Can Indeterminacy Resolve the Cross-Country Correlation Puzzle?*

Wei Xiao†

University of New Orleans

Abstract

Are productivity shocks the only driving force of international business fluctuations? In this paper I argue that another source of uncertainty - changes in market expectations or "sunspots" - is also important. One major shortcoming of existing IRBC models is the "cross-country correlation puzzle": models tend to generate cross-country consumption correlations that are too high and output, investment and employment correlations that are too low when compared to the data. I show that with empirically supported level of increasing returns, an otherwise standard model possesses multiple, indeterminate convergent paths to the steady state, which allows for sunspots to influence the economy. The model displays time series properties that in many ways match the data better than the conventional model. It is especially successful in generating realistic consumption and output correlations.

JEL classification: E3, E13, E32, F4

Keywords: indeterminacy; multiple equilibria; real business cycle; international business cycle; sunspots.

*I am grateful to Jess Benhabib, Dean Corbae, David DeJong, Wouter J. den Haan, John Duffy, Roger Farmer, G. Vaughan Parker, Chris Telmer, James Warnick, Yi Wen, Gerald Whitney and three anonymous referees for valuable comments and suggestions.

†Wei Xiao, Department of Economics and Finance, 2000 Lakeshore Drive, New Orleans, LA 70148. Tel: 504-280-7147. Fax: 504-280-6397. Email: wxiao@uno.edu.

1 Introduction

As noted in the pioneering work of Backus, Kehoe, and Kydland (1992 and 1993), an international real business cycle (IRBC) model typically fails to account for the cross-country correlations of economic aggregates. It generates cross-country consumption correlations that are too high and output, investment and employment correlations that are too low when compared to the data. In Backus, Kehoe and Kydland (1993), the model predicts that cross-country consumption correlations are in the range of 0.88 and 0.95, but output, investment and employment correlations are all negative. In contrast the data shows that cross-country correlations of all four variables are positive, and consumption correlation is typically lower than output correlation. Table 1 shows this anomaly in more detail.

A striking contrast between the model and the data is that the model's predictions for cross-country correlations of output, investment and employment all have the "wrong" sign. This is closely related to the propagation of productivity shocks in a world of integrated financial and commodity markets. In a two-country model, for example, a positive productivity shock in one country raises its factor payoffs, which attracts flows of resources from the other country to "make hay where the sun shines." Production activities hence move in opposite directions in the two countries. Consumption is still positively correlated because agents can pool country-specific risks in world financial markets.

Recent research in this area has advanced in two major directions. One direction is to introduce restrictions on asset trade (Baxter and Crucini, 1995; Kollmann, 1996; Kehoe and Perri, 2000). This is based on the intuition that restricting the degree of risk-sharing may potentially weaken the international link of consumption growth and reduce cross-border resource flows. Baxter and Crucini (1995) consider a single-bond economy, and find that the cross-country correlation of consumption is lower than that of output if productivity shocks are a random walk rather than trend stationary. But in their model, consumption, investment and employment correlations are negative. Kollmann

(1996) also studies a bond-economy and is able to produce positive output correlations. But in his model, investment and labor are negatively correlated internationally. Kehoe and Perri (2000) mainly seek to endogenize the incomplete asset market assumption.

Another direction of research considers demand shocks as an alternative impulse mechanism (Stockman and Tesar, 1995; Wen, 2002; Kollmann, 2001). This is based on the view that fluctuations in aggregate demand are important for business cycles. Stockman and Tesar (1995) add "taste shocks" to an IRBC model with non-tradable goods. Wen (2002) adds taste shocks to an IRBC economy with habit-persistence consumer preferences. Kollmann (2001) considers a sticky price environment in which the economy is subject to monetary policy shocks. These authors show that adding an aggregate demand "flavor" to an IRBC model yields much more realistic cross-country correlations of economic aggregates.

In this paper, I argue that another source of market uncertainty - self-fulfilling expectations - is also important for international business cycles. Such uncertainty is commonly referred to as "sunspots" or "animal spirits." Optimistic or pessimistic expectations cause investors and consumers to adjust spending, and thus generate business cycles. This paper argues that fluctuations of expectations can propagate through financial markets, and have a contagious impact on investors and consumers of other countries who adjust their spending accordingly. This added uncertainty yields more correlated international business cycles. Animal spirits were the cornerstone of the Keynesian explanation of recessions. Recently this idea has been revitalized by a number of researchers within the rational expectation framework.[1] Most of these works, however, focus on closed-economies. In this paper I extend the sunspot model into an international environment, and examine its implications for the international comovements of economic aggregates.

Sunspots do not exist in standard IRBC models. To generate sunspots, I assume there are mild

[1]Sunspot dynamics in RBC models have been studied by Benhabib and Farmer (1994), Farmer and Guo (1994), Schmitt-Grohé (1997), Wen, (1998), and Benhabib and Wen (2002), among others.

increasing returns in the production technology. This gives rise to multiple convergent paths to the steady state equilibrium, and allows sunspots to become an alternative source of uncertainty.[2] Past work has shown that the level of increasing returns needed to generate indeterminacy is in the upper range of empirical estimates. The recent works of Weder (2001) and Wen (1998), however, suggest that if we allow capacity utilization to be endogenously determined, the level of increasing returns can be dramatically reduced. This mechanism can be expected to work the same way in an open-economy framework, hence I incorporate it into this paper.

The main findings are as follows. With the assumption of complete asset-markets, the model with indeterminate equilibria and sunspot shocks can create positive international correlations of output, employment and investment. With the alternative assumption of incomplete asset-markets, the model makes realistic predictions of international correlations of economic aggregates. Most importantly, it predicts that the international correlation of consumption will fall below that of output. These results depend crucially on two factors. One is the size of the sunspot shocks relative to productivity shocks. In order to generate realistic international correlations, sunspot shocks have to be the dominant source of economic fluctuations. The second factor is incompleteness of the asset markets. Without it, consumption is always more correlated internationally than output. This finding is consistent with the spirit of Baxter and Crucini (1995) and Kollmann (1996). Unlike their models, however, multiple equilibria allow sunspot shocks to enter the bond pricing equation directly.

Shifts in beliefs induce people to increase or decrease aggregate demand. In this respect they are similar to the taste shocks considered by Stockman and Tesar (1995) and Wen (2002). However, some

[2] Increasing returns to scale is crucial in generating indeterminate equilibria. The intuition is as follows: from any given equilibrium, if agents have some optimistic expectations about future returns, they will increase their investment. Their expectations will be "self-fulfilling" only if future rates of returns on investment indeed increase. Constant-returns-to-scale technology does not deliver such results. Increasing returns, if strong enough, can produce a rise in the future rates of return, which justifies agents' earlier expectations. See Benhabib (1998) for a more detailed explanation.

important differences separate these two sources of uncertainty. Firstly, taste shocks are fundamental shocks and are exogenously imposed. They represent unanticipated changes in national health, birth rates, social structure, and other similar factors that affect the marginal rate of substitution across different market goods. They are often assumed to be serially correlated. Sunspot shocks are non-fundamental in that they are extrinsic to preferences and technology. They arise endogenously from forecast errors of agents under rational expectation (explained in section 3), and must be i.i.d. random variables. Secondly, sunspots arise in multiple, indeterminate equilibria, which can generate system dynamics that are richer than those of the typical RBC models. For example, a transitory sunspot shock can generate persistent, hump-shaped responses in output and consumption (See Farmer and Guo, 1994). To generate similar dynamics, models with a determinate equilibrium usually resort to other mechanisms, such as habit persistence utility functions (Wen, 2002).

There are few papers that study indeterminacy in an open-economy environment. Guo and Sturzenegger (1998)'s work is the closest to this paper. Differences between the two are as follows. Firstly, their version of the sunspot-driven economy does not allow agents to share risk in the international asset market. This paper examines alternative economies in which contingent claims and bonds are traded, even when the economy is subject to only sunspot shocks. Secondly, their model allows installed capital stocks, instead of investments, to be perfectly mobile across countries, forcing capital rental rates to be always equalized internationally. This paper keeps the more standard assumption of mobile investment but immobile capital stock. Thirdly, their model assumes fixed capital utilization, while this paper allows the capital utilization rate to vary endogenously. Fourthly, their model requires an increasing return of more than 1.7, which is no longer supported by empirical findings, to generate indeterminacy. In this paper, I show that with dynamic utilization and non-separable utility, it is possible to generate indeterminacy with much lower levels of increasing returns. Finally, their model predicts almost perfect output correlation across countries.

My model generates more realistic international comovements.

The rest of the paper is organized as follows. Part 2 presents the model environment. Part 3 describes the calibration of parameters and explores the dynamic properties of the model. Part 4 presents and discusses the final results, and part 5 concludes.

2 A Two-Country Model

There are two countries that produce a homogeneous good. Each country is represented by a single agent. The agents' preferences and technologies have the same structure and parameter values. However, domestic and home production differ in two important respects: in each country the labor input consists only of domestic labor, and production is subject to country-specific technology shocks. I describe the domestic economic environment below. The foreign environment can be defined analogously.

2.1 Households

The preferences of the representative consumer are characterized by an expected utility function[3]

$$E\sum_{t=0}^{\infty}\rho^t\frac{[C_t\exp(-N_t)]^{1-\sigma}}{1-\sigma}, \tag{1}$$

where $0 < \rho < 1$ is the discount factor, and C and N are consumption and hours worked in the home country. To ensure concavity of the utility function, we need to have $\sigma \geq 1$ (King, Plosser and Rebelo, 1988). When $\sigma = 1$, it collapses to Hansen's indivisible-labor utility function commonly used in the RBC literature.

