

Policy Inertia and Equilibrium Determinacy in a New Keynesian Model with Investment

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Abstract

Carlstrom and Fuerst (2005) demonstrate that when investment is added to a new Keynesian model, forward-looking interest rate rules almost always lead to equilibrium indeterminacy, even when the central bank responds strongly to expected inflation. In this paper, we show that equilibrium determinacy can be retained with forward-looking rules, as long as there is policy inertia. Strong response to expected inflation is still essential in guaranteeing macroeconomic stability.

Keywords: nominal rigidities, determinacy, interest rate rules, Taylor principle

JEL Classifications: E32, E43, E52

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1 Introduction

Recently there have been considerable amount of research interests in designing interest rate rules to ensure equilibrium determinacy in a general equilibrium model with nominal rigidities. A classic result is that to avoid real indeterminacy central banks should respond strongly (more than one-for-one) to either expected or current inflation (Bernanke and Woodford, 1997; Clarida et al., 2000; Bullard and Mitra, 2002). This is often referred to as the *Taylor principle*. In these models investment is usually deemed nonessential. In their latest contribution, however, Carlstrom and Fuerst (2005) point out that investment can be crucial in evaluating interest rate rules. If investment is added to a new Keynesian model, the Taylor principle no longer guarantees equilibrium determinacy: forward-looking rules almost always lead to real indeterminacy, even when the central bank responds strongly to expected inflation. Current-looking rules, on the other hand, still relies on the Taylor principle to ensure determinacy. Since indeterminacy can cause excessive volatility, Carlstrom and Fuerst deliver a warning for policy conduct - “there is clear danger to any policy that is forward-looking,” as they forcefully put. Their recipe is to favor current-looking rules, which leads to a determinate equilibrium as long as the policymakers react aggressively to current inflation.

In reality, forward-looking behavior is quite common among central banks. Information about people’s inflation forecasts is regularly collected and analyzed,¹ and is often used as a basis for formulating interest rate policies. Evidence of such policy conduct can be found not only among speeches and testimonies of central bankers, but also in empirical research of policy rules. For example, Clarida et al. (1998 and 2000) find that central banks in G7 countries have typically targeted anticipated inflation instead of lagged inflation since 1979. Orphanides (2001) reports

¹For example, the Federal Reserve publishes a quarterly report of inflation forecasts from the Survey of Professional Forecasters. The European Central Bank, Bank of England, and several other central banks collect data from similar surveys. Many central banks also use prices of inflation-indexed government securities to infer about inflation expectations (Orphanides and Williams, 2005).

that the Federal Reserve's policy could be better described as a forward-looking rule, as opposed to the conventional Taylor rule which uses ex post data on GDP and inflation. Forward-looking behavior is also evident in central banks' implementation of *preemptive* policies, the history of which can be traced back to the founding days of the Federal Reserve.² Such preemptive policies are forward-looking by nature.

Theory predicts that these forward-looking policies can harm the economy by inducing excessive volatility. In reality, however, macroeconomic volatility has not increased with the adoptions of forward-looking policies. Indeed, some evidence even suggests that during the period which forward-looking policies became more popular (1979 - present), the volatility of macroeconomic variables have reduced compared with the period prior to that (Clarida et al., 1998 and 2000). One naturally wonders why there seems to be a gap between theoretical predictions and reality. In this paper, we propose one possible way to reconcile this seeming contradiction. We find that there is no contradiction if another realistic aspect of central bank behavior is modeled: policy inertia. Policy inertia refers to the fact that central banks tend to purposely smooth out interest rate movement by letting the current interest rate respond to lagged interest rate as well as economic conditions. Rudebusch (1995), among others, has provided statistical evidence for this phenomenon. We demonstrate that in the model economy, if the central bank targets forecasted inflation but has policy inertia, the Taylor principle again becomes a necessary and sufficient condition for equilibrium determinacy. Moreover, the result is robust to three extensions of Carlstrom and Fuerst's original model: nonlinear labor supply, policy rules that target both inflation and output, and capital adjustment costs.

