Learning About Monetary Policy Rules When the Housing Market Matters

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Abstract

In this paper we study a general equilibrium model with a housing market, and use stability under adaptive learning as a criterion to evaluate monetary policy rules. An important feature of the model is that there exist credit-constrained borrowers who use their housing assets as collateral to finance purchases. We evaluate both conventional Taylor rules and rules that incorporate other targets such as housing prices. We find that the effect of responding to housing prices, in addition to output and inflation, depends critically on the assumed information structure of the economy.

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1 Introduction

In the literature of monetary policy design, "learnability" has become an important criterion to evaluate interest rate policies. A rational expectations equilibrium (REE) is learnable (or expectationally stable (E-stable) in the sense of Evans and Honkapohja, 2001) if it can be eventually reached when agents who do not initially possess knowledge of the true law of motion of the economy use adaptive learning techniques such as least squares to acquire such knowledge. If a policy rule is properly designed, it should facilitate the convergence of the economy to a learnable REE. Bullard and Mitra (2002) and Evans and Honkapohja (2003 and 2006) provide some landmark results on the learnability of REEs under instrumental and optimal rules. Since the publication of their works, there has been a burgeoning literature that contributes to this topic.

The workhorse for this research is the new Keynesian model, i.e., dynamic stochastic general equilibrium models with imperfect competition and staggered price setting. The model provides a compact and tractable framework for E-stability analysis. However, what is notably missing from the model is the asset market. Typically, neither the financial equity market nor real asset markets such as the housing market is considered in this model. While this omission is harmless when analyzing most targeted issues, it does render the model useless in addressing certain important policy questions. One such question that we are particularly interested in is: how should the central bank respond to housing market volatility? The question has long been a challenging one for academics and policymakers. Recent development in the economy has inevitably sparked a new round of intensive debate. Newer evidence suggests that housing wealth redistribution has a significant effect on consumer expenditure and the business cycle, which was largely neglected in earlier research (Leamer, 2007). This effect is enhanced when housing assets can be used as collateral in a well-functioning credit market (Muellbauer, 2008). To our knowledge, no work has yet been done in the adaptive learning literature to incorporate these new findings and analyze proper policy responses to housing market volatility. This is the goal of this paper. We construct an extended version of the new Keynesian model in which the housing market is specifically characterized, and study the E-stability of REEs under different Taylor-type interest rate rules. As in similar studies, an interest rate policy is deemed plausible if it is conducive to a learnable REE. We also examine the related concept of determinacy of an REE – a determinate REE is locally unique and free from self-fulfilling fluctuations. We consider not only conventional policy rules that target inflation and output, but also more innovative rules that target the housing prices and the amount of total outstanding credit. We hope to shed light on the question of whether central banks should directly target the housing market, in addition to output gaps and inflation.

We need a model that not only has a housing market, but also has the proper channel via which asset market fluctuations can have real effects on the economy. To this end we take advantage of recent development in dynamic general equilibrium models and modify and extend a housing model studied by Iacoviello (2005). Our model has two consumers, a patient lender and an impatient borrower, who trade housing assets. The credit-constrained borrower must use her housing assets as collateral to borrow from the lender. A housing boom raises the borrower's net worth, and enables her to expand purchases, which leads to an increase in aggregate demand. Consequently, a change in the interest rate policy can stabilize the economy via a "credit channel" – higher interest rate lowers the borrower's net worth, reduces her ability to borrow, and decreases aggregate demand. This channel does not exist in a standard new Keynesian model. The critical component of the model that triggers this channel is the collateral constraint.

Equipped with a working model, we put different interest rate rules to a test. We begin with a conventional Taylor rule that targets inflation and output. Our task is to investigate whether or not the established results in the literature are overturned when the underlined economic environment undergo major changes that we describe above. Precedents suggest that this is not unlikely. Carlstrom and Fuerst (2005), for example, show that the determinacy property of REEs can be very sensitive to the introduction of a new state variable – in their case, physical capital. After obtaining some results with the Taylor rule, we add housing price to the policy rule and examine what changes it can bring to the model's E-stability properties. This analysis helps us understand the merits or demerits of housing price targeting, from a learning perspective. Finally, we consider a third policy rule that includes total outstanding credit in its target set. What motivates us to consider this rule is the model's transmission mechanism. The rationale is that if the credit channel is critical for the relationship between asset prices and the real economy, then targeting total credits could become a viable approach to stabilize the economy.

Our results help us draw three interesting conclusions. First, with conventional Taylor rules, the benchmark result of Bullard and Mitra (2002) still holds, i.e., Taylor's principle of a more-than-proportional rise in the central bank's target interest rate in response to higher inflation is still a necessary condition for the existence of a learnable and determinate REE. We believe this result is instructive, in that it highlights the fact that some established conclusions are indeed robust to model environment changes. Second, the effect of responding to housing prices, in addition to output and inflation, depends critically on the assumed information structure of the economy. If agents and the central bank do not possess current data of inflation and output and must forecast them, but do observe current housing prices, then responding to housing prices is *stabilizing,* as it makes it more likely for REEs to be E-stable and determinate given empirically plausible policy parameters. If current housing price data is not available, then the conclusion is reversed – responding to (forecasted) housing prices makes it less likely that an REE is learnable and determinate. Finally, if current data on all three variables are available, responding to housing prices is redundant. The result suggests that in the debate of whether monetary policy should target housing prices, one potentially critical factor is the availability and quality of housing market data itself. Reliable housing market data improves equilibrium stability by offering an information gain to policymakers, while unreliable data does the opposite. Our third conclusion is that responding to total outstanding credit imposes further restrictions on policymakers' reaction to inflation and output gaps, and makes a learnable REE more difficult to obtain.

2 Related Literature

Our paper can be considered as an extension to the adaptive learning literature that studies the relationship between monetary policies and the E-stability of REEs. We provide a short review of the closely related papers below. See Bullard (2006) and Evans and Honkapohja (2008) for a comprehensive review of recent research in this area.

Bullard and Mitra (2002) analyze determinacy and E-stability of REEs under Taylor-type instrumental rules in a standard new Keynesian model. They find that when the central banks possess current data of output gaps and inflation, A determinate REE is always E-stable and an E-stable REE is also determinate, as long as the Taylor principle is adopted. If forward-looking or lagged data rules are used, the situation is more complicated. The Taylor principle is no longer sufficient for determinacy and E-stability. Instead, careful choices of policy reaction parameters are needed.

