(1-n) + m(t/\tau)^{n-1}]. \tag{3}$$

The condition that $S(t)$ may have a peak at $t = t_{cr}$ [$\mathrm{d}S(t)/\mathrm{d}t = 0$] leads to the ratio $x_{cr} = t_{cr}/\tau$ satisfying the following equation

$$(1-n)^2 x_{cr}^2 - (1-n)(1-2m-n)x_{cr}^{n+1} + m^2 x_{cr}^{2n} = 0. \tag{4}$$

The solution of Eq.(4) can be exactly obtained as

$$x_{cr} = t_{cr}/\tau = (\xi/2)^{1/(1-n)}, \tag{5}$$

with

$$\xi = [1-2m-n+(1-n)^{1/2}(1-4m-n)^{1/2}]/(1-n), \tag{6}$$

where $4m+n < 1$. Note that the value of x_{cr} is uniquely determined only by the values of n and m. When $m = 0$, $x_{cr} = 1$ (or $t_{cr} = \tau$), which is independent of n. Figure 1(a) shows the contour plot of x_{cr} in the (n,m) plane with $-0.02 \le m \le 0.08$ and $0.2 \le n \le 0.9$, where the points having the same x_{cr} are connected by each solid line. The value of x_{cr} is lower than 1 for $m > 0$, is equal to 1 for $m = 0$ irrespective of n, and is larger than 1 for $m < 0$. Figure 1(b) shows a plot of x_{cr} as a function of m at various fixed n. The maximum value of $S(t)$ at $t = t_{cr}$ is given by

$$\begin{aligned} S_{\max} &= A\tau^{-m}2^{-1+\frac{m}{1-n}} \\ &\exp[-\frac{1}{2} + \frac{m}{1-n} - \frac{\sqrt{1-4m-n}}{2\sqrt{1-n}}](1-n)^{\frac{m}{1-n}} \\ &\times(1-n+\sqrt{1-n}\sqrt{1-4m-n}) \\ &\times(1-2m-n+\sqrt{1-n}\sqrt{1-4m-n})^{\frac{-m}{1-n}}. \end{aligned} \tag{7}$$

When $m = 0$, S_{max} is equal to S_{max}^0 [$= A(1-n)/e$] with $e = 2.7182$.

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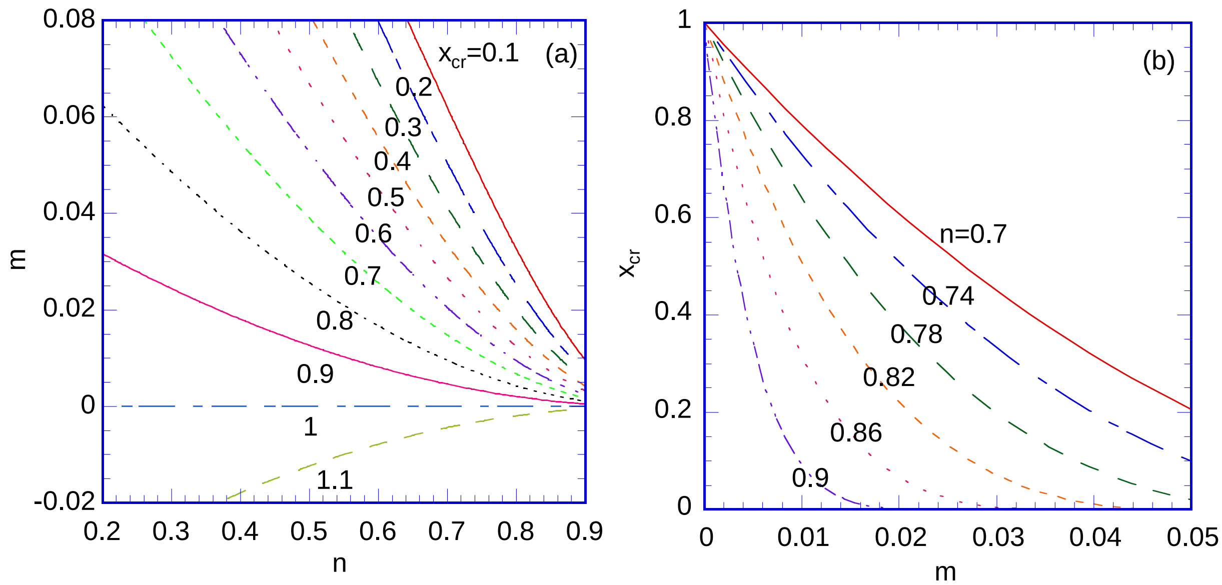

FIG. 1: (Color online)(a) Contour plot of x_{cr} ($0.1 \leq x_{cr} \leq 1.1$) in the (n, m) plane, where $x_{cr} = t_{cr}/\tau$, and the points with the same x_{cr} are connected by the same solid line. The definition of t_{cr} and τ is given in the text. (b) Plot of x_{cr} vs m at various n. The expression for x_{cr} is given by Eqs.(5) and (6).

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