In a new Keynesian model without capital, an active forward-looking rule prevents indeter-

²See Orphanides (forthcoming).

minacy and sunspot-driven fluctuations, because the sharp rise in nominal rates reduces the real interest rate and aggregate demand, which puts a downward pressure on prices and makes such any inflation fear not self-fulfilling. The addition of capital to the model, as in Carlstrom and Fuerst (2005), may change this result substantially depending on what policy rules are used. The difference is that there is now an arbitrage relationship between the returns on bonds and physical capital, which ensures that a rise in the nominal rate also leads to a rise in the future rental rate of capital. The latter is in turn positively related to the future real marginal cost and prices. If this channel dominates the effect of lower demand, a rise in the nominal rate will cause the real marginal cost to rise by so much that a rise in prices is warranted, which exactly fulfills the earlier inflation expectations. A pure forward-looking policy strengthens this mechanism by letting the nominal rate and hence future rental rate and real marginal cost move very closely with inflation expectations. In our model with policy inertia, the interest rate moves not only with inflation expectations, but also with past values of interest rates. This backward-looking feature weakens the latter mechanism and makes a determinate equilibrium easier to reach. This explains the differences in theoretical conclusions in Carlstrom and Fuerst (2005) and ours.

This paper is not the first to explore the benefits of policy inertia. Rotemberg and Woodford (1999), Woodford (2000), and Bullard and Mitra (2005) are earlier contributors to this literature. Their models do not have productive capital, as in most first-generation new Keynesian models. Sveen and Weinke (2005) do consider capital and also find that interest rate smoothing is conducive to equilibrium determinacy. There are two major differences between their work and ours. First, in their model capital is firm specific and cannot be traded, and our model keeps the conventional assumption of a rental market for capital, as in Carlstrom and Fuerst (2005).³ Second, their results

³The more conventional assumption of a rental market, however, is not necessarily more plausible or realistic, as argued by Woodford (2003).

are all numerical, while we analytically derive the necessary and sufficient conditions for determinacy. The parallel work of Kurozumi and Van Zandweghe (2007), which were done independently and simultaneously, also recommend interest smoothing as a remedy for the indeterminacy problem. Their model is quite similar to ours and so does their conclusion. The difference is that they simplify Carlstrom and Fuerst (2005)'s utility function by letting the cross differentials between consumption and real balances be zero, while we do not rely on such assumption to derive our analytical result. We derive and compare the necessary conditions for determinacy for forward and current-looking policy rules, which they do not do.⁴

The rest of the paper is structured as follows. In section 2, we briefly lay out Carlstrom and Fuerst (2005)'s model, and introduce policy inertia. In section 3, we prove analytically that the Taylor principle is both sufficient and necessary for equilibrium determinacy under forward-looking and current-looking Taylor rules. In section 4, we discuss three extensions to the model, and study the conditions for determinacy. Section 5 concludes.

2 A sticky price model with policy inertia

To contrast our result with those of Carlstrom and Fuerst (2005), we work with the exact same model that they work with, which we lay out briefly below. The only novelty of our model is the interest rate rule.

⁴They also recommend two other remedies: one is for the policy to target current economic activities such as consumption and output, and the other is using E-stability as a selection criterion for multiple equilibria.

There are a large number of households who try to maximize life-time utility:

$$\begin{aligned} & \max E_0 \sum_{t=0}^{\infty} \beta^t [U(C_t, M_{t+1}/P_t) - L_t], \\ \text{s.t. } M_{t+1} &= M_t^s (G_t - 1) + B_{t-1} R_{t-1} - B_t + P_t \{W_t L_t + [r_t + (1 - \delta)] K_t\} - P_t C_t - P_t K_{t+1} + \Pi_t, \end{aligned}$$

where C_t is consumption, M_t is cash balances, L_t is labor hours worked, B_t is the quantity of bond holdings, and K_t is accumulated capital. M_t^s denotes per capita money supply and G_t is the gross money growth rate. R_t is the nominal interest rate of bonds, while r_t is the real rate of return for renting capital. W_t is the wage rate. P_t represents the general price level. Π_t is the profit flow from firms.