[3]This is a version of the general form utility function of Bennett and Farmer (2001).

2.1.1 Complete markets

In one version of the economy, I assume there is an integrated global financial markets in which agents can trade ownership shares and a full set of contingent claims against states of nature. Define $B(s_t)$ as the domestic agents' holding of one-period claims, each of which pays out one unit of consumption good contingent on the state of nature being s_t, and $q(s_{t+1}, s_t)$ as the price of such a claim whose payoff occurs in period $t+1$. Property ownerships are modeled as follows: at the beginning of each period, the domestic representative agent owns A_t^H shares (fraction) of the domestic firms' output, and A_t^F shares of the foreign firms' output (with their foreign counterparts being A_t^{H*} and A_t^{F*}). Total ownership shares are normalized to be 1 for each country. In each period, the agents can trade these shares in the global asset market. Let p_t and and p_t^* be the market values, and D_t and D_t^* be the dividend yields of home and foreign shares. Finally, let w_t denote the real wage rate. The domestic budget constraint is[4]

$$C_t + \sum_{s_{t+1}} B(s_{t+1})q(s_{t+1}, s_t) + A_{t+1}^H p_t + A_{t+1}^F p_t^* \leq w_t N_t + B(s_t) + A_t^H(p_t + D_t) + A_t^F(p_t^* + D_t^*)] \quad (2)$$

The agent chooses consumption, leisure, home and foreign share holdings, and state-contingent bond holdings to maximize her objective. This yields the following necessary conditions:

$$C_t^{-\sigma}[\exp(-N_t)]^{1-\sigma} = \lambda_t \quad (3)$$

$$C_t^{1-\sigma}[\exp(-N_t)]^{-\sigma} = \lambda_t w_t \quad (4)$$

$$p_t = E_t \rho \frac{\lambda_{t+1}}{\lambda_t}(p_{t+1} + D_{t+1}) \quad (5)$$

[4]All variables are state contingent. To simplify notation, I suppress the symbols for states of nature, except for the price and quantity of state contingent claims.

$$p_t^* = E_t \rho \frac{\lambda_{t+1}}{\lambda_t}(p_{t+1}^* + D_{t+1}^*) \tag{6}$$

$$q(s_{t+1}, s_t) = \rho \frac{\lambda_{t+1}}{\lambda_t} \tag{7}$$

Equation (3) equates the marginal utility of consumption with its opportunity cost (shadow price of capital). Equation (4) equates the marginal utility gained from a unit of leisure with its utility-measured cost. Equations (5) and (6) are the consumption-based asset pricing equations for domestic and foreign stocks. Equation (7) is the pricing equation for the state-contingent claims.

The foreign country's necessary conditions can be obtained analogously. What is particularly important is the foreign counterpart of (7), which implies that in every state of nature,

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{\lambda_{t+1}^*}{\lambda_t^*} \tag{8}$$

That is, risk-pooling will necessarily equalize the two countries' marginal rates of substitution.[5]

2.1.2 Incomplete markets

To study the implications of restricted asset trade, I consider a version of the Baxter and Crucini (1995) incomplete-market economy, in which the only way of international risk-sharing is through the trading of a riskless bond. Ownership shares can only be traded domestically. The domestic budget constraint becomes

$$C_t + b_{t+1} p_t^b + A_{t+1}^H p_t \leq w_t N_t + b_t + A_t^H (p_t + D_t), \tag{9}$$

[5] See Obstfeld (1994) for a more elaborate derivation.

where b_{t+1} and p_t^b are the quantity and price of the riskless bond, respectively. The agent's optimal conditions still include (3) - (5). The pricing equation for bonds is

$$p_t^b = \rho E_t \frac{\lambda_{t+1}}{\lambda_t} \tag{10}$$

Combining the home and foreign countries' bond pricing equations, we get the critical relationship

$$E_t \frac{\lambda_{t+1}}{\lambda_t} = E_t \frac{\lambda_{t+1}^*}{\lambda_t^*}, \tag{11}$$

which says that the intertemporal marginal rates of substitution are no longer equalized between the two countries. Instead their *expected values* are now equalized.

2.2 Firms

The production sector is similar to Jermann (1998)'s asset pricing economy with production. The home country has a large number of identical firms. In each period, managers maximize the value of the firm to its owners, which is equal to the present discounted value of all current and future dividends (D_t).

$$E_t \sum_{i=0}^{\infty} \rho^t \frac{\lambda_{t+i}}{\lambda_t} D_{t+i},$$

where ρ is the discount factor, and $\frac{\lambda_{t+i}}{\lambda_t}$ is the marginal rate of intertemporal substitution of the owners.[6] The dividend yields of the firm D_t are defined as output (Y_t) less labor cost ($w_t N_t$) and investment (I_t):

$$D_t = Y_t - w_t N_t - I_t$$

[6] It appears that the firms are only maximizing domestic owners' values. But in the complete-market version $\frac{\lambda_{t+1}}{\lambda_t} = \frac{\lambda_{t+1}^*}{\lambda_t^*}$ in every state of nature. The domestic and foreign owners' values are simultaneously maximized. In the incomplete market version, foreigners do not own domestic firms.

The firm's production technology is

$$Y_t = Z_t(u_t K_t)^a N_t^{1-a} Q_t, \tag{12}$$

where $0 < a < 1$, Z_t is a stochastic shock to productivity, K_t is capital stock, $u_t \in (0, 1)$ is the rate of capital utilization, and Q_t is a measure of production externalities defined as

$$Q_t = (\overline{u}_t \overline{K}_t)^{a\theta_1} \overline{N}_t^{(1-a)\theta_2} \tag{13}$$

$\overline{K}_t$, $\overline{u}_t$ and $\overline{N}_t$ are the average economy-wide levels of capital, utilization rate and labor, which are exogenous from the point of view of each firm.

Investment drives the process of capital accumulation through the dynamic constraint

$$I_t = K_{t+1} - (1 - \delta_t)K_t \tag{14}$$

where δ_t is the rate of depreciation of capital stock defined as an increasing function of capital utilization rate u_t:

$$\delta_t = \frac{1}{\eta} u_t^{\eta},\ \eta > 1 \tag{15}$$

The speed of capital depreciation is therefore endogenously determined.

The firm's necessary conditions for profit maximization are

$$u_t^{\eta} = a\frac{Y_t}{K_t} \tag{16}$$

$$w_t = (1-a)\frac{Y_t}{N_t} \tag{17}$$

$$\lambda_t = E_t \rho \lambda_{t+1}(a\frac{Y_{t+1}}{K_{t+1}} + 1 - \delta_{t+1}) \tag{18}$$

Equation (16) determines the efficient level of capacity utilization. Equation (17) defines the real wage rate. Equation (18) equalizes the current and future (shadow) values of capital investment.

Since all firms are identical, in equilibrium we have $N_t = \overline{N}_t$, $K_t = \overline{K}_t$, and $u_t = \overline{u}_t$. By making the parameter substitutions

$$\alpha = a(1+\theta_1) > 0 \tag{19}$$

$$\beta = (1-a)(1+\theta_2) > 0 \tag{20}$$

we obtain the aggregate production function

$$Y_t = Z_t(u_t K_t)^{\alpha} N_t^{\beta} \tag{21}$$

When $\theta_1 > 0$ or $\theta_2 > 0$, the aggregate production function has increasing returns to scale ($\alpha+\beta > 1$).

2.3 Equilibrium

A dynamic equilibrium consists of a set of prices and quantities such that the agents maximize their objectives as described above, and all markets clear.

If markets are complete, the asset market clearing conditions are

$$\begin{aligned} B(s_t) + B(s_t)^* &= 0 \\ A_t^H + A_t^{H*} &= 1 \\ A_t^F + A_t^{F*} &= 1 \end{aligned} \tag{22}$$

The world goods market clearing condition is given by

$$C_t + C_t^* + I_t + I_t^* = Y_t + Y_t^* \tag{23}$$

If markets are incomplete, the market clearing conditions are given by (23) and

$$\begin{aligned} b_t + b_t^* &= 0 \\ A_t^H &= 1 \\ A_t^{F*} &= 1 \end{aligned} \tag{24}$$

3 System dynamics and calibration

3.1 Sunspot dynamics

To analyze the short-run dynamics of the model, I linearize the first order conditions around the steady state as in King, Plosser and Rebelo (1988). Dynamics of the model are determined by the set of linear expectational difference equations[7]

$$\begin{bmatrix} S_{t+1} \\ \Lambda_{t+1} \end{bmatrix} = J \begin{bmatrix} S_t \\ \Lambda_t \end{bmatrix} + R \begin{bmatrix} \epsilon_{t+1}^z \\ \epsilon_{t+1}^s \end{bmatrix}, \tag{25}$$

where $S_t = (K_t, K_t^*, Z_t, Z_t^*)'$ is the state vector, Λ_t is the co-state vector, ϵ^z is the vector of technological shocks, and ϵ^s is the vector of one step-ahead forecast error, or sunspots:[8]

[7] All other variables can be expressed as functions of the state and co-state vectors.