Evans and Honkapohja (2003, 2006) also analyze E-stability of REEs in a standard new Keynesian model. Their emphasis is on optimal monetary policy rules. They show that with a fundamental-based rule, the REE is invariably unstable under learning. If the central bank uses forecasted output and inflation data to formulate its policy, on the other hand, the resulting REE is always learnable.

A number of extensions have been considered in the literature after the publication of the above landmark results. Models with physical capital are examined by Kurozumi and Van Zandweghe (2008) and Duffy and Xiao (2009). Learnability of sunspot equilibria are studied by Honkapohja and Mitra (2004) and Evans and McGough (2005). When different agents have heterogeneous expectations, the E-stability result may change. This issue is considered by Honkapohja and Mitra (2006) and Branch and Evans (2006). Open economy models have also been studied, for example, in Bullard and Schaling (2006). The general message from these papers is that when learnability is adopted as a criterion to measure the success of monetary policies, no simple rule-of-thumb suffices to guarantee the stability of REEs. The Taylor principle, for example, is usually necessary but not sufficient to ensure E-stability.

To our knowledge, our paper is the first one that studies the learnability of REEs in a DSGE model with a housing market. As we explained in the previous section, a critical monetary transmission mechanism in this model is the credit channel. In the learning literature, the importance of the credit channel has been noticed by some researchers. Assenza and Berardi (2009) study the farmer-gatherer model of Kiyotaki and Moore (1997), and find that while the standard learning analysis often yields stability results, adding the assumption of heterogeneous expectations easily gives rise to E-unstable REEs. In Kiyotaki and Moore (1997), the credit channel arises due to the existence of a collateral constraint – a feature that our model shares.

Our paper can also be viewed as a contribution to the dialogue on how central banks ought to respond to asset market volatilities. In this respect we offer a unique angle to address the issue – the learnability of REEs. The model we use, a DSGE model with collateral constraints, has its root in this literature: Bernanke and Gertler (1989) and Williamson (1987) provide the earlier works that consider financial intermediation and agency costs as propagation mechanisms for aggregate shocks in DSGE models. Kiyotaki and Moore (1997) make important progress by bringing credit constrained agents and collateralized debts into DSGE models. Bernanke et. al. (1999) embed a financial accelerator in a sticky price environment, and make monetary policy analysis possible. Recent works, such as those by Iacoviello (2005) and Monacelli (2009), specifically incorporate a housing sector into general equilibrium models with collateral constraint. Our model is a modified version of Iacoviello (2005).

Earlier discussions of policy related issues often fall under the larger context of "asset market and monetary policy." Most research suggests that it is not necessary for infation-targeting and "Taylor rule" regimes to respond to asset prices, on the ground that asset price movements and inflation tend to move in the same direction, and excess volatilities can be taken care of by the regime (Batini and Nelson, 2000, and Bernanke and Gertler, 2001). Iacoviello (2005) confirms that the same conclusion can be drawn when the housing market is introduced into a modified version of the model by Bernanke and Gertler (2001). More recent research has emphasized that the housing sector and housing prices have important real effect on the economy, and policies need to take it into consideration. Leamer (2007), for example, shows that housing weakness is a critical part of U.S. recessions, and residential investment is usually the first to recover after a typical recession. He proposes that monetary policies should prevent housing price fluctuations from causing large wealth distributions and potential bubbles. Muellbauer (2008) analyzes multi-country data, and argues that credit liberalization contributes to the size of the wealth effect caused by housing price appreciations. In countries with efficient credit markets, rising housing prices have a large positive effect on consumer expenditure as the value of collateral increase. Taylor (2007) argues that the nominal interest rate responded too weakly to inflation and housing prices from 2002 to 2005, and has contributed to the housing boom and bust cycle.

The rest of the paper is organized as follows. Section 3 lays out the model framework and derives the equilibrium conditions. Section 4 analyzes conditions for learnable REEs. Section 5 presents the numeric results. Section 6 concludes.

3 The model

Since a new Keynesian model with housing is unconventional, we present some model details below before turning to learning analysis. The model is based on Iacoviello (2005), which is in turn related to earlier works of Bernanke et. al. (1999) and Kiyotaki and Moore (1997). Unlike in Iacoviello (2005), we assume housing assets do not enter the production function, and we do not model any entrepreneurs or physical capital investment.

Consider a discrete time, infinite horizon economy where a patient household (lender), an impatient household (borrower), a wholesaler firm, and some retailers reside. The borrower is less patient than the lender as she discounts the future more heavily. Both households consume, work and demand a housing asset. The borrower uses her housing assets as collateral to borrow from the lender, and her capacity to borrow is limited by the expected future value of her discounted asset holdings. The wholesaler hires labor from both households to produce a homogeneous intermediate good. There are a large number of monopolistically competitive retailers who buy the intermediate good and differentiate it into consumption goods, and sell to the households. As in Bernanke et. al. (1999), the retailers are Calvo-type price setters that are the source of sticky prices.

3.1 Patient household/lender

The lender maximizes a lifetime utility function given by:

$$E_0 \sum_{t=0}^{\infty} \beta_1^t (\ln C_{1t} + \ln h_{1t} - \frac{L_{1t}^{\eta}}{\eta}),$$

subject to the constraint

$$C_{1t} + q_t h_{1t} + \frac{R_{t-1} b_{1t-1}}{\pi_t} = b_{1t} + q_t h_{1t-1} + w_{1t} L_{1t} + F_t, \tag{1}$$

where E_0 is the expectation operator, β_1 is the discount factor, C_{1t} is consumption, h_{1t} is her holding of housing asset, and L_{1t} is hours worked. $b_t = \frac{B_t}{P_t}$ represents real holdings of one period loan, R_{t-1} is the nominal interest rate, $q_t = \frac{Q_t}{P_t}$ is real housing price, $w_{1t} = \frac{W_{1t}}{P_t}$ is real wage, $\pi_t = \frac{P_t}{P_{t-1}}$ is inflation rate, and F_t represents real profits received from the retailers. P_t is the general price level at time t. Note that capital letters represent the nominal counterparts of the defined real variables (except for C, L and F), and the subscript "1" is used to tag all variables of the patient household. The subscript "2" will shortly be used to denote variables of the impatient household.

Note that by putting total housing stock into the utility function, we implicitly assume that housing services are proportional to the housing stock.