Solving the consumers' problem yields the following first order conditions

$$W_t U_c(C_t, M_{t+1}/P_t) = 1, \quad (1)$$

$$U_c(C_t, M_{t+1}/P_t) = \beta E_t U_c(C_{t+1}, M_{t+2}/P_{t+1}) (r_{t+1} + 1 - \delta), \quad (2)$$

$$U_m(C_t, M_{t+1}/P_t) = U_c(C_t, M_{t+1}/P_t) \frac{R_t - 1}{R_t}, \quad (3)$$

$$U_c(C_t, M_{t+1}/P_t) = \beta E_t U_c(C_{t+1}, M_{t+2}/P_{t+1}) R_t \frac{P_t}{P_{t+1}}. \quad (4)$$

The CES production function for final goods in the competitive industry is

$$Y_t = \left(\int_0^1 y_{it}^{\frac{\eta-1}{\eta}} di \right)^{\frac{\eta}{\eta-1}}, \quad (5)$$

where Y_t stands for final goods, and y_{it} denotes the continuum of intermediate goods, each indexed by i .

Intermediate goods are produced by monopolistic competitors who utilize the production tech-

nology

$$y_{it} = K_{it}^\alpha L_{it}^{1-\alpha}. \quad (6)$$

Monopolistic competitors can set a new price at probability $1 - v$. The optimization problem is

$$\max_{P_{it}} E_0 \sum_{j=0}^{\infty} (v\beta)^j \frac{U_c(C_{t+j}, M_{t+j+1}/P_{t+j})}{U_c(C_t, M_{t+1}/P_t)} \frac{P_t}{P_{t+j}} \left(\frac{P_{it}}{P_t}\right)^{-\eta} Y_t \left(\frac{P_{it}}{P_t} - Z_t\right),$$

where Z_t denotes marginal cost.

The model is solved as in Carlstrom and Fuerst (2005). After substituting out a few variables, the model is reduced to four dynamic equations with four variables $X_t = L_t/K_t$, K_t , Z_t and inflation rate $\pi_t = P_t/P_{t-1}$. For local dynamics analysis, we linearize the system around the steady state and use lower case letters to denote the linearized variables.

The model is closed by specifying the central bank's forward-looking interest rate rule as follows:

$$i_t = \rho i_{t-1} + (1 - \rho)\tau E_t \pi_{t+1}, \quad (7)$$

where i_t stands for the deviation of R_t from its steady state value. The parameter $\rho > 0$ reflects the central bank's desire to smooth out changes in the interest rate. This policy rule captures the empirical fact that interest rate changes typically occur through a series of small adjustments in the same direction, drawn out over a period of months, rather than through an immediate once-and-for-all response to new changes in inflation (Rudebusch, 1995). In the case of current-looking policy rules, the equation becomes

$$i_t = \rho i_{t-1} + (1 - \rho)\tau \pi_t. \quad (8)$$

The system of linearized equations can be organized in matrix form as

$$E_t S_{t+1} = WS. \tag{9}$$

Since past interest rate has entered the system, we have two state variables $(k_t, i_{t-1})'$ and three endogenous variables $(c_t, z_t, \pi_t)'$. For determinacy, we require two eigenvalues of W to be within the unit circle, and three to be outside. We establish sufficient and necessary conditions for determinacy in the following section.

3 Equilibrium determinacy

Proposition 1 *Suppose monetary policy is given by the forward-looking Taylor rule (7). A necessary condition for determinacy is*

$$\tau > 1.$$

Furthermore, if $6J_3 + 2J_2 < 0$, where $J_3 = \beta a$, $J_2 = b\kappa[\tau(1-\rho) - 1] - [1 + (1+\rho)\beta]a$, $a = 1 - \beta(1-\delta)$, and $b = [1 - \beta(1-\alpha)(1-\delta)]$, then $\tau > 1$ is both a sufficient and necessary condition for determinacy.