[8] In a recent paper, Lubik and Schorfheide (2003) point out that when fundamental shocks and sunspot shocks both exist in a model, the dynamic effect of the fundamental shocks cannot be uniquely determined without specific assumptions about how forecast errors are determined. The definition of sunspots here makes it clear that forecast errors are directly determined by sunspots and not affected by productivity shocks. The dynamic effect of the latter is therefore uniquely pinned down. Most previous works in this field adopted this approach. See, for example, Farmer and Guo (1994), Schimitt-Grohé (1997), and Wen (1998).

$$\epsilon_{t+1}^s = \Lambda_{t+1} - E_t \Lambda_{t+1} \tag{26}$$

With complete markets, the marginal rates of substitution of the two countries are equalized state by state (see equation 8). Since this also implies

$$\lambda_t / \lambda_{t-1} = \lambda_t^* / \lambda_{t-1}^*, \tag{27}$$

we can multiply both sides of (8) by those of (27), and iterate in this fashion to get

$$\lambda_t / \lambda_0 = \lambda_t^* / \lambda_0^*$$

Because we consider symmetric economies where $\lambda_0 = \lambda_0^*$, this will equalize the two $\lambda' s$ of the two countries at each state:[9]

$$\lambda_t = \lambda_t^* \tag{28}$$

Therefore, in the complete-market economy, there is only one *independent* co-state variable. Let this be the domestic co-state variable λ_t. When solving the system (25), we need to substitute out λ_t^* so that $\Lambda_t = \lambda_t$, and $\epsilon_{t+1}^s = \lambda_{t+1} - E_t \lambda_{t+1}$. That is, the entire economy shares the same shadow price of capital and is subject to only one single sunspot shock. In a fully integrated financial market, a shift of investor beliefs affect both countries simultaneously.

With incomplete markets, the marginal rates of substitution of the two countries are only equalized in expectations (see equation 11), and the above iteration cannot be made. We have $\Lambda_t = (\lambda_t, \lambda_t^*)'$, and $\epsilon_{t+1}^s = (e_{t+1}^s, e_{t+1}^{s*})' = (\lambda_{t+1} - E_t \lambda_{t+1}, \lambda_{t+1}^* - E_t \lambda_{t+1}^*)'$. Each economy is subject to a distinct sunspot shock.

[9]Even when the economy is not symmetric, the linearized version of the equations are still not affected by initial values.

The impact of sunspot shocks, however, depends critically on how many roots of the Jacobian matrix J in equation (25) lie inside the unit circle. If the number of stable roots is equal to the number of predetermined variables (K and Z), the system has a unique "saddle-path" solution. When eliminating the unstable roots, the co-state vector will be solved as functions of the state vector, and the expectation error term ϵ_{t+1}^{s} does not have any effect on the equilibrium. This is the case of a standard IRBC economy.

If the number of stable roots of J is greater than the number of predetermined variables, however, the co-state vector will no longer be uniquely pinned down by the state vector. There are multiple equilibria because each realization of the expectation errors will put the economy on a different path. The system becomes a "sink". The expectation errors matter because on each different path there are different realizations of consumption and output. In this paper, when the level of increasing returns to scale is sufficiently large, the economic system will turn from a saddle-path to a sink.

Besides admitting sunspot shocks, sink equilibria differ from a saddle path equilibrium in another important way: they generate much richer system dynamics. The dynamics of the saddle-path economy is determined by a subset of (25)

$$S_{t+1} = BS_t + D\epsilon_{t+1}^{z}, \tag{29}$$

where B and D are parameter matrices. The co-state vector is solved as a function of S_t and ϵ_{t+1}^{z}. The law of motion of the sink-path economy, on the other hand, is the entire system of (25), which naturally displays richer dynamics than the subset (29). In particular, the co-state vector now fluctuates independently. In practice, the sink system often has complex roots, which causes (25) to fluctuate cyclically in response to a transitory shock.

Finally, I shall stress the importance of goods market frictions in generating sunspot dynamics. In this economy, agents trade contingent claims against states of nature. These states not only include ones that have different fundamentals (technology shocks), but also ones where fundamentals are identical but sunspot shocks differ. The latter are called a set of "sunspot states." As proved in the seminal work of Cass and Shell (1983), when markets are complete, sunspots "do not matter" (in terms of affecting allocations) unless there are violations of the Arrow-Debreu welfare theorems. The critical assumption of this model is that production externalities violate the welfare theorems. Therefore sunspots do matter even if there are contingent claims against the set of sunspot states.[10]

3.2 Calibration of parameters

It remains to calibrate the parameters of the economy before the model be examined numerically. Except for the production technology, most parameter values are chosen to conform with standard RBC models. The steady state real interest rate is set equal to 1 percent per quarter, which is close to the average rate of return on capital over the past century. This implies a discount factor of 0.99. The depreciation rate is consistent with properties of quarterly data (2.5% per quarter or 10% per annum). The values for capital and labor shares (a and $1-a$) are set at 0.3 and 0.7, respectively.

The key issue is to generate indeterminate equilibria with empirically plausible increasing returns. A number of authors have estimated the returns to scale in US industries. The lower bounds of the estimates have decreased over time as econometricians refine their estimation techniques. Most authors now base their model calibration on the findings of Basu and Fernald (1997), who conclude that an average firm has approximately constant returns to scale (1.02), except for manufacturing firms (1.26, and 1.33 for manufacturing durables), and that the seemingly large increasing returns to scale at the aggregate level (1.72) are caused by aggregation of heterogeneous sectors. However, they also point out that this does not suggest that returns to scale in one sector models, such as $\alpha+\beta$ in this paper, should be confined to the estimated firm level parameter. To them a multi-sector

[10] A similar point was made by Woodford (1991).

model is suitable for matching their empirical results.

As shown in Bennett and Farmer (2001), the level of required increasing returns to generate indeterminacy also depends on the utility curvature parameter σ. The higher σ is, the higher the required increasing returns. Empirical estimates of this parameter, such as those in Hansen and Singleton (1983), fall into a broad range of 0 and 3. I did a numeric search for the required levels of $\alpha + \beta$ with each value of σ, and found the following. If $\sigma = 1$, the required level of returns to scale is about 1.1; If $\sigma = 2$, the required level is 1.44; If $\sigma = 3$, the required level becomes 1.54. As a calibration exercise, I use $\sigma = 1.1$ in my simulation of the sunspot model so that the level of increasing returns is empirically plausible.[11]

When $\sigma = 1.1$, there are still many combinations of empirically plausible levels of externalities to choose from. To calibrate this, I proceed as follows. Since the average US business cycle has a periodicity of 20 quarters or a frequency of 0.05 cycles per quarter, I pick the value of α and β such that the model economy will also fluctuate at such frequencies.[12] This yields $\alpha = 0.34$, $\beta = 0.83$ (or $\theta_1 = 0.13$ and $\theta_2 = 0.186$). The aggregate level of returns to scale is therefore 1.17.

Conventional measures of Solow residuals are inappropriate for this model because they are estimated based on the assumption of fixed utilization rates. In a recent empirical work, Burnside, Eichenbaum and Rebelo (1996) re-estimate the properties of productivity shocks by employing electricity use as a proxy for capacity utilization. Two interesting conclusions arise from their work: (1) the standard deviation of productivity shocks relative to output is only 0.36, a 70% drop from the conventional measures. This requires that theoretical models provide a much bigger amplification mechanism for the shock; and (2) the hypothesis that the growth rate of productivity is uncorrelated with the growth rate of output cannot be rejected. I later use these as criteria for measuring the

[11] I do not use $\sigma = 1$ because that will always produce perfect international consumption correlations for the productivity-driven IRBC economy. It is a standard practice to use a non-separable utility function to study international business cycles.

[12] The system has two complex roots. The frequency of the roots $x + yi$ is $\cos^{-1}(\frac{x}{\sqrt{x^2+y^2}})\frac{1}{2\pi}$. I search for the level of α and β such that the frequency defined above is equal to 0.05.

performance of the model.

To calibrate the properties of productivity shocks, I estimate a transition matrix using the Solow residuals of the US and an aggregate of European countries. The US data is directly taken from Burnside, Eichenbaum and Rebelo (1996). The European capacity utilization rate data is taken from *European Economy* supplement B, published by the European Communities. The data spans the first quarter of 1974 and the last quarter of 1998. European output data are obtained from OECD's *Quarterly National Accounts.* To proxy for labor input, I use the civilian employment index obtained from the OECD database.[13] With the assumption of constant returns to scale, the VAR(1) estimation yields

$$\begin{bmatrix} \log Z_t \\ \log Z_t^* \end{bmatrix} = \begin{bmatrix} 0.986_{(0.0301)} & 0.02_{(0.0167)} \\ 0.026_{(0.0304)} & 0.946_{(0.0168)} \end{bmatrix} \begin{bmatrix} \log Z_{t-1} \\ \log Z_{t-1}^* \end{bmatrix} + \begin{bmatrix} e_t^z \\ e_t^{z*} \end{bmatrix}, \tag{30}$$

where the standard errors are reported in the parentheses. A VAR was also run under the assumption of increasing returns to scale, and a similar result was obtained.

For simulation, a symmetric transition matrix is used so that the results are not affected by country sizes. This will also facilitate exposition as I only have to report the simulation results for one country. The symmetric matrix is chosen such that its eigenvalues match those in (30). This yields a persistent parameter of 0.966 and a "spill-over" parameter of 0.03. The estimated correlation between e_t^z and e_t^{z*} is about 0.11. With the assumption of increasing returns, the two coefficients are 0.96 and 0.034, and the correlation between e_t^z and e_t^{z*} is 0.10.