This is a fairly standard household problem that can be solved to yield the following first order conditions

$$
\begin{aligned}
\frac{1}{C_{1t}} w_{1t} &= L_{1t}^{\eta-1}, & (2)\\
\frac{q_t}{C_{1t}} &= \frac{1}{h_{1t}} + \beta_1 E_t \frac{q_{t+1}}{C_{1t+1}}, & (3)\\
\frac{1}{C_{1t}} &= \beta_1 E_t \frac{R_t}{\pi_{t+1} C_{1t+1}}. & (4)
\end{aligned}
$$

3.2 Impatient household/borrower

The impatient household's problem is similar to that of the patient household, with two differences. First, the impatient household does not own any retailers and does not receive profits, and second, her borrowing capacity is constrained by the discounted future value of the collateral - her housing assets. The problem thus is

$$
\max E_0 \sum_{t=0}^{\infty} \beta_2^t (\ln C_{2t} + \ln h_{2t} - \frac{L_{2t}^{\eta}}{\eta}),
$$

subject to

$$
C_{2t} + q_t h_{2t} + \frac{R_{t-1} b_{2t-1}}{\pi_t} = b_{2t} + q_t h_{2t-1} + w_t L_{2t}, \tag{5}
$$

$$
b_{2t} \leq m_2 E_t (\frac{q_{t+1} h_{2t} \pi_{t+1}}{R_t}). \tag{6}
$$

A requirement that $\beta_2 < \beta_1$ ensures that this household is more impatient than the lender and will need to borrow from her. The amount that the debtor can borrow, in nominal terms, is bounded by $m_2 E_t(\frac{Q_{t+1} h_{2t}}{R_t})$, where $0 < m_2 < 1$. In other words, a fraction $1 - m_2$ of the housing value cannot be used as collateral. One can broadly think of $1 - m_2$ as the down-payment rate, or think of m_2

as the loan-to-value ratio.

Solving this problem yields the following conditions:

$$\begin{aligned} \frac{1}{C_{2t}} w_{2t} &= L_{2t}^{\eta-1}, & (7) \\ \frac{q_t}{C_{2t}} &= \frac{1}{h_{2t}} + \beta_2 E_t \frac{q_{t+1}}{C_{2t+1}} + \lambda_t E_t m_2 \pi_{t+1} q_{t+1}, & (8) \\ \frac{1}{C_{2t}} &= \beta_2 E_t \frac{R_t}{\pi_{t+1} C_{2t+1}} + \lambda_t R_t, & (9) \end{aligned}$$

where λ_t is the Lagrangian multiplier associated with the borrowing constraint.

3.3 Intermediate goods firm

The intermediate goods (wholesaler) firm hires labor from both households as inputs to produce a homogeneous good Y_t:

$$Y_t = A_t L_{1t}^{\alpha} L_{2t}^{1-\alpha}, \tag{10}$$

where $0 < \alpha < 1$, and A_t is a temporary shock to productivity.

After the intermediate goods are produced, retailers purchase them at the wholesale price P_t^w, and transform them into final goods and sell them at the price P_t. We denote the markup of final over intermediate goods as $X_t = \frac{P_t}{P_t^w}$.

The producer maximizes her profit

$$Y_t / X_t - w_{1t} L_{1t} - w_{2t} L_{2t}. \tag{11}$$

subject to (10).

3.4 Retailers

There are a continuum of retailers indexed by i. Retailer i buys the intermediate good in a competitive market, differentiates it at no cost into $Y_t(i)$, and sells it at $P_t(i)$. Total final goods are aggregated from each individual final good as

$$Y_t^f = [\int_0^1 Y_t(i)^{\frac{\varepsilon-1}{\varepsilon}} di]^{\frac{\varepsilon}{\varepsilon-1}}, \tag{12}$$

where $\varepsilon > 1$. Each retailer's demand curve is

$$Y_t(i) = [\frac{P_t(i)}{P_t}]^{-\varepsilon} Y_t^f. \tag{13}$$

The price index of final goods is

$$P_t = [\int_0^1 P_t(i)^{1-\varepsilon} di]^{\frac{1}{1-\varepsilon}}. \tag{14}$$

We assume Calvo-type pricing for retailers. Each retailer can only change the price with probability $1-\theta$, where $0 < \theta < 1$. The optimal pricing decision is

$$\max_{P_t^o} \sum_{k=0}^{\infty} \theta^k E_t \{\beta_1 \frac{C_{1t}}{C_{1t+k}} [\frac{P_t^o - P_{t+k}^w}{P_{t+k}} Y_{t+k}(i)]\}$$

subject to (13). P_t^o represents the optimal price chosen by the retailer to maximize the objective. The retailers use the lender's discount factor because they are owned by her. Differentiating with respect to P_t^o implies that the optimal set price must satisfy:

$$\sum_{k=0}^{\infty} \theta^k E_t \{\beta_1^k \frac{C_{1t}}{C_{1t+k}} [\frac{P_t^o(i)}{P_{t+k}} - \frac{X}{X_{t+k}}) Y_{t+k}(i)]\} = 0. \tag{15}$$

Given that the fraction θ of retailers do not change their price in period t, the aggregate price evolves according to

$$P_t = [\theta P_{t-1}^{1-\varepsilon} + (1-\theta) P_t^{o1-\varepsilon}]^{\frac{1}{1-\varepsilon}}. \tag{16}$$

These two conditions can be combined to create the new Phillips curve in the linearized version of the model.

3.5 Equilibrium

The equilibrium of the model is a sequence of prices $\{q_t, R_t, P_t, X_{t,}w_{1t}, w_{2t},\}$, and an allocation $\{h_{1t}, h_{2t}, L_{1t}, L_{2t}, Y_t, C_{1t}, C_{2t}, b_{1t}, b_{2t},\}$, such that all first order conditions and constraints hold, and all markets clear.

Goods market clear when[1]

$$C_{1t} + C_{2t} = Y_t. \tag{17}$$

It is straightforward to show that the retailer profit is equal to

$$F_t = \frac{X_t - 1}{X_t} Y_t. \tag{18}$$

The loans market equilibrium is

$$b_{1t} + b_{2t} = 0. \tag{19}$$

As in Iacoviello (2005), housing assets are assumed to have a fixed total supply H, which leads

[1] As in Iacoviello (2005), total output can be approximated by $Y_t^f = \int_0^1 Y_t(i)di \approx Y_t$.

to the trivial market clearing condition

$$h_{1t} + h_{2t} = H. \tag{20}$$

3.6 Interest rate rules

We assume there is a central bank that implements a Taylor-type interest rate rule that targets output gap, inflation, and possibly housing prices and outstanding credit. The interest rate rule is best presented in log-linear form, where all lower case variables represent percentage deviations from the steady state equilibrium.