Proof. See Appendix 1. ■

In Carlstrom and Fuerst (2005)'s original model, with forward-looking rules the area of determinacy is reduced to an extremely narrow region around $\tau = 1$, and they conclude that the presence of capital market has made determinacy “essentially impossible.” In Proposition 1, we prove that when there is policy inertia, the area of determinacy has been dramatically increased. The sufficient condition for determinacy has imposed restrictions on the parameter ρ and τ . For $\tau > 1$ to

guarantee a determinate equilibrium, it requires that

$$1 - \beta(2 - \rho) > \frac{b\kappa[\tau(1 - \rho) - 1]}{a},$$

where ρ is the interest rate smoothing parameter. This condition states that a (nonlinear) combination of ρ and τ must be higher than a certain value to ensure determinacy, which generally requires ρ to be high enough by itself. See Figure 1 for an example.

What explains the difference in this result and that in Carlstrom and Fuerst (2005)? Plug (2) into (4) and express the real return on capital as a function of the marginal product of capital and real marginal cost:

$$\frac{R_t}{\pi_{t+1}} = \alpha z_{t+1} \left(\frac{K_{t+1}}{L_{t+1}} \right)^{\alpha-1} + 1 - \delta. \quad (10)$$

This is the arbitrage relationship between real returns on bonds and physical capital. The linearized version is

$$i_t - \pi_{t+1} = z_{t+1} + (\alpha - 1)(k_{t+1} - l_{t+1}). \quad (11)$$

Without interest rate smoothing, the interest rate rule is

$$i_t = \tau \pi_{t+1}. \quad (12)$$

Plugging the rule into (11), we have an expression that contains only forward-looking terms. As Carlstrom and Fuerst (2005) point out, this implies that there must be a zero eigenvalue in the system. Since capital is the only state variable, for determinacy all other eigenvalues must be greater than 1 in absolute values. The zero eigenvalue must then force capital to become a jump variable – it must adjust to the steady state at time 0, and stay there. This equilibrium is extremely

difficult to reach.

With interest rate smoothing, the interest rate rule becomes (7). While the arbitrage relationship still exists and there still is a zero eigenvalue, the arbitrage equation is no longer purely forward-looking. There are now two state variables k_t and i_{t-1} in the system. For determinacy we need another nonzero stable root and three other explosive roots. In this case, the zero eigenvalue means that capital needs to adjust at each time t to maintain a certain relationship with i_{t-1} , so that the arbitrage condition can be satisfied, but it no longer has to jump to the steady state at time 0. This equilibrium is evidently much easier to reach.

We next consider the condition for determinacy under current-looking rules with interest rate smoothing.

Proposition 2 *Suppose monetary policy is given by the current-looking Taylor rule (8). A necessary condition for determinacy is*

$$\tau > 1.$$

Furthermore, if $6q_3 + 2q_2 < 0$, where $q_3 = \beta a$, $q_2 = -b\kappa - [1 + (1 + \rho)\beta]a$, and $a = 1 - \beta(1 - \delta)$, then $\tau > 1$ is both a sufficient and necessary condition for determinacy.

Proof. See Appendix 2 ■

This proposition states that a current-looking interest rate rule with inertia also leads to a determinate equilibrium, as long as the central bank responds strong enough to current levels of inflation. Note that the sufficient condition imposes a restriction on the parameter ρ :

$$\rho > 5 - \frac{2b\kappa}{\beta[1 - \beta(1 - \delta)]} - \frac{1}{\beta}.$$

Unlike the forward-looking case, this condition does not demand any relationship between ρ and

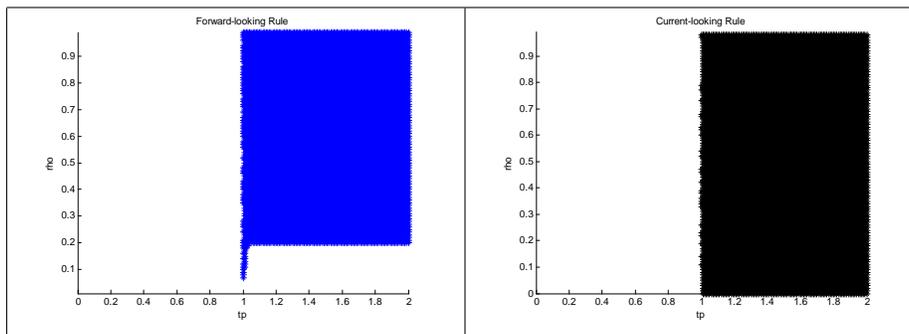


Figure 1: Equilibrium Determinacy with Policy Inertia

τ . Instead, it only requires ρ to be greater than a certain value. This condition indicates that it is easier to reach a determinate equilibrium with a current-looking rule than with a forward-looking rule. In fact, with reasonably calibrated parameters, the term on the right-hand-side is negative, which makes $\tau > 1$ a sufficient condition for determinacy for all values of $\rho \in (0, 1)$. See Figure 1.