Sunspot shocks are i.i.d. random variables, so I only have to specify their volatilities. As a

[13] A caveat follows. The Burnside, Christiano and Eichenbaum (1996) estimation is based on industry level data. Labor hours and electricity usage are used to proxy N_t and u_t. Since the same data is not available for European countries, I use employment indices and a general utilization rate as proxies. Therefore the estimation cannot be exactly accurate. However, it can serve as a useful approximation. In fact the estimation results are quite consistent with other authors' findings that productivity shocks are persistent and have small spill-over (Backus, Kehoe and Kydland, 1993 and Baxter and Crucini, 1995). Note also that when the economy is driven only by sunspots, this matrix has no effect on system dynamics.

calibration exercise, in each simulation the standard deviation for sunspots is picked to match the volatility of US output. I also assume that the sunspot shocks are independent between countries in the benchmark experiments.

All calibrated parameter values are reported in table 2.

4 Simulation Results

To study the implications of the model for international business cycles, I run numeric simulations to produce artificial time series data, and then compare their properties with those of the real data as a criterion for measuring model performances. For every version of the model economy, I report the means of various statistics across 100 simulations, each for 150 periods.

4.1 The saddle-path IRBC economy

I first examine the model's performance when the equilibrium is determinate and sunspots do not exist. This will provide some benchmark results to compare with the indeterminacy model. The simulation results are presented in table 3. The parameter values used for the simulations are those in panel A of table 2 unless otherwise described. Since the variance of shocks is calibrated to match output volatility of the US, I only report relative volatility of aggregate variables. For each experiment, the volatility of shocks is reported in the last row of the table.

A standard IRBC model typically has complete asset markets, constant returns to scale and fixed utilization rates. We begin by looking at the behavior of this benchmark economy (labeled "benchmark" in table 3). The model statistics display the typical properties of the puzzle: international consumption correlation is high (0.49), and output, investment, and labor correlations are all sharply negative (-0.94, -1, and -0.99). Compared with the results of Backus, Kehoe and Kydland (1993) in table 1, the consumption correlation is relatively low, but the negative correlations are more extreme in magnitude. This is primarily because in their simulation the cross-country correlation of productivity shocks is as high as 0.26, while in this paper it is estimated to be 0.11.

In section 1 the negative correlations are explained as being caused by volatile cross-border flows of physical investment in pursuit of higher returns. This is evident from the excessively high volatility of investment (17 times the standard deviation of output, as opposed to 3.27 In the data), and the negative correlation between domestic investment and domestic output (-0.08).

Next I relax the assumption of fixed capacity utilization (labeled "dynamic utilization" in table 3). This simulation uses the same set of parameter values as in the previous experiment. With flexible utilization, I expect the model to perform somewhat differently: consider a technology shock at home but no shock in the foreign country. With constant utilization, investment will definitely flow from the foreign country to home to pursue higher returns; with dynamic utilization, an alternative is for the home utilization rate to increase. If the increase is strong enough, home interest rate will drop quickly to shut down foreign investment inflows. In that case the cross-country output correlation will be less negative. The cost of increasing home utilization rate, however, is the foregone consumption. The extent that this mechanism works therefore depends on agents' balancing of the marginal costs and benefits of altering utilization rates.

As the simulation shows, this mechanism is very weak in the calibrated economy. Compared with the benchmark case, the model predicts a slightly less negative international output correlation, and a pro-cyclical investment with a small coefficient. Otherwise it behaves almost identically to the benchmark model. To examine the robustness of this finding, a sensitivity analysis with respect to the parameter σ and the cross-country correlation coefficient of productivity shocks is performed. I report the results in Table 4. In one set of analyses I fix the cross-country correlation of productivity shocks at the benchmark value and let the curvature parameter σ vary from 1 to 4 (panel A of table 4). In the other set of analyses, I fix σ and allow the correlation coefficients to vary (panel B of table

4). With high values of σ and high correlations of productivity shocks, output does become more less negatively correlated internationally. But two facts remain true: 1) international consumption correlation is much higher than that of output; and 2) output, employment and investment are negatively correlated internationally. The essential features of the puzzle are still present.

As a third experiment, the assumption of constant returns to scale is relaxed for the dynamic-utilization economy. I set $\theta_1 = \theta_2 = 0.1$ so that the level of increasing returns is $\alpha + \beta = 1.1$. This value is carefully chosen such that it is close to the value calibrated for the benchmark sunspot economy (1.17), but is not high enough to generate indeterminate equilibria. All other parameter values are still those in panel A of table 2. The results are labeled "increasing returns" in table 3. Again the model's predictions are no more accurate than those of the benchmark model. This is not surprising - without any changes in the impulse or propagation mechanism, a small increasing return of 1.1 is not likely to make the model behave much differently.

As a last experiment, I consider the implications of restricted asset trade in the constant-returns-to-scale IRBC model with dynamic utilization (labeled "incomplete markets" in table 3). If risk-pooling is responsible for high consumption correlation and excessive resource flows, it seems intuitive that introducing incomplete asset markets should go a long way towards resolving the puzzle. But surprisingly, the results are almost identical to those of the complete-market version (column 2) of the economy. Baxter and Crucini (1995) have an extensive analysis of this result. In the next section I show that with sunspot shocks, the incomplete-market economy will behave quite differently from the complete-market one.

In order to have a consistent comparison, all simulations of table 3 use the benchmark calibration (constant returns and dynamic utilization) for productivity shocks. By doing this, however, we lose some degree of precision for the "benchmark" and the "increasing returns" model, since in these two cases the estimated Solow residuals are not consistent with their respective features of fixed utilization and increasing returns. So we re-estimate Solow residuals for these two models, and report the simulation results for cross-country correlations in the last panel of table 4. As expected, all major features of the puzzle persist.[14]

The key is to understand the economic mechanisms behind the simulation results. The working of the standard IRBC model is well-known. Productivity shocks in one country attract resources from the other, which drives one country's production above trend and the other's below trend. Figure 1 plots the impulse response functions of each country to one unit of productivity shock in the home country. On impact, the home country's output, employment and investment increase while the foreign country reacts in the opposite way. Consumption in both countries trends up since risk is pooled in the financial market. Technically, the level of risk-pooling is determined by the equation

$$\widehat{\lambda}_{t+1} - \widehat{\lambda}_t = \widehat{\lambda}^*_{t+1} - \widehat{\lambda}^*_t, \tag{31}$$

which is the log-linear version of (8). The correlation of λ_t and λ_t^* is therefore always 1. With a non-separable utility function, the international consumption correlation will be high but not equal to 1, as λ and λ^* also incorporate country-specific changes in leisure. In the incomplete-market economy the risk-pooling condition becomes

$$E_t\widehat{\lambda}_{t+1} - \widehat{\lambda}_t = E_t\widehat{\lambda}^*_{t+1} - \widehat{\lambda}^*_t, \tag{32}$$

which is the log-linear form of (11). Given that the expected terms are eventually determined only by fundamentals (the expectation error $\widehat{\lambda}_{t+1} - E_t\widehat{\lambda}_{t+1}$ has no effect on system dynamics), the working of this economy is not very different from the complete-market one. In the next section I show this is not the case when the equilibrium is indeterminate.

[14]Both estimated Solow residuals are trend stationary with small spill-overs. To simply exposition, I do not report the estimated parameters in the tables.

4.2 Indeterminacy in the IRBC economy

4.2.1 Sunspot-driven economies

Let's now examine the model's predictions when the scale of increasing returns is high enough to yield indeterminate equilibria. First I simulate an economy with complete asset markets and dynamic utilization rate. I assume the only source of uncertainty comes from i.i.d. sunspots. The parameter values used for this experiment are those of panel B of table 2 (except for the parameters for productivity shocks, of course). The volatility of the shocks are calibrated to match the volatility of US output. The results are reported in column 2 of table 5.

It is immediately evident that there are important differences between this economy and the saddle-path IRBC economy. With sunspots being the only uncertainty and with complete markets, the model predicts that the two countries' business cycles are completely synchronized and the international correlations of all variables are equal to 1. This extreme case is unrealistic, since the two economies are subject to a single sunspot shock (explained in section 3.1). But it exactly highlights the important effect of sunspots: they tend to force the economic variables to "move together" across countries. If the economy is subject to multiple shocks, the effect of sunspot shocks would be to produce more positively correlated business cycles - the very feature that productivity shocks fail to deliver.

The model makes distinct predictions for investment dynamics. Recall that in the standard model the enormous amount of cross-border resource flows drive up the volatility of investment to more than 10 times that of output, and sometimes render domestic investment counter-cyclical. The indeterminacy model predicts a much lower standard deviation for investment (4.4 relative to output), and that investment is highly pro-cyclical (0.99).

Another striking feature of the model is perhaps the persistent oscillations of all variables in response to a purely transitory shock. Figure 2 plots the impulse response function of both countries to a unit of sunspot shock in the home country. On impact, all economic variables respond positively to the shock. Subsequently, the economy exhibits the type of hump-shaped dynamics that we typically observe in the data. The model displays cycles with a half-life of about 10 quarters. An economic expansion is followed by a recession, which in turn is followed by another expansion. Essentially, there is a multiplier-accelerator mechanism in play: a rise in consumption demand stimulates capital accumulation, which reduces its marginal product. As a result employment goes down and output shrinks, which drives up the marginal products and triggers another round of expansion (see Farmer and Guo, 1994).