We consider three classes of interest rate rules. In the first class, we assume agents and policymakers can observe current data for all variables. The rule is therefore described by the equation

$$i_t = \tau_y y_t + \tau_\pi \pi_t + \tau_q q_t + \tau_b b_{2t}, \tag{21}$$

where the τ's are the reaction parameters of the central bank. Note that b_{2t} is the amount of loans borrowed, and q_t is housing price. In our subsequent analysis, we will start by assuming $\tau_q = 0$ and $\tau_b = 0$, which will reduce the rule to the conventional Taylor rule. After examining the characteristics of the REE under learning, we will then relax the two restrictions to see what changes they might bring to the conclusion.

In reality, current data for output and inflation are rarely available in real time, and policymakers often resort to lagged or forecasted data for policy decisions – a point emphasized by McCallum (1999). We therefore consider forward-looking rules of the form

$$i_t = \tau_y E_t^* y_{t+1} + \tau_\pi E_t^* \pi_{t+1} + \tau_q E_t^* q_{t+1} + \tau_b E_t^* b_{2t+1}, \tag{22}$$

where the "E_t^*" represents expectations that are not necessarily rational. Policymakers must use lagged data to make forecasts, and then use forecasted data as inputs in the interest rate rule.

In reality, while aggregate data is difficult to obtain in real time, asset price data is usually not subject to the same limitation. They are updated more frequently and timely, and are much more likely to be available to policymakers at decision time. This motivates us to consider the following hybrid rule:

$$i_t = \tau_y E_t^* y_{t+1} + \tau_\pi E_t^* \pi_{t+1} + \tau_q q_t, \tag{23}$$

where the central bank uses forecasted output and inflation but current levels of housing prices to formulate its policy. Our purpose is to understand whether current housing data can provide the monetary authority with any *information gain*, so that it can better guide the economy towards a stable equilibrium.

3.7 Transmission mechanism

The model has a shock propagation mechanism that does not exist in a standard new Keynesian model. Consider a rise in the housing price. Higher housing price increases the net worth of the borrower, and expands her capacity to borrow. The borrower would borrow more, consume more, and acquire even more housing assets. This raises aggregate demand and leads to an economic boom. This "accelerator" mechanism ensures that housing prices have real effects on the economy. It also provides a rationale for policymakers to pay close attention to asset prices when formulating policy rules.

Since this mechanism is now fairly well-known, we do not elaborate on it here. We refer readers to Kiyotaki and Moore (1997) for the theoretical foundation, and Iacoviello (2005) for its application in DSGE models with a housing market.

4 Methodology and Calibration

4.1 Methodology

Our methodology for learning analysis follows the standard approach of Evans and Honkapohja (2001). We sketch the general methodology below.

We take log-linear approximation of the equilibrium conditions around the steady state, and reduce the economy to a system of linear dynamic equations. The linearized system consists of seven expectational dynamic equations. We can write the system as

$$x_t = \beta E_t^* x_{t+1} + \delta x_{t-1} + \kappa a_t, \tag{24}$$

where $x_t = (y_t, \pi_t, X_t, q_t, b_{2t}, h_{2t}, i_t)'$ and a_t is a vector of technology shocks that follows

$$a_t = \rho a_{t-1} + e_t.$$

e_t is white noise, and all roots of ρ lie inside the unit circle.

For adaptive learning analysis, assume the perceived law of motion (PLM) is

$$x_t = a + b y_{t-1} + c a_t. \tag{25}$$

Given the PLM, agents form their forecasts as

$$E_t^* x_{t+1} = (I+b)a + b^2 y_{t-1} + (bc + c\rho)a_t.$$

Inserting this into (24), we obtain the actual law of motion and T-mapping from the PLM to the ALM as

$$T(a,b,c) = [\beta(I+b)a, \beta b^2 + \delta, \beta bc + \beta c\rho].$$

As Evans and Honkapohja (2001), the critical differential equation that determines the E-stability conditions of the system is

$$d\xi/dt = T(\xi) - \xi,$$

where $\xi = (a,b,c)'$. The Jacobian matrix of the equation must have eigenvalues that all have negative real parts for the REE to be E-stable.

For determinacy analysis, the system is slightly modified and written as

$$E_t s_{t+1} = J s_t + \kappa a_t,$$

where s_t represents a vector of control and state variables of the model. The REE is determinate if the number of stable eigenvalues of J is equal to the number of predetermined variables of the system.

4.2 Calibration

Ideally, we would like to provide analytic results concerning the learnability of REE under various interest rate rules. Unfortunately, except in a few special cases analytic results are not possible. In our case the model has produced a seven-dimensional system, which necessitates that we adopt a numerical approach.

We calibrate our parameters to conform with standard values used in the literature or with empirically estimated values in recent research. The discount factor is set at 0.99 for the lender and 0.98 for the more impatient borrower, as in Iacoviello (2005). The elasticity of substitution across final goods, ε, is set a 4, a value commonly used in the literature. The inverse of the elasticity of labor supply, η, is set to 1.01, as in Iacoviello (2005), which makes the labor supply curve virtually flat. The fraction of firms that keep their prices unchanged, θ, is given a value of 0.75, which corresponds to an average price duration of about one year. The share of the lender's labor in the production, α, is set at 0.5. The borrower's downpayment rate, $1 - m_2$, is given a benchmark value of 0.1.

5 Learnability of REE under various interest rate rules

5.1 Conventional Taylor Rules

We start by examining the learnability of REEs under the conventional Taylor rule

$$i_t = \tau_y y_t + \tau_\pi \pi_t.$$

Our numeric approach is as follows. In all simulation exercises, we vary the policy weights τ_π,τ_y in the interest rate rule. The ranges allowed for these weights cover all empirically relevant cases; in particular we search over a fine grid of values between 0 and 4 for both τ_y and τ_π. We use an increment stepsize of .02. For each possible pair of weights (τ_π, τ_y) in this grid, we check whether the eigenvalues satisfy the conditions for 1) determinacy and 2) E-stability. If both conditions are satisfied, we indicate this in the figures below using a blue color. We use white color to denote weight pairs that lead to indeterminate and E-unstable equilibria.