We next present the plot from a numerical analysis to help readers visualize the effects of interest rate smoothing on equilibrium determinacy. The parameters are calibrated as in Carlstrom and Fuerst: $\alpha = \kappa = 1/3$, $\delta = 0.02$, and $\beta = 0.99$. We let τ and ρ vary and plot the area of determinacy allowed by a combination of the two parameters. In Figure 1, the left panel shows the simulation results for the forward-looking rule, and the right panel for the current-looking rule. The dark regions mark the area of determinacy. As the two propositions predict, $\tau > 1$ is necessary for determinacy for both interest rate rules. For the forward-looking rule, determinacy is only possible when the level of policy inertia (measured by ρ) is high enough. For the current-looking rule, the sufficient condition for determinacy is always met, even when ρ is equal to zero.

4 Extensions

Carlstrom and Fuerst (2005)'s model captures the essence of a new Keynesian model with investment, and is simple by design to facilitate analytical solutions. To check the robustness of our results, however, we need to extend the model in several dimensions.

4.1 Three extensions of the model

Our first extension is to remove the requirement of linear labor supply and let the utility function become

$$U = U(c_t, \frac{M_{t+1}}{P_t}) - \frac{L_t^{1-\chi}}{1-\chi}.$$

Adding a curvature to labor supply not only changes the labor supply relationship (1), but also makes it impossible to summarize the dynamics of K and L by a single variable x .

Our second extension is to allow the central bank to respond to forecasted output. The interest rate rule becomes

$$i_t = \rho i_{t-1} + (1 - \rho)(\tau_\pi E_t \pi_{t+1} + \tau_y E_t y_{t+1}). \quad (13)$$

Letting the interest rate respond to output gaps is considered by a number of authors. For the new Keynesian model with capital, adding output to the interest rule may change the determinacy results considerably, as pointed out by Sveen and Weinke (2005) for a model with firm-specific capital.

Our third extension is to add an adjustment cost to the capital accumulation equation, as suggested by Woodford (2004). Investment is now defined as

$$I_t = I\left(\frac{K_{t+1}}{K_t}\right)K_t, \quad (14)$$

where the function $I(\bullet)$ is assumed to satisfy the following: $I(1) = \delta$, $I'(1) = 1$, and $I''(1) = \varepsilon_\psi$. ε_ψ represents the elasticity of the invest-capital ratio with respect to Tobin's q , evaluated in the steady state. Carlstrom and Fuerst (2005) add adjustment cost to their model, and find that it will only change the determinacy result if the level of adjustment costs is at the high end. It is therefore interesting to check if the same conclusion will hold with interest rate smoothing.

For each of the three extensions, analytical results are difficult to obtain. So we resort to numerical analysis. To do this, we need a functional form for the utility function, which we assume to take the form

$$U(C_t, \frac{M_{t+1}}{P_t}) = \frac{C_t^{1-\sigma}}{1-\sigma} + \gamma \frac{(M_t/P_t)^{1-b}}{1-b},$$

where $\sigma, \gamma, b > 0$. This is a utility function that is widely used for this type of analysis.

4.2 Equilibrium determinacy for the extensions

We examine the equilibrium determinacy of our extended models. We consider each extension separately. That is, we do not combine extensions in our model so that the effect of each extension can be isolated.

The additional parameters are calibrated as follows. The curvature for the utility function σ is equal to 1 - a value often used by the real business cycle literature. γ and b do not have any impact on system dynamics, and we do not need to calibrate them. ε_ψ is set at 3, which is justified by Woodford (2003), but we also consider $\varepsilon_\psi = 1$ and $\varepsilon_\psi = 10$ for a sensitivity analysis. Similarly, we also consider three values for χ : 0, 1, and 6, and consider three values for τ_y : 0.1, 0.4 and 0.8. We focus on the forward-looking case in the following analysis.