An undesirable feature of the sunspot model is that the predicted consumption fluctuations seem to be too smooth (a relative volatility of about 0.1). This is less than the prediction of the standard IRBC model (0.17), and is much lower than that of the data (0.75). Wen (1998) reported the same result for a closed-economy with only sunspot shocks. But he also showed that in a more realistic model with other demand shocks, the volatility of consumption relative to output will increase.[15]

With complete markets, the sunspot economy does not resolve the cross-country correlation puzzle since there is only one shock. In the next experiment, I examine a version of the sunspot economy in which the only internationally traded asset is a riskless bond. Each economy is subject to its own sunspot shocks. The two shocks are independent to each other.

A significant difference turns up when we look at the cross-country consumption correlations (the last column of table 5). With incomplete markets, it decreases to a value of 0.15. What is more, the cross-country correlations of output (0.35), investment (0.2) and labor (0.37) are all positive and higher than consumption correlation. This is the most empirically plausible prediction in all experiments so far. What has proved to be difficult to explain is why most IRBC models cannot reverse the rank of international consumption and output correlations. This experiment shows

[15] Indeterminacy and other demand shocks, such as taste shocks, are not mutually exclusive. Researchers (Benhabib and Wen, 2002 and Wen, 1998) have established that the combination of the two can resolve several puzzles in the RBC literature.

that an IRBC model will predict such features if both goods market and asset market frictions are considered.

What accounts for the distinct predictions of this economy? The first important element is the demand-driven feature of the economy. Sunspots are essentially shocks to the co-state variable λ. Unlike productivity shocks, whose initial impact is on the supply side, a shock to λ shifts the marginal utility of consumption, and has an initial impact on aggregate demand. An increase in demand will naturally drive up productive activities in both countries, through the purchase of home goods and the imports of the foreign goods. This mechanism alone determines that the two economies will have positively correlated business cycles.

What reverses the rank of international consumption and output correlations? The critical condition is the asset-market relationship (32), which can be equivalently written as

$$\widehat{\lambda}_{t+1} - \widehat{\lambda}_t + e^s_{t+1} = \widehat{\lambda}^*_{t+1} - \widehat{\lambda}^*_t + e^{s*}_{t+1}, \tag{33}$$

where $e^s_{t+1} = E_t\widehat{\lambda}_{t+1} - \widehat{\lambda}_{t+1}$ are sunspots. Unlike in the saddle-path economy, indeterminacy allows sunspots to specifically enter the risk-pooling equation. With idiosyncratic shocks, there is now no guarantee that the two countries' marginal rates of intertemporal substitution are equalized. It is even more helpful to consider the extreme case where $\sigma = 1$, so that the utility function is separable in consumption and leisure. In that case (33) is reduced to

$$\widehat{c}_{t+1} - \widehat{c}_t + e^s_{t+1} = \widehat{c}^*_{t+1} - \widehat{c}^*_t + e^{s*}_{t+1} \tag{34}$$

Without the two error terms, the international consumption correlation will surely be 1. But with the two error terms, this correlation depends on how the errors are related. In the model we assume that the two error terms are independent, which is sufficient to generate enough "diverging" consumption movements such that the international correlation are lower than that of output. When the utility function is non-separable in consumption and leisure, it will be even easier to generate low consumption correlations. For example, I later show (table 6) that international consumption correlation is lower than output correlation even when the two shocks are moderately correlated.

Baxter and Crucini (1995) and Kollmann (1996) both study economies in which the risk-pooling condition (32) holds. They find that this modification makes very little improvement over the standard IRBC model. But in both models there are only determinate, saddle-path equilibria. The major difference between this paper and theirs is the existence of indeterminacy, which allows sunspots to enter the system in the manner of (33). This accounts for the different model predictions.

Finally, I examine the robustness of the results for the incomplete-market model. In the previous experiments I assume that the sunspots shocks are not correlated between the two countries. In reality, there is some evidence that they are positively correlated (Guo and Sturzenegger, 1998). I let the correlation coefficient vary from 0.1 to 0.3, and also consider the less likely cases where it is equal to 0.6, -0.2 and -0.4, respectively. As table 6 shows, high correlation of shocks implies high international correlations of all variables. If the shocks are negatively correlated, economic aggregates also become negatively correlated. In all cases, consumption is less correlated internationally than output.

The level of increasing returns affects the dynamic properties of the linearized system. I next examine the sensitivity of the results with respect to the level of increasing returns. The crucial parameters are the sizes of externality θ_1 and θ_2. Within the empirically relevant levels, there are a large number of combinations of the two parameters to choose from. To pick some representative numbers, we assume $\theta_1 = \theta_2$ and let the value vary from 0.18 to 0.30 at an increment of 0.02 (so the aggregate returns to scale will vary from 1.18 to 1.3). As table 6 shows, the cross-country correlations are quite robust under different levels of externality. In all cases consumption and output are positively correlated across countries, and the ranking of correlations are unchanged. The only exception is investment, which tends to be negatively correlated when the level of increasing returns becomes higher than 1.2.

4.2.2 Incorporating productivity shocks

Having only sunspot uncertainty is a somewhat extreme assumption. In the next set of experiments, productivity shocks enter the model. The goal is to answer the following two questions: can the model still resolve the cross-country correlation puzzle when productivity shocks are incorporated? and can the model provide a good amplification mechanism for productivity shocks? The latter question is relevant because the endogenous fluctuation feature of the model seems to be a good candidate for the amplification of productivity shocks. According to Burnside, Christiano and Eichenbaum (1996), productivity shocks are small ($\sigma^z/\sigma^y = 0.36$), and the hypothesis that it is uncorrelated with output cannot be rejected ($corr(\sigma^z, \sigma^y) = 0.2$ with a standard error of 0.17). Can the current model reproduce such results?

I simulate the model with both sunspot and productivity shocks. The parameter values are taken from panel B of table 2. Since the model's performances are likely to depend on the relative importance of sunspot shocks, and there is little empirical evidence to guide its calibration, I simply experiment with different values. Specifically, I experiment with the ratio $\sigma^z/(\sigma^s + \sigma^z)$, where σ^z and σ^s stand for the standard deviation of productivity and sunspots respectively. To examine whether or not the correlation between the two shocks are important, I also consider the case where the two shocks are equally volatile and are correlated at 0.8 and -0.8. High values are used to underline the effect of such correlation. I do this for each assumption of the asset markets. For the incomplete-market economy the two sunspot shocks are assumed to be independent.

The answer to the first question depends on how much productivity shocks will "distort" the results presented in the previous section. The results can be summarized as follows (see table 7):

in all experiments, the cross-country correlations of output, consumption, labor and investment are positive. Consumption correlation never goes below output correlation in the complete-market economy, but will do so if the asset markets are incomplete, and sunspot shocks are the dominate source of fluctuations.[16] The absolute values of correlation presented in the table seem to be lower than what we observe in the data. But that is because sunspot shocks are not correlated internationally, and productivity correlation is only 0.1 between the two countries. As the sensitivity analysis of table 6 shows, when we consider correlated sunspot shocks, the cross-country correlations of economic aggregates become even more realistic. We therefore conclude that the model's performances provide a positive answer to the first question.

Why do we need dominant sunspot shocks to resolve the puzzle? As explained in the previous section, a sunspot shock works on the demand side and tends to increase output in both countries, while a productivity shock works on the supply side and tends to increase output in one country but decrease it in the other. For the two economies to have positively correlated business fluctuations, the demand-side shock will therefore have to dominate the supply-side shock. A remaining question is whether or not we can support this result empirically. Guo and Sturzenegger (1998) use consumer confidence indicators as a measure of sunspot shocks. They estimate the standard deviation of sunspot shocks to be 0.049, which is 6.7 times the standard deviation of the productivity shocks estimated by Burnside, Christiano and Eichenbaum (1996). We view this as preliminary evidence that supports the theoretical result of this paper.

With regard to the second question, the model also produces some encouraging results. First of all, in all experiments (including the case where productivity shocks are the major source of uncertainty), the relative volatility of productivity to output is well below 0.15, and the correlation between productivity and output is near 0. These results are in sharp contrast with the standard

[16] My experiments show that $\sigma^z/(\sigma^s+\sigma^z) < 0.25$ is required for the ranking of international consumption and output correlation to reverse. In this sense I should probably characterize sunspots as being "strongly dominant," rather than simply "dominant.".

IRBC model, where the correlation of productivity and output is never below 0.5. In a closed-economy model, this correlation is typically higher than 0.9.

There are two mechanisms underlining these results. Firstly, dynamic utilization makes output more responsive to productivity shocks. This mechanism works the same in a saddle-path economy. For example, King and Rebelo (1999) conclude that variable utilization helps create amplification, but "only in a modest manner." Secondly, the dynamic system (25) fluctuates cyclically once disturbed from the steady state, as explained in the previous section. This provides extra amplification that the saddle-path economy does not have.