Determinacy and E-stability findings using the current data rule are shown in Figure 1. Values of the monetary policy rule weight τ_π are indicated on the horizontal axis and values of the monetary policy rule weight τ_y are indicated on the vertical axis in this and all subsequent figures. Two conclusions can be drawn from the figure. One, with the current data rule, a determinate equilibrium is always learnable and a learnable REE is always determinate. An indeterminate REE, on the other hand, is always E-unstable, and an E-unstable REE is always indeterminate. Two, in order to ensure a learnable and determinate REE, the central bank must respond strongly to inflation. This usually implies that the reaction parameter to inflation, τ_π, be bigger than 1. Or, in borderline cases, when τ_π is slightly less than 1, a stronger response to output is required. These two results are nearly identical to Bullard and Mitra (2002)'s finding using a two-equation standard new Keynesian model. In particular, they show that conclusion two can be summarized as a long-run "Taylor principle": the long-run response of nominal interest rates to inflation must be more than one-for-one. Our result confirms that the Taylor principle prevail even when the inclusion of housing markets causes major changes in the model environment .

Conclusion one above is ideal for policymakers, because adhering to the Taylor rule can easily guarantee an E-stable and unique equilibrium, and the central bank does not have to worry about self-fulfilling fluctuations since all indeterminate REEs cannot be reached under learning. Unfortunately, the current data rule is often criticized as being unrealistic, since policymakers do not have the luxury of current data in real-life policy-making scenarios. A more realistic rule would be for policymakers to set the nominal rate in response to changes in *forecasted data.* We consider the forward-looking rule

$$i_t = \tau_\pi E_t^* \pi_{t+1} + \tau_y E_t^* y_{t+1},$$

where the "E_t^*" represents forecasts based on adaptive learning techniques such as least squares.

Figure 2 reports the result. We have used a green color to denote policy weight pairs that give rise to an indeterminate REE that is learnable. The result shows that sticking to the long-run Taylor-principle no longer guarantees a determinate and learnable REE. Instead, with some policy weight combinations, the economy could converge to an indeterminate equilibrium that suffers from self-fulfilling fluctuations and therefore excess volatility. A determinate REE is not always learnable, and a learnable REE is not always determinate – this result is again similar to Bullard and Mitra (2002)'s benchmark result. The Difference is that the indeterminacy problem is not as acute in our model. In their model, a very moderate response to output (less than 0.5) is required to ensure a learnable and determinate REE, while in ours, the admissible region for output response is much larger.

A general message policymakers can derive from the result is that simply sticking to the Taylor principle is not sufficient to guarantee equilibrium stability; a more carefully calculated response to output and inflation is required. But on the other hand, the Taylor principle is still *a necessary condition* for E-stability. If it is violated, no learnable REEs exist.

5.2 Policy rules that respond to housing prices

Should the central bank respond to housing prices? If it does, will the REE become more or less likely to be learnable? Is the Taylor principle still necessary for E-stability and determinacy? With these questions in mind, we postulate a policy rule of the form

$$i_t = \tau_\pi \pi_t + \tau_y y_t + \tau_q q_t,$$

where as a starting point, we again assume all current data are available to the policymakers. We present our result in Figure 3. Since this time we have three policy reaction parameters, we have to check E-stability and determinacy conditions for all possible policy weight *triples*. The upper and lower bounds for τ_y and τ_π remain the same. We let τ_q vary between 0 and 1. When presenting our result, we still choose to plot two-dimensional graphs, since they are easier to read than 3-D ones. We will plot a series of them when necessary to reflect changes brought about by the third policy parameter τ_q.

In the current data case, it turns out that given the same combinations of τ_y and τ_π, variations in τ_q between 0 and 1 do not result in any visible changes in the E-stability and determinacy properties of REEs. This is why we only show a representative plot in Figure 3. The plot is produced with $\tau_q = 0.5$. Note that it looks nearly identical to Figure 1, where $\tau_q = 0$. The implication is that responding to housing prices is neither more stabilizing nor less stabilizing; if the central bank is already responding to inflation and output, and is sticking to the Taylor principle, then responding to housing prices is redundant.

This result is reminiscent of Bernanke and Gertler (2001) and Iacoviello (2005). Both works conclude that if the interest rate rule targets inflation and output, and the Taylor principle is satisfied, then responding to asset prices is redundant. The difference is that their measuring criterion is the volatility of the central bank's loss function, while our criterion is the E-stability of the REE. Also, Bernanke and Gertler (2001) focus on the general asset market instead of the housing market.

This result is of course based on the (unrealistic) assumption that current data is available for all three variables. As we argued above, contemporaneous macro data such as output and inflation are often not readily available. The same limitation, however, need not apply to asset price data, because in reality asset prices are much more frequently updated and collected, and therefore easier to observe. Consequently, having current asset price data may provide the central bank with potential *information gains*. We next investigate if such gains can more easily lead to E-stable and determinate REEs. We consider the rule

$$i_t = \tau_\pi E_t^* \pi_{t+1} + \tau_y E_t^* y_{t+1} + \tau_q q_t,$$

where the underlined assumption is that the central bank cannot observe π_t and y_t, but does observe q_t. It must use lagged values of output and inflation to forecast their future values. In Figure 4, we plot a series of four two-dimensional plots, each with a different policy weight on τ_q. This will highlight the differences that τ_q makes when it varies.

Interestingly, the result in Figure 4 confirms our intuition. While responding to housing prices does not change the fundamental picture, it does lead to some sizable changes in the permissible policy parameters for output. Specifically, as the policy weight on housing prices increases, the region of (τ_y, τ_π) combinations that lead to determinate and learnable REEs gradually enlarges, and the region that leads to learnable and indeterminate REEs gradually moves up and out of the empirically plausible area. This essentially enlarges the "comfort zone" for policymakers, for as they respond more strongly to housing prices, they can also be more liberal in their responses to output gaps and do not have to worry about REEs driven by self-fulfilling expectations. This is certainly a desirable situation for policymakers.

A natural question to ask at this point is what happens if current housing price data is not available or not accurate. Would it remove the case for housing-price targeting? We consider a rule of the form

$$i_t = \tau_y E_t^* y_{t+1} + \tau_\pi E_t^* \pi_{t+1} + \tau_q E_t^* q_{t+1}.$$

The underlined assumption is that no current data is available, and the central bank must use t-1 information to forecast all three, and set the nominal rate accordingly. We present the result in Figure 5. When housing prices must be forecasted in the policy rule, responding to housing prices no longer provides policymakers with more freedom. Instead, it imposes more restrictions on other dimensions. As the policy weight on housing prices increases, the combinations of policy parameters (τ_y, τ_π) that ensures a determinate and Learnable REE shrinks. The blue region in the lower right panel ($\tau_q = 1$) is only half of the one in the upper left panel ($\tau_q = 0$). Essentially, we now face a trade-off in policy reactions: if the response to housing price is strong, the response to output must be reduced to maintain stability.