Figure 2 compares the determinacy areas when the curvature parameter χ changes from 0 to

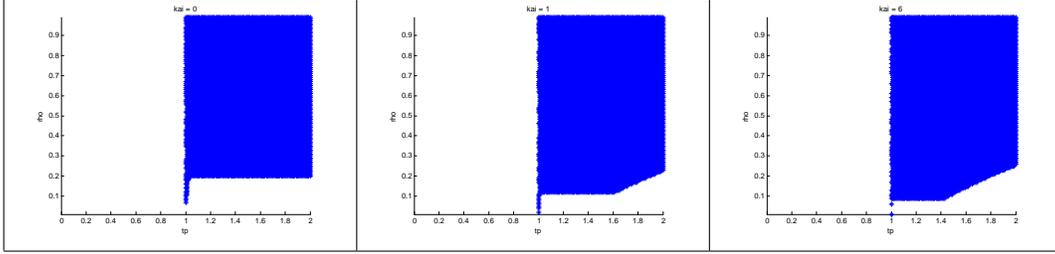


Figure 2: Equilibrium Determinacy with Elastic Labor Supply

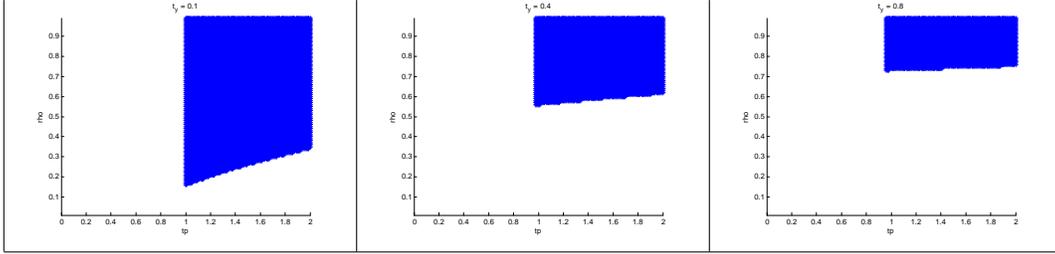


Figure 3: Output as a Policy Target

1 and 6. A higher value of χ enlarges the determinacy area by reducing the requirement for the level of policy inertia (ρ) in order to get determinacy. However, when ρ is close to 0, the area of determinacy remains very small. This is true even when χ is as high as 20 (not shown in Figure 2).

We next consider the case in which the interest rate rule targets forecasted output. As Figure 3 shows, when the reaction parameter to output increases from 0.1 to 0.4 and 0.8 (from left to right), the area of determinacy actually shrinks. In other words, higher levels of policy inertia is required to reach a determinate equilibrium when the interest rate policy responds strongly to forecasted output. This result should not be surprising, since what the policy targets is not *current* output but forecasted *future* output. Including future output in the policy rule does not change the pure forward-looking nature of the arbitrage condition (10), and therefore is not conducive to determinacy. If we let the interest rate policy targets current output, the result will change dramatically. The plot will look very similar to the case of current-looking policy rules in Figure 1.

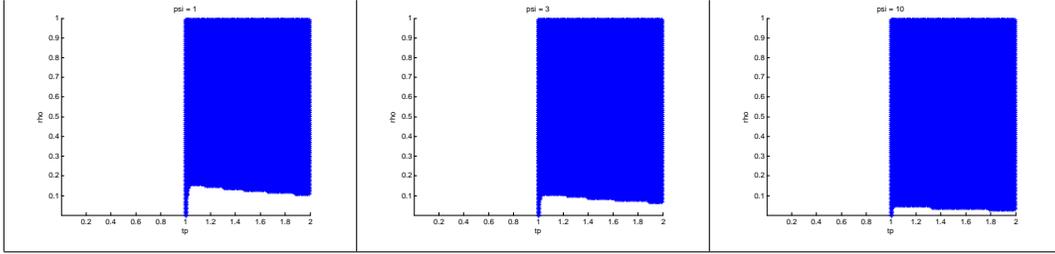


Figure 4: Equilibrium Determinacy With Adjustment Costs

Finally, we consider the case of capital adjustment costs. From left to right in Figure 4, we let the invest-capital ratio elasticity ε_ψ change from 1 to 3 and 10, and plot the determinacy area. A higher elasticity enlarges the determinacy area. But in all cases, the determinacy area is extremely limited when there is no interest rate smoothing.