The second mechanism is no doubt also responsible for the small output-productivity correlation. Consider how the economic system converges back to the steady state on impact of a shock at period 1. When the equilibrium is a "saddle", the system follows a smooth path back to the origin (see figure 1). The nature of the productivity process determines the shapes of all other variables' response curves. For example, it is well-known that if the productivity shock is (not) persistent, then the response of output is also (not) persistent. Therefore output must be highly correlated with productivity. When the equilibrium is a "sink" with complex roots, however, the system exhibits dampened cycles when disturbed by a temporary shock (see figure 3). Output "fluctuates by itself" - its response curve does not depend on the shape of the shock process. In this case, output-productivity correlation will surely be smaller.[17]

If measured quantitatively, however, the amplification effect seems to be too strong. For example, if we use Burnside, Christiano and Eichenbaum (1996)'s estimate of the standard deviation of productivity shocks (0.007), and shut down all sunspot shocks, the simulated volatility (standard deviation) of output is as high as 20% in the incomplete-market version of the economy. If we

[17] When markets are complete, the productivity-output correlation is small but has a surprising negative sign. My intuition is that when the endogenous comovements of the two economies are very strong, as in the complete-market case, country-specific productivity shocks become almost irrelevant. In this case productivity-output correlation may have a negative sign, especially when the coefficient is small and not significant.

use King and Rebelo (1999)'s estimate (0.0012) instead, output volatility is reduced to 3.4 percent, which is still twice as volatile as US output. One way to remedy this is to increase the agents' coefficient of risk aversion, σ. A numeric search shows that if σ is close to 2.7, the simulated output volatility will match the data. Although $\sigma = 2.7$ is still in the range of empirically plausible values, the required level of increasing returns will be as high as 1.5.

Therefore we conclude that the model provides a qualitatively interesting answer to the second question, but the current setup of the model does not produce a quantitative match with empirical data.

5 Conclusion

This paper extends the analytical basis for understanding international business cycles by considering the effect of indeterminacy on system dynamics. The results of the study confirm the intuition that indeterminacy helps to explain the consumption correlation puzzle. In our model with increasing returns, the cross-country consumption correlation is lower than that of output, and international business cycles become positively correlated. Two factors contribute to this result. One is the arising of self-fulfilling sunspot shocks that cause fluctuations in aggregate demand. The other is incomplete asset markets that keep economic agents from fully insuring against country-specific risks.

Given the results in this paper, opportunities now exist to enrich our understanding of international business cycles in other aspects. One possible extension of the current model is to add another good to study the relative price changes across-countries. Backus, Kehoe and Kydland (1993) show that variability of relative prices in a standard model is too low compared to the data. They call this discrepancy between the model and the data the "price anomaly." The two features of the sunspot model - multiple equilibria and endogenous fluctuations - can be exploited to study this anomaly. Imagine that price levels are different on each equilibrium path regardless of fundamentals. When sunspot shocks shift the economy from path to path, price levels can potentially exhibit volatile fluctuations, which will in turn render the terms of trade more volatile. We leave this topic for future research.

References

[1] Backus, D.K., P.J. Kehoe and F. Kydland (1992), "International Real Business Cycles," *Journal of Political Economy,* 100, 745-775.

[2] Backus, D.K., P.J. Kehoe and F. Kydland (1993), "International Business Cycles: Theory vs. Evidence," *Federal Reserve bank of Minneapolis Quarterly Review,* Fall 1993, 14-29.

[3] Basu and J. Fernald (1997), "Returns to scale in US production: Estimates and implications," *Journal of Political Economy*, 105, 248-283.

[4] Baxter, M. and M. Crucini (1995), "Business Cycles and the Asset Structure of Foreign Trade," *International Economic Review,* 36, 821-854.

[5] Benhabib, J. and R. Farmer (1994), "Indeterminacy and Increasing Returns," *Journal of Economic Theory,* 63, 19-41.

[6] Benhabib, J. (1998), "Introduction to Sunspots in Macroeconomics," *Journal of Economic Theory,* 81, 1-6.

[7] Benhabib and Wen (2002), "Indeterminacy, Aggregate Demand, and the Real Business Cycle," NYU working paper.

[8] Bennett, R.L. and R. Farmer (2000), "Indeterminacy with Non-separable Utility," *Journal of Economic Theory, 93, 118-143.*

[9] Burnside, C., M. Eichenbaum, and S. Rebelo (1996), "Sectoral Solow Residuals," *European Economic Review 40, 861 - 869.*

[10] Cass, D. and K. Shell (1983), "Do Sunspots Matter?" *Journal of Political Economy*, 91, 193 - 227.

[11] Farmer, R. and J-T. Guo (1994), "Real Business Cycles and the Animal Spirits Hypothesis," *Journal of Economic Theory,* 63, 42-72.

[12] Guo, J-T, and F. Sturzenegger (1998), "Crazy Explanations of International Business Cycles," *International Economic Review,* 39, 111-133.

[13] Hansen, L.P., and K. Singleton (1983), "Stochastic consumption, risk aversion and the temporal behavior of asset returns," *Journal of Political Economy 91, 249 - 265.*

[14] Jermann, U. (1998), "Asset Pricing in Production Economies," *Journal of Monetary Economics*, 257-275

[15] Kehoe, P. and W. Perri (2000), "International Business Cycles with Endogenous Incomplete Markets," *Federal Reserve Bank of Minneapolis Staff Report*, 26.

[16] King, R., C. Plosser and S. Rebelo (1988), "Production, Growth and Business Cycles: I. the Basic Neoclassical Model," *Journal of Monetary Economics*, 21: 195 - 232.

[17] King, R. and S. Rebelo (1999), "Resuscitating Real Business Cycles," *Handbook of Macroeconomics,* Elsevier Science Publications, Chapter 14: 927-1007.

[18] Kollmann, R. (1996), "Incomplete Asset Markets and Cross-Country Consumption Correlation," *Journal of Economic Dynamics and Control,* 20, 945-961.

[19] Kollmann, R. (2001), "Explaining International Comovements of Output and Asset Returns: The Role of Money and Nominal Rigidities," *Journal of Economic Dynamics and Control*, 25, 1547 - 1583.

[20] Lubik, T. and F. Schorfheide (2003), "Computing Sunspot Equilibria in Linear Rational Expectations Models," *Journal of Economic Dynamics and Control*, 28, issue 2, 273 - 285.

[21] Obstfeld, M. (1994), "Are Industrial-Country Consumption Risks Globally Diversified?" in *Capital Mobility: The Impact on Consumption, Investment and Growth*, edited by L. Leiderman and A. Razin, 13-44, Cambridge University Press: New York.

[22] Obstfeld, M. and K. Rogoff (1996), "Foundations of International Macroeconomics," MIT Press.

[23] Schmitt-Grohé, S. (1997), "Comparing Four Models of Aggregate Fluctuations due to Self-Fulfilling Expectations," *Journal of Economic Theory, 72, 96-147.*

[24] Stockman, A.C. and L.L. Tesar *(1995)*, "Tastes and Technology in a Two Country Model of the Business Cycle: Explaining International Comovements," *American Economic Review, 85, 168 - 185.*

[25] Weder, M. (2001), "On the Plausibility of Sunspot Equilibria," Humboldt University Working Paper.

[26] Wen, Y. (1998), "Capacity Utilization under Increasing Returns to Scale," *Journal of Economic Theory,* 81, 7-36.

[27] Wen, Y. (2002), "Fickle Consumers vs. Random Technology: Explaining Domestic and International Comovements," Cornell University working paper.

[28] Woodford, M. (1991), "Self-Fulfilling Expectations and Fluctuations in Aggregate Demand," in *New Keynesian Economics,* vol. 1, 77 - 112, the MIT Press.

[29] Xiao, W. (2002), "Aggregate Demand, Indeterminacy, and International Business Cycles," unpublished manuscripts.

Table 1. Cross-Country Correlations in the Data and in the Model

	Output	Consumption	Investment	Employment
Data	0.66	0.51	0.53	0.33
Model	-0.21	0.88	-0.94	-0.78

Note: table contents are from Backus, Kehoe, and Kydland (1993). "Data" refers to the data of US and an European aggregate, "model" refers to their benchmark IRBC model.

Table 2. Calibrated Parameter Values

A. Parameters used in the benchmark IRBC economy

ρ	δ	a	σ	θ_1	θ_2	α	β	Transition Matrix of Productivity
0.99	0.025	0.3	1.1	0	0	0.3	0.7	Persistence: 0.966 Spill-over: 0.03 International Correlation: 0.11

B. Parameters used in the benchmark sunspot economy

ρ	δ	a	σ	θ_1	θ_2	α	β	Transition Matrix of Productivity
0.99	0.025	0.3	1.1	0.13	0.186	0.34	0.83	Persistence: 0.96 Spill-over: 0.034 International Correlation: 0.1

Note: the standard deviations of sunspot and productivity shocks are model-specific. In each version of the model they are calibrated so that the volatility of simulated output matches the volatility of US output. They are reported in tables 3 and 5 for each simulation.

Table 3. Predictions of the Saddle-path IRBC Model

	Data	Benchmark	Dynamic Utilization	Increasing Returns	Incomplete Markets
International Correlations					
$corr(C,C^*)$	0.51	0.49	0.53	0.58	0.54
$corr(Y,Y^*)$	0.66	-0.94	-0.92	-0.91	-0.91
$corr(I,I^*)$	0.53	-1	-0.99	-0.99	-0.99
$corr(N,N^*)$	0.33	-0.99	-0.98	-0.98	-0.98
$corr(u,u^*)$	0.11	-	-0.06	-0.79	-0.01
Volatility (% of σ_Y)					
Consumption (C)	0.75	0.16	0.17	0.19	0.17
Investment (I)	3.27	17	16	29	15
Employment (N)	0.61	0.92	0.91	0.91	0.91
Utilization (u)	1.05	-	0.16	0.25	0.16
Productivity (Z)	0.36	0.15	0.12	0.13	0.13
Correlations with Y					
Consumption (C)	0.82	0.58	0.6	0.58	0.6
Investment (I)	0.94	-0.08	0.02	-0.48	0.03
Employment (N)	0.88	0.99	0.98	0.98	0.99
Utilization (u)	0.2	-	0.14	0.4	0.14
Productivity (Z)	0.2	0.54	0.6	-0.07	0.6
Standard dev. of shocks	-	0.0012	0.0007	0.0005	0.0008

Note: "Benchmark" refers to the economy with constant returns to scale and fixed utilization rate. "Dynamic utilization" refers to the economy of dynamic utilization rate and constant returns to scale. "Increasing Returns" refers to the economy with increasing return of 1.1 and dynamic utilization. "Incomplete Market" refers to the economy with a single bond traded internationally.