The result is quite intuitive. For learning agents who rely on available data to estimate the model structure, information is crucial. Current housing price data facilitates their estimation, while removing current data adds another dimension of data uncertainty, and raises the difficulty of their estimation. The accuracy of their estimation directly affects their ability to learn the REE. Therefore, for the REE to be learned, more restrictions must be imposed on policy parameters when there is less data available, and vice versa. This result is consistent with some existing works. In Kurozumi and Van Zandeveghe (2005), for example, a forward-looking Taylor rule that targets current output instead of future output enlarges the policy scope for learnable REEs in a new Keynesian model with physical capital.

In standard stochastic experiments with rational expectations, this kind of conclusions do not emerge, because the true model is already known to agents. In this respect our result highlights the value-added of adaptive learning analysis. The result has non-trivial real-life implications. It suggests that when addressing the question of whether or not central banks should target housing prices, policymakers must not overlook a critical factor: the availability and quality of the housing market data itself.

5.3 Policy rules that target total outstanding credit

In this section, we postulate policy rules that not only target output and inflation, but also total outstanding credit b_{2t}. This experiment is inspired by recent empirical findings that the credit market plays a crucial role in the transmission of housing market shocks. Muellbauer (2008), for example, argues that the effect of housing appreciation on consumer spending has opposite signs in economies with inefficient and efficient credit markets. In economies with inefficient credit markets, high housing prices depress consumption because of higher downpayment requirements. In economies with efficient credit markets, the positive effect from collateralized borrowing and lower downpayment rate more than offset the former, and tend to raise consumption. This credit channel is the most important component of our theoretical model. The goal of formulating this policy rule is to investigate if targeting total amount of credit itself is a good way to stabilize the economy.

We replace q_t with b_{2t} in both the current data and forward-looking rules, so the rules become

$$i_t = \tau_\pi \pi_t + \tau_y y_t + \tau_b b_{2t}$$

and

$$i_t = \tau_y E_t^* y_{t+1} + \tau_\pi E_t^* \pi_{t+1} + \tau_b E_t^* b_{2t+1}.$$

The results are reported in Figures 6 and 7.

Figure 6 shows the results for the current data rule. As the policy weight on credit increases, the determinate and E-stable region gradually shrinks and moves up. Some determinate REEs become E-unstable (red). As τ_b continues to increase, the Taylor rule no longer ensures any learnable REEs. Instead, a strong response to output is required for a determinate REE to be E-stable. On the other hand, the required response to inflation is slightly eased as the permissible region for inflation gradually enlarges.

The result in Figure 7 shows what happens with the forward-looking rule. Responding to total credit results in a larger and larger region of policy parameter combinations that will lead to E-unstable REEs, and the scope for determinate and E-stable REEs shrinks. Essentially, strong responses to credit requires the policy rule to reduce its corresponding response to output and inflation.

In summary, responding to total outstanding credits can still lead to E-stable REEs. However, the stronger the reaction parameter for credit is, the more restrictions it imposes on the reaction parameters for inflation and output. Policy parameters that are usually considered empirically plausible can easily lead to unstable REEs. We conclude that responding to total credit is not a desirable policy, as it creates more uncertainty and imposes more restrictions on policy-marking.

6 Conclusion

In this paper we study a general equilibrium model with a housing market, and use stability under adaptive learning as a criterion to evaluate monetary policy rules. We find that the Taylor principle remains an important necessary condition for learnable and determinate REEs, although it is not a sufficient condition in most cases we studied. The effect of responding to housing prices, in addition to output and inflation, depends critically on the assumed information structure of the economy. If agents and the central bank do not possess current data of inflation and output and must forecast them, but do observe current housing prices, then responding to housing prices is stabilizing. If current housing data is not observable, then responding to housing prices can be destabilizing. If all variables are observed in real time, then responding to housing prices is redundant. We also find that responding to total outstanding credit is generally not a desirable policy as it makes it more likely for policies to generate unstable REEs.

Given the results in this paper, opportunities now exist to extend our understanding of this issue. A major problem of housing price fluctuations is not the volatilities per se, but volatilities that are "unnecessary" as they do not reflect changes in fundamentals. A possible extension is to study a model with learning agents, but distinguish between fundamental and non-fundamental housing market shocks. An interesting question to investigate is whether or not the existence of non-fundamental shocks will change the conclusion of this paper. Another straightforward extension is to study other types of monetary policies, for example, optimal policy rules. We leave these for future research.

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7 Figures

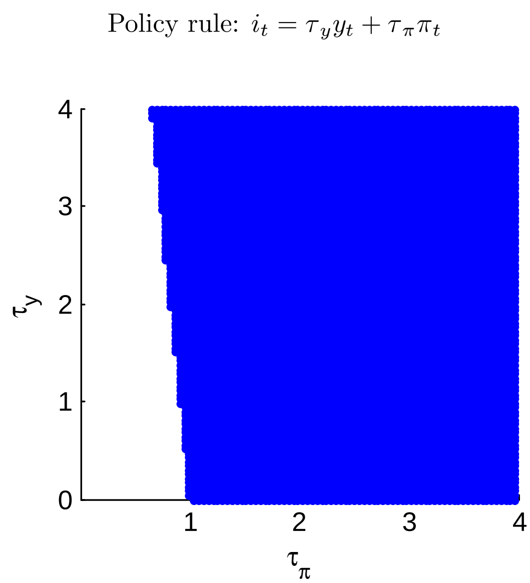

Figure 1: E-stable and determinate results under the current data rule. Blue: determinate and E-stable. White: indeterminate, E-unstable