In summary, for all three extensions of the benchmark model, we have found that interest rate smoothing is important for the model economy to reach a determinate equilibrium. For most cases in our calibrated analysis, when the level of interest rate smoothing ρ is higher than 0.2, the Taylor principle is sufficient to ensure equilibrium determinacy.

5 Conclusion

Carlstrom and Fuerst (2005) demonstrate that when investment is added to a new Keynesian model, forward-looking interest rate rules almost always lead to equilibrium indeterminacy, even when the central bank follows the Taylor principle. In this paper, we show that equilibrium determinacy can be retained with forward-looking rules, as long as there is policy inertia. With policy inertia, the Taylor principle is again a necessary and sufficient condition for equilibrium determinacy. This result is robust to three extensions of Carlstrom and Fuerst's model.

As McCallum (1999) points out, central banks usually do not have complete information on

output and inflation of the same quarter that a policy decision must be made, and this makes the current-looking rule quite unrealistic. Our theoretical result suggests that a central bank need not resort to current-looking rules to avoid excessive volatility, as long as it purposely smooths out interest rate movements.

6 Appendix

The linearized system consists of four dynamic equations. The first equation is the new Phillips curve obtained by solving the firms' pricing problem:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa z_t, \quad (15)$$

where we use lower case letters to denote percentage deviation from the steady state of a variable.

π_t is inflation rate. The second equation is the linearized version of (2):

$$\alpha x_t - z_t = \{1 - (1 - \alpha)[1 - \beta(1 - \delta)]\} E_t x_{t+1} - \beta(1 - \delta) E_t z_{t+1}. \quad (16)$$

The third equation is the linearized version of (4):

$$\alpha x_t - z_t = \alpha E_t x_{t+1} - E_t z_{t+1} + (i_t - E_t \pi_{t+1}), \quad (17)$$

where i_t denotes the nominal interest rate.

The last equation is the linearized version of the economy's resource constraint

$$k_{t+1} = a_1 k_t + a_2 z_t + a_3 x_t + a_4 i_t, \quad (18)$$

where $a_1 = 1 + \frac{c}{k}$, $a_2 = -\frac{c}{k}\gamma_z$, $a_3 = (1 - \alpha)\frac{y}{k} - \gamma_x\frac{c}{k}$, and $a_4 = -\frac{c}{k}\gamma_i$. $\frac{y}{k}$ and $\frac{c}{k}$ are output-capital ratio and consumption-capital ratio in the steady state computed as $\frac{y}{k} = \frac{1-\beta(1-\delta)}{\alpha\beta}$ and $\frac{c}{k} = \frac{y}{k} - \delta$. γ_z , γ_x and γ_i are parameters associated with the relationship $c_t = \gamma_x x_t + \gamma_z z_t + \gamma_i i_t$, which is derived by combining (1) and (3).

Proposition 1 *Suppose monetary policy is given by the forward-looking Taylor rule (7). A necessary condition for determinacy is $\tau > 1$. Furthermore, if $6J_3 + 2J_2 < 0$, where $J_3 = \beta a$, $J_2 = b\kappa[\tau(1 - \rho) - 1] - [1 + (1 + \rho)\beta]a$, $a = 1 - \beta(1 - \delta)$, and $b = [1 - \beta(1 - \alpha)(1 - \delta)]$, then $\tau > 1$ is both a sufficient and necessary condition for determinacy.*