Table 4. Sensitivity Analyses for the Saddle-path IRBC Economy

Sensitivity Experiments	$corr(C,C^*)$	$corr(Y,Y^*)$	$corr(I,I^*)$	$corr(N,N^*)$
A. Variations in σ				
σ =1.0	1	-0.96	-0.99	-0.98
σ =1.5	0.48	-0.54	-0.99	-0.9
σ =2.0	0.53	-0.26	-0.98	-0.85
σ =3.0	0.46	-0.16	-0.98	-0.82
B. Variations in $corr(e^z, e^{z^*})$				
$corr(e^z, e^{z^*})$ =0.1	0.53	-0.92	-0.99	-0.98
$corr(e^z, e^{z^*})$ =0.2	0.61	-0.89	-0.99	-0.98
$corr(e^z, e^{z^*})$ =0.3	0.65	-0.88	-0.99	-0.98
$corr(e^z, e^{z^*})$ =0.6	0.8	-0.76	-0.99	-0.96
C. Alternative Solow Residuals				
Benchmark Model	0.75	-0.84	-0.99	-0.97
Increasing Returns Model	0.49	-0.91	-0.99	-0.98

Panel A and B: the sensitivity analyses are conducted using the saddle-path economy with constant returns and dynamic utilization. σ is the curvature of the utility function and $corr(e^z, e^{z^*})$ is the correlation between domestic and foreign productivity shocks. In panel A, we set $corr(e^z, e^{z^*})$ at the benchmark value of 0.11, and let σ vary; in panel B, we fix σ at the benchmark of 1.1 and the let $corr(e^z, e^{z^*})$ vary. Panel C: instead of using the specifications in table 2, Solow residuals are re-estimated according to the theoretical assumptions of each model, namely, fixed utilization and increasing returns to scale of 1.1.

Table 5. Predictions of the IRBC Economy with Sunspots

	Data	Complete Markets	Incomplete Markets
International Correlations			
$corr(C,C^*)$	0.51	1	0.15
$corr(Y,Y^*)$	0.66	1	0.35
$corr(I,I^*)$	0.53	1	0.2
$corr(N,N^*)$	0.33	1	0.37
$corr(u,u^*)$	0.11	1	0.44
Volatility (% of σ_Y)			
Consumption (C)	0.75	0.09	0.1
Investment (I)	3.27	4.4	4.6
Employment (N)	0.61	0.99	0.9
Utilization (u)	1.05	0.97	0.73
Correlations with Y			
Consumption (C)	0.82	0.95	0.93
Investment (I)	0.94	0.99	0.97
Employment (N)	0.88	0.99	0.99
Utilization (u)	0.2	0.97	0.95
Standard Dev. of Shocks	-	0.0014	0.0003

Note: this table contains simulation results of the benchmark indeterminacy model driven by sunspot shocks alone. "Complete Markets" refers to the economy with a contingent claims market. "Incomplete Markets" refers to the economy with only a single bond traded internationally.

Table 6. Sensitivity Analyses for the Sunspot Economy with Incomplete Markets

	$corr(C,C^*)$	$corr(Y,Y^*)$	$corr(I,I^*)$	$corr(N,N^*)$
$corr(e^S,e^{S*})$				
0.1	0.23	0.4	0.26	0.42
0.2	0.34	0.51	0.38	0.53
0.3	0.39	0.57	0.47	0.59
0.6	0.65	0.76	0.68	0.77
-0.2	-0.15	0.1	-0.04	0.12
-0.4	-0.35	-0.12	-0.25	-0.1
Returns to Scale				
1.18	0.19	0.41	0.13	0.43
1.2	0.26	0.53	0.04	0.58
1.22	0.35	0.57	-0.05	0.62
1.24	0.3	0.5	-0.2	0.56
1.26	0.38	0.54	-0.2	0.6
1.3	0.38	0.53	-0.24	0.6

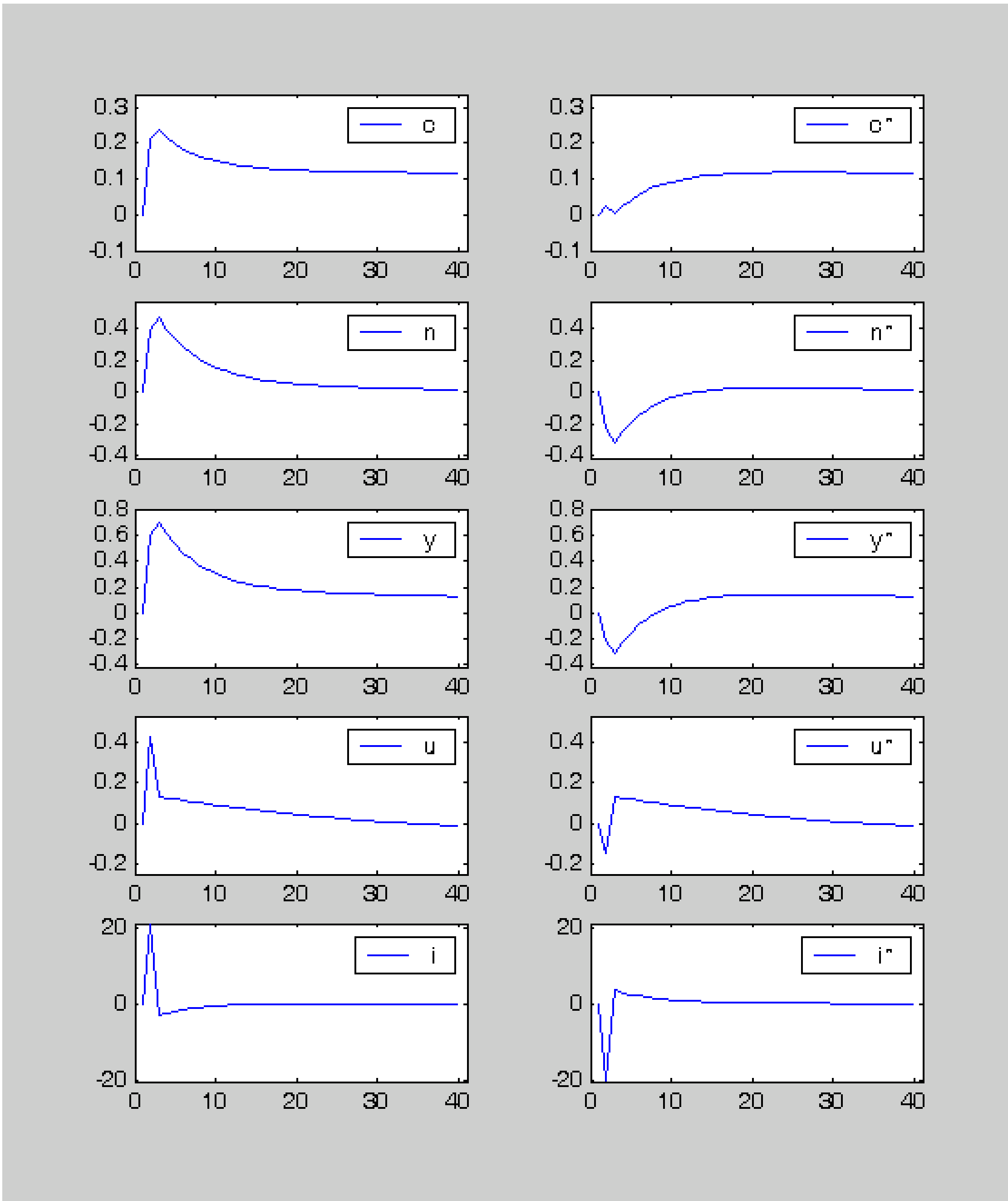
Note: This table contains results of sensitivity analysis for the incomplete-market model driven by sunspot shocks alone. $corr(e^S,e^{S*})$ is the correlation of domestic and foreign sunspot shocks. In the lower panel, we vary returns to scale by letting $\theta_1 = \theta_2$ vary from 0.18 to 0.3 at an increment of 0.02.

Table 7. The Sunspot Economy Driven by Two Shocks

Models	σ_Z / σ_Y	$corr(Z, Y^*)$	$corr(C, C^*)$	$corr(Y, Y^*)$	$corr(I, I^*)$	$corr(N, N^*)$
Complete Markets						
$\sigma_Z /(\sigma_Z + \sigma_S) = 0.5$	0.02	-0.12	0.78	0.68	0.6	0.67
$\sigma_Z /(\sigma_Z + \sigma_S) = 0.8$	0.03	-0.13	0.68	0.5	0.4	0.5
$\sigma_Z /(\sigma_Z + \sigma_S) = 0.2$	0.01	-0.03	0.96	0.94	0.92	0.94
$corr(\sigma^Z, \sigma^S) = 0.8$	0.02	-0.12	0.86	0.8	0.74	0.79
$corr(\sigma^Z, \sigma^S) = -0.8$	0.04	-0.07	0.68	0.47	0.33	0.46
Incomplete Markets						
$\sigma_Z /(\sigma_Z + \sigma_S) = 0.5$	0.09	0.04	0.29	0.14	0.36	0.15
$\sigma_Z /(\sigma_Z + \sigma_S) = 0.8$	0.1	0.07	0.18	0.08	0.48	0.1
$\sigma_Z /(\sigma_Z + \sigma_S) = 0.2$	0.05	0.02	0.15	0.23	0.2	0.26
$corr(\sigma^Z, \sigma^S) = 0.8$	0.09	0.07	0.35	0.23	0.41	0.24
$corr(\sigma^Z, \sigma^S) = -0.8$	0.12	0.2	0.02	-0.2	0.23	-0.18

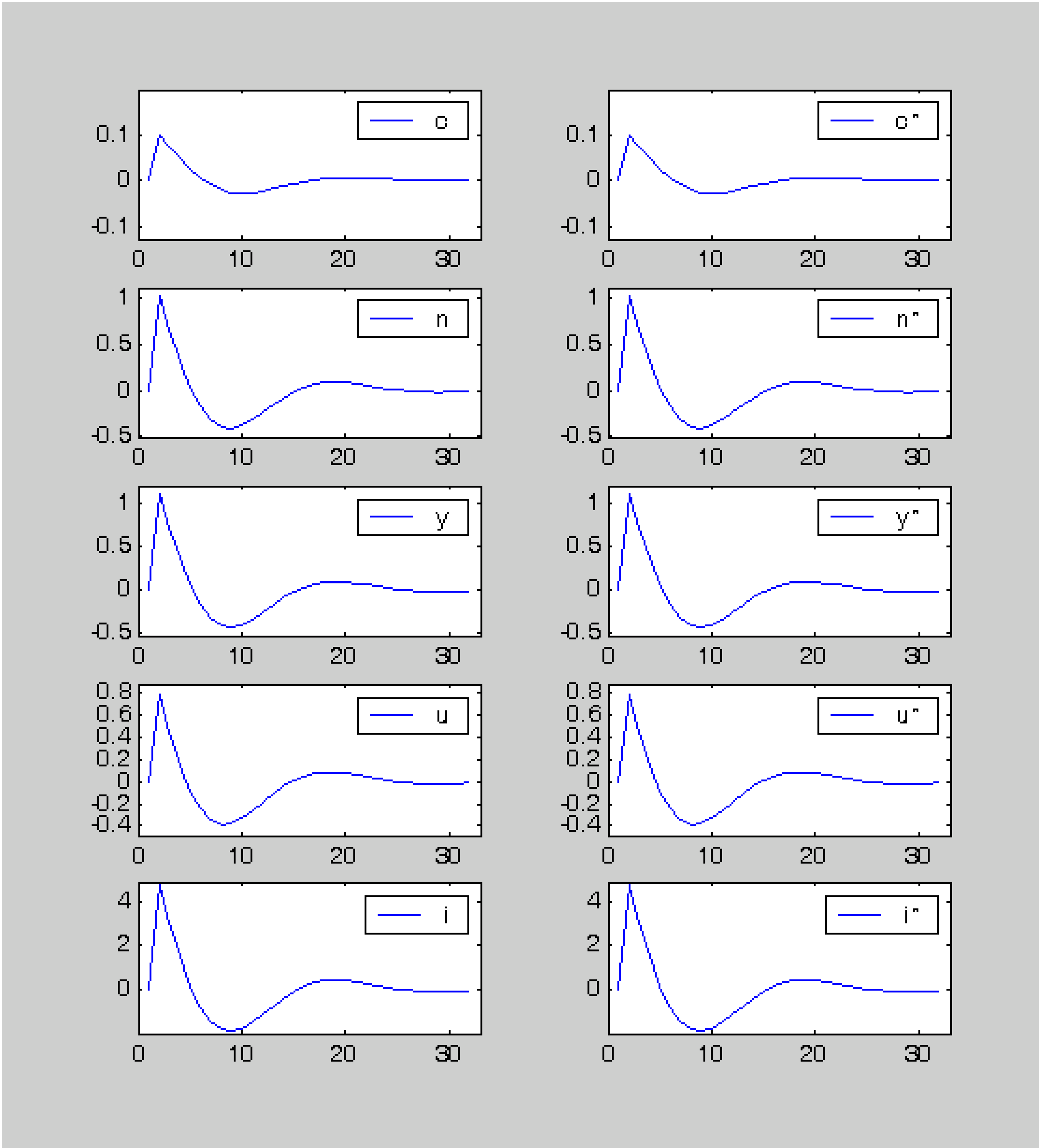
Note: This table contains simulation results of the indeterminacy model driven by both sunspot shocks and productivity shocks. σ_Z represents the standard deviation of productivity shocks; σ_S represents standard deviation of sunspot shocks; and σ_Y represents standard deviation of output.

Figure 1. Dynamic Response in the Standard IRBC Economy



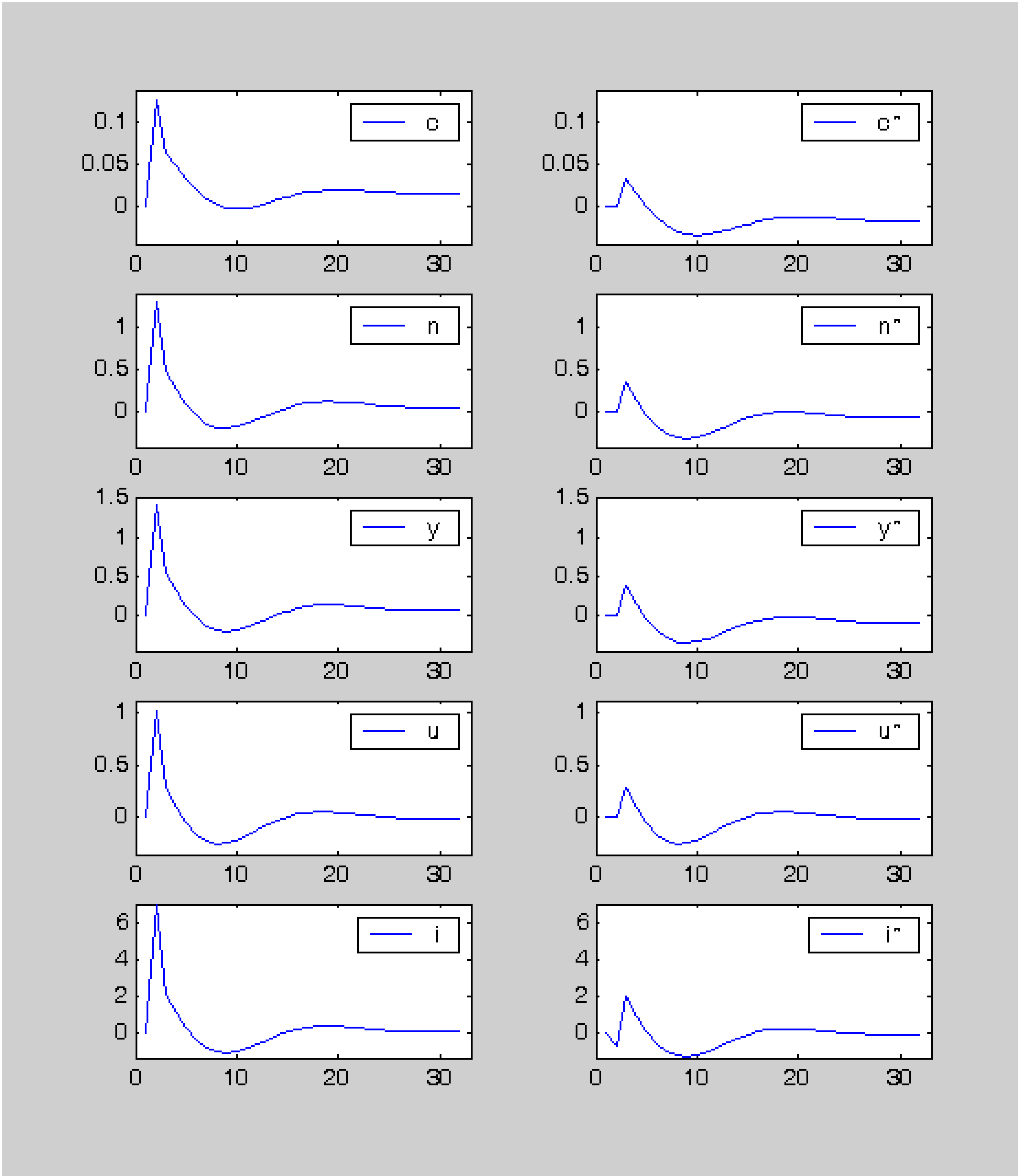
Note: the figure plots the impulse response of the economy to a unit of productivity shock. The left column lists variables for the home country, and the right column for the foreign country. The variables are (from top to bottom): consumption, labor, output, utilization rate, and investment.

Figure 2. Dynamic Response of the Sunspot Economy with Complete Markets



Note: the figure plots the impulse response of the economy to a unit of sunspot shock. The left column lists variables for the home country, and the right column for the foreign country. The variables are (from top to bottom): consumption, labor, output, utilization rate, and investment.

Figure 3. Dynamic Response of the Sunspot Economy with Incomplete Markets



Note: the figure plots the impulse response of the economy to a unit of sunspot shock. The left column lists variables for the home country, and the right column for the foreign country. The variables are (from top to bottom): consumption, labor, output, utilization rate, and investment.