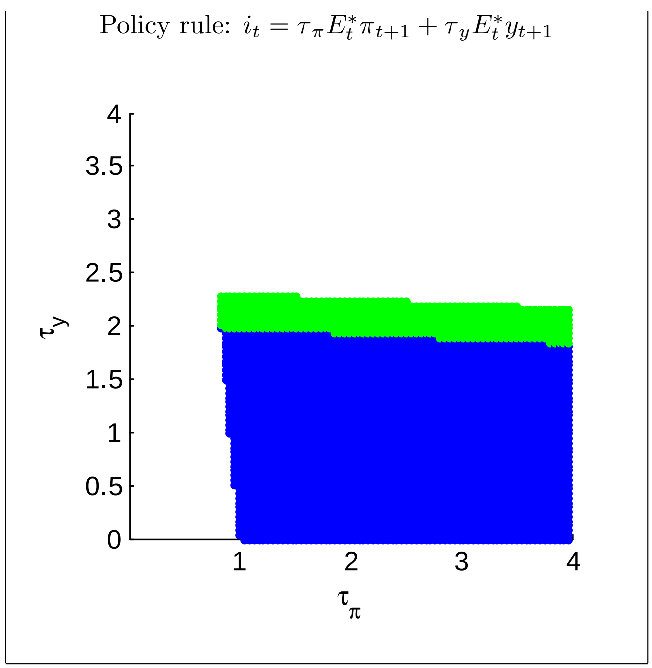

Figure 2: E-stable and determinate results under the forward-looking rule. Blue: determinate and E-stable. Green: indeterminate and E-stable. White: indeterminate, E-unstable

Policy rule: $i_t = \tau_y y_t + \tau_\pi \pi_t + 0.5 q_t$

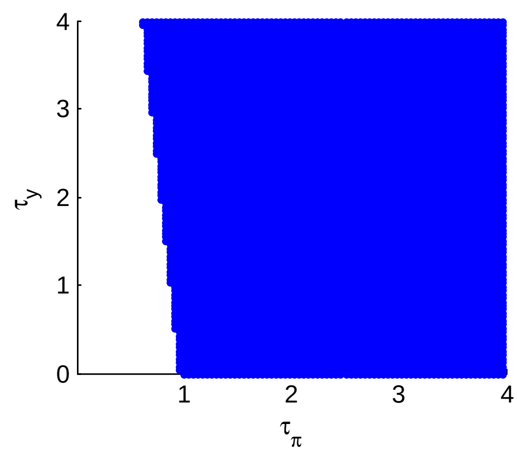

Figure 3: E-stability and determinacy results under the current data rule with housing price targeting. Blue: determinate and E-stable. White: indeterminate, E-unstable.

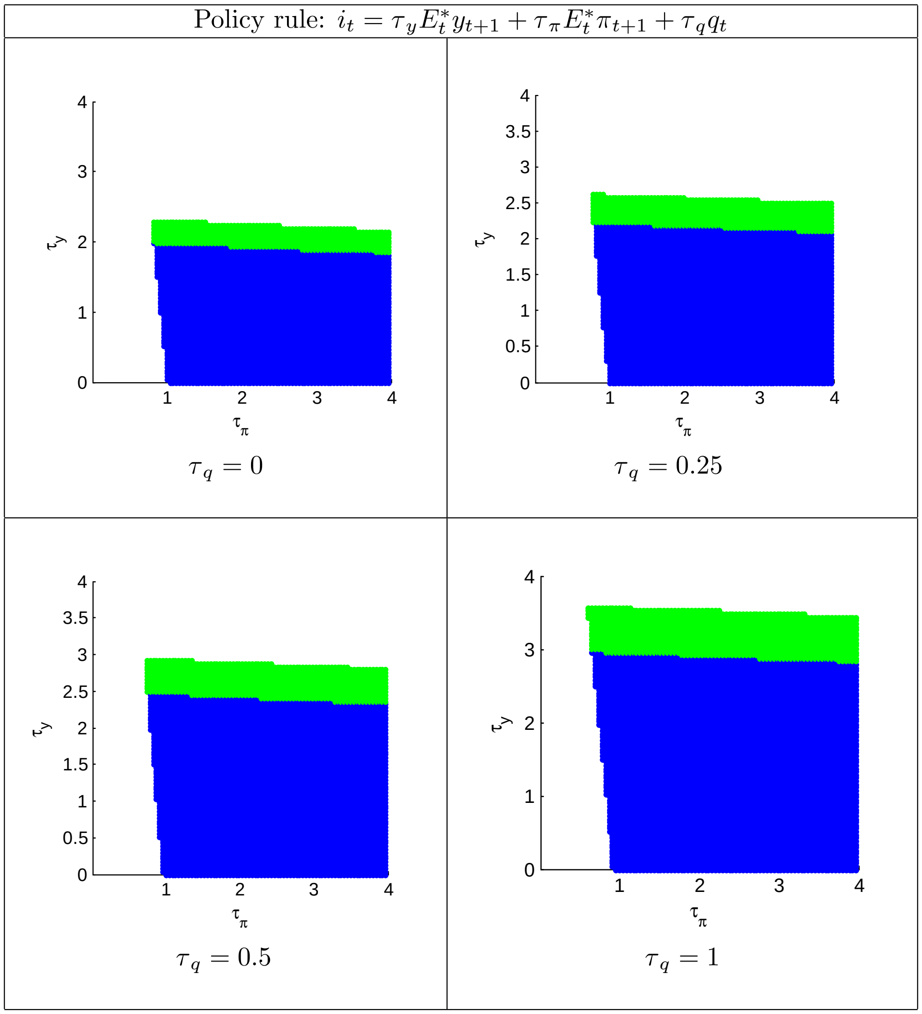

Figure 4: E-stability and Determinacy results under the hybrid rule with housing price targeting. Blue: determinate and E-stable. Green: indeterminate and E-stable. White: indeterminate, E-unstable

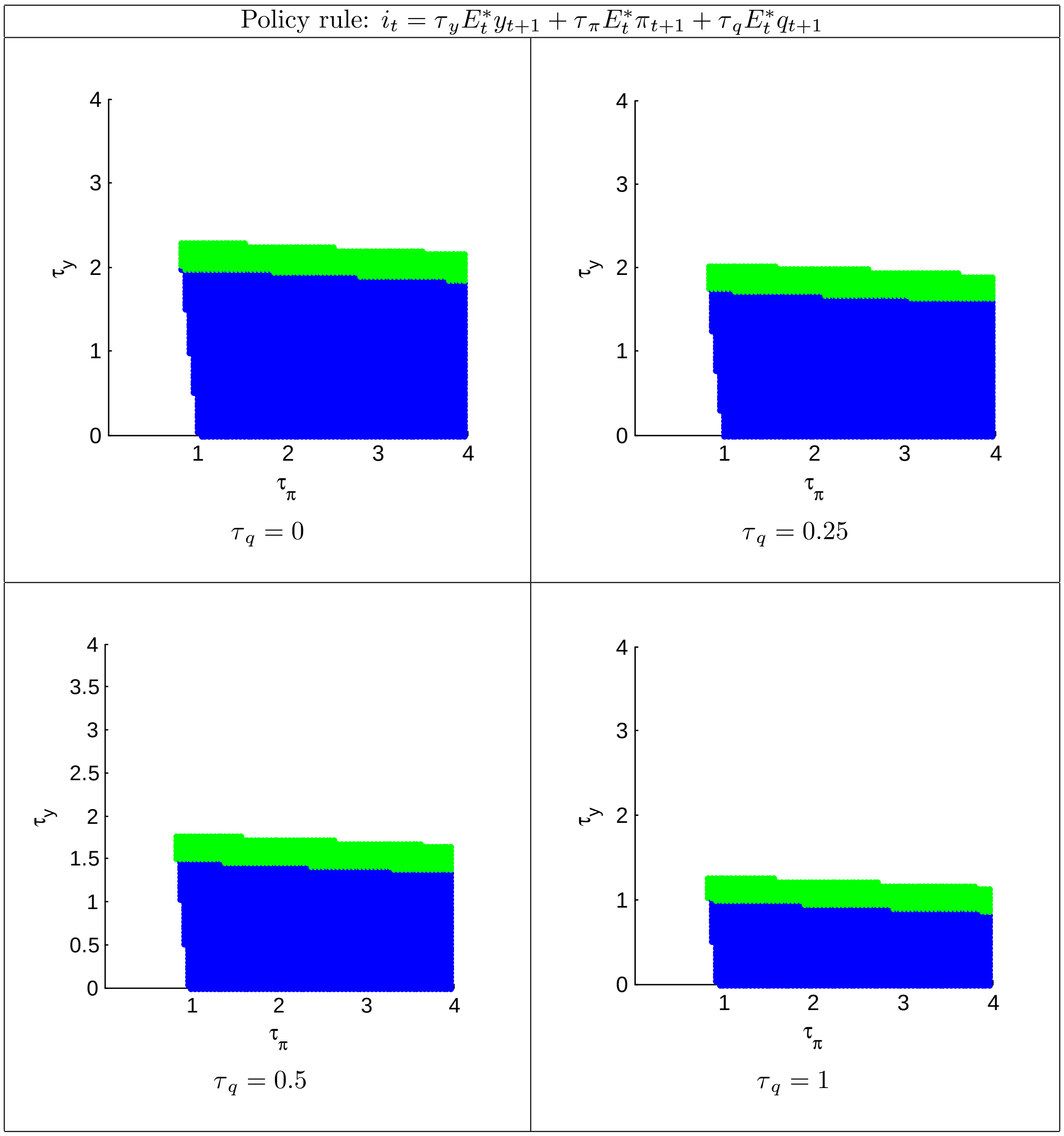

Figure 5: E-stability and Determinacy results under the forward-looking rule with housing price targeting. Blue: determinate and E-stable. Green: indeterminate and E-stable. White: indeterminate, E-unstable

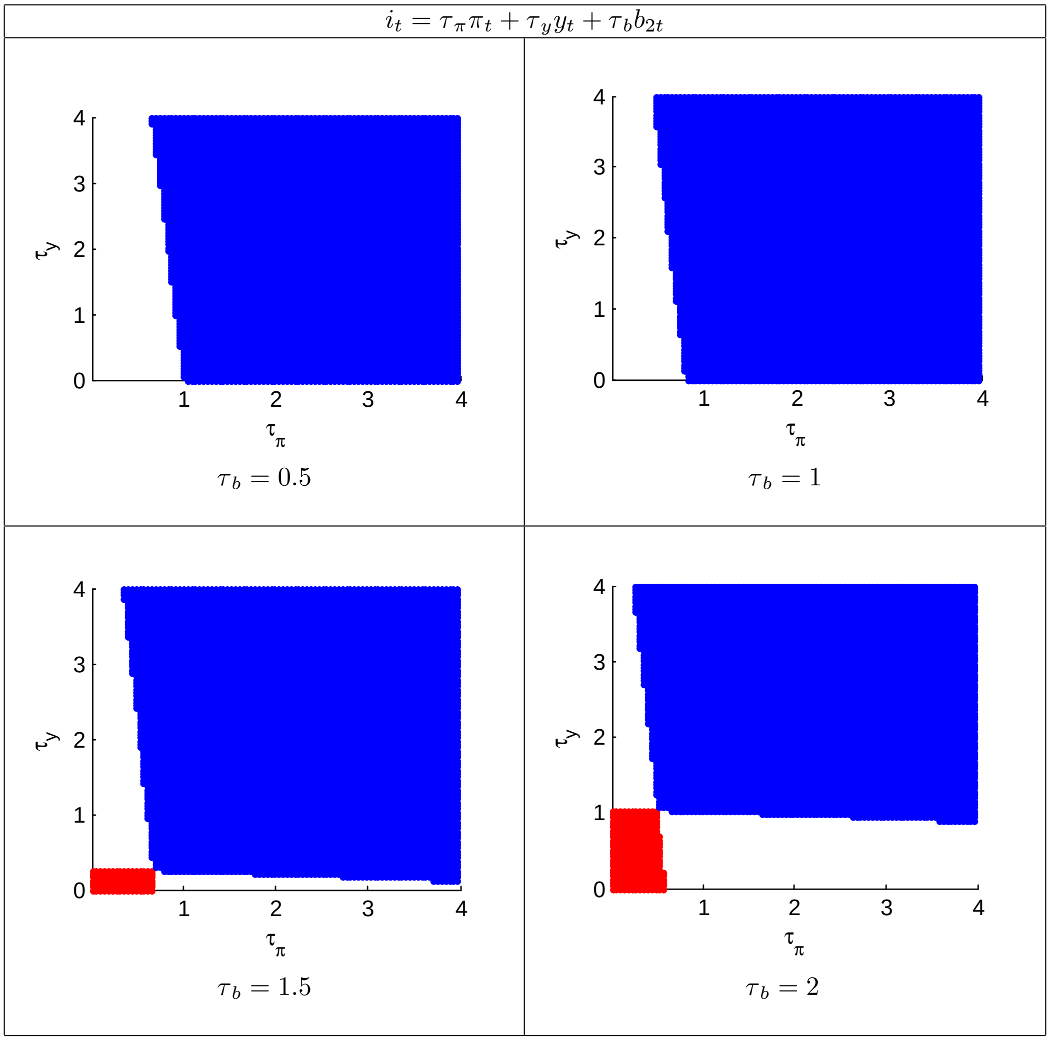

Figure 6: E-stability and Determinacy results under the current data rule with credit targeting. Blue: determinate and E-stable. Red: determinate and E-unstable. White: indeterminate, E-unstable