Proof. The linearized system can be written as

$$E_t S_{t+1} = W S_t,$$

where $S_t = (x_t, \pi_t, z_t, k_t)'$. For determinacy we need three eigenvalues to be outside the unit circle, and two to be inside. It is straightforward to show that one eigenvalue is equal to 0, and another is equal to $a_1 = 1 + \frac{c}{k} > 1$. To obtain the other eigenvalues, we compute the characteristic polynomial for the remaining part of the matrix as

$$J = J_3\lambda^3 + J_2\lambda^2 + J_1\lambda + J_0, \text{ where} \quad (19)$$

$$J_3 = \beta a, \quad (20)$$

$$J_2 = b\kappa[\tau(1 - \rho) - 1] - [1 + (1 + \rho)\beta]a, \quad (21)$$

$$J_1 = -\alpha\kappa[\tau(1 - \rho) - 1] + [1 + (1 + \rho)\beta]a + b\kappa\rho, \quad (22)$$

$$J_0 = -\rho a - \alpha\kappa\rho, \quad (23)$$

where $a = 1 - \beta(1 - \delta)$, and $b = [1 - \beta(1 - \alpha)(1 - \delta)]$. For determinacy, we need two explosive roots and one stable root in the remaining three eigenvalues. We can compute

$$J(0) = J_0 = -\rho[1 - \beta(1 - \delta) + \alpha\kappa], \quad (24)$$

$$J(1) = \kappa(\tau - 1)(1 - \rho)[1 - \beta(1 - \delta)](1 - \alpha). \quad (25)$$

Since $0 < \rho < 1$, $1 - \beta(1 - \delta) > 0$, and $\alpha\kappa > 0$, it must be true that $J(0) < 0$. In $J(1)$, all terms except for $\tau - 1$ are positive, therefore $J(1) > 0$ only if $\tau > 1$. When $q(0) < 0$ and $q(1) > 0$, there must be at least one eigenvalue $0 < \lambda^* < 1$ that makes $J(\lambda^*) = 0$. We conclude that $\tau > 1$ is a necessary condition for determinacy.

If $6J_3 + 2J_2 < 0$, J is concave at 1. Since the cubic expression J will eventually goes to infinity as λ becomes larger, this implies that there will be two more roots of J that are greater than unity. In other words, J will intercept the horizontal axis twice to the right of 1, one from above, and one from below. Two explosive roots plus one stable root will exactly ensure determinacy. Therefore, in this case $\tau > 1$ is both sufficient and necessary for determinacy. ■

Proposition 2 *Suppose monetary policy is given by the current-looking Taylor rule (8). A necessary condition for determinacy is*

$$\tau > 1.$$

Furthermore, if $6q_3 + 2q_2 < 0$, where $q_3 = \beta a$, $q_2 = -b\kappa - [1 + (1 + \rho)\beta]a$, and $a = 1 - \beta(1 - \delta)$, then $\tau > 1$ is both a sufficient and necessary condition for determinacy.

Proof. It is again straightforward to show that one eigenvalue is equal to 0, and another is equal to $a_1 = 1 + \frac{c}{k} > 1$. To obtain the other eigenvalues, we compute the characteristic polynomial for

the remaining part of the matrix as

$$q = q_3\lambda^3 + q_2\lambda^2 + q_1\lambda + q_0, \text{ where} \quad (26)$$

$$q_3 = \beta a, \quad (27)$$

$$q_2 = -b\kappa - [1 + (1 + \rho)\beta]a, \quad (28)$$

$$q_1 = [1 + (1 + \rho)\beta]a + b\kappa\tau(1 - \rho) + \kappa(\alpha + b\rho), \quad (29)$$

$$q_0 = -\rho a - \alpha\kappa[\rho + \tau(1 - \rho)]. \quad (30)$$

Obviously, $q(0) = q_0 < 0$. When $\lambda = 1$,

$$q(1) = (1 - \alpha)[1 - \beta(1 - \delta)]\kappa(\tau - 1)(1 - \rho).$$

This expression is exactly identical to $J(1)$ for the forward-looking case. Therefore $q(1) > 0$ only if $\tau > 1$, a necessary condition for determinacy. For q to be concave at 1, we must have $6q_3 + 2q_2 < 0$, in which case $\tau > 1$ is both necessary and sufficient for determinacy. ■

7 References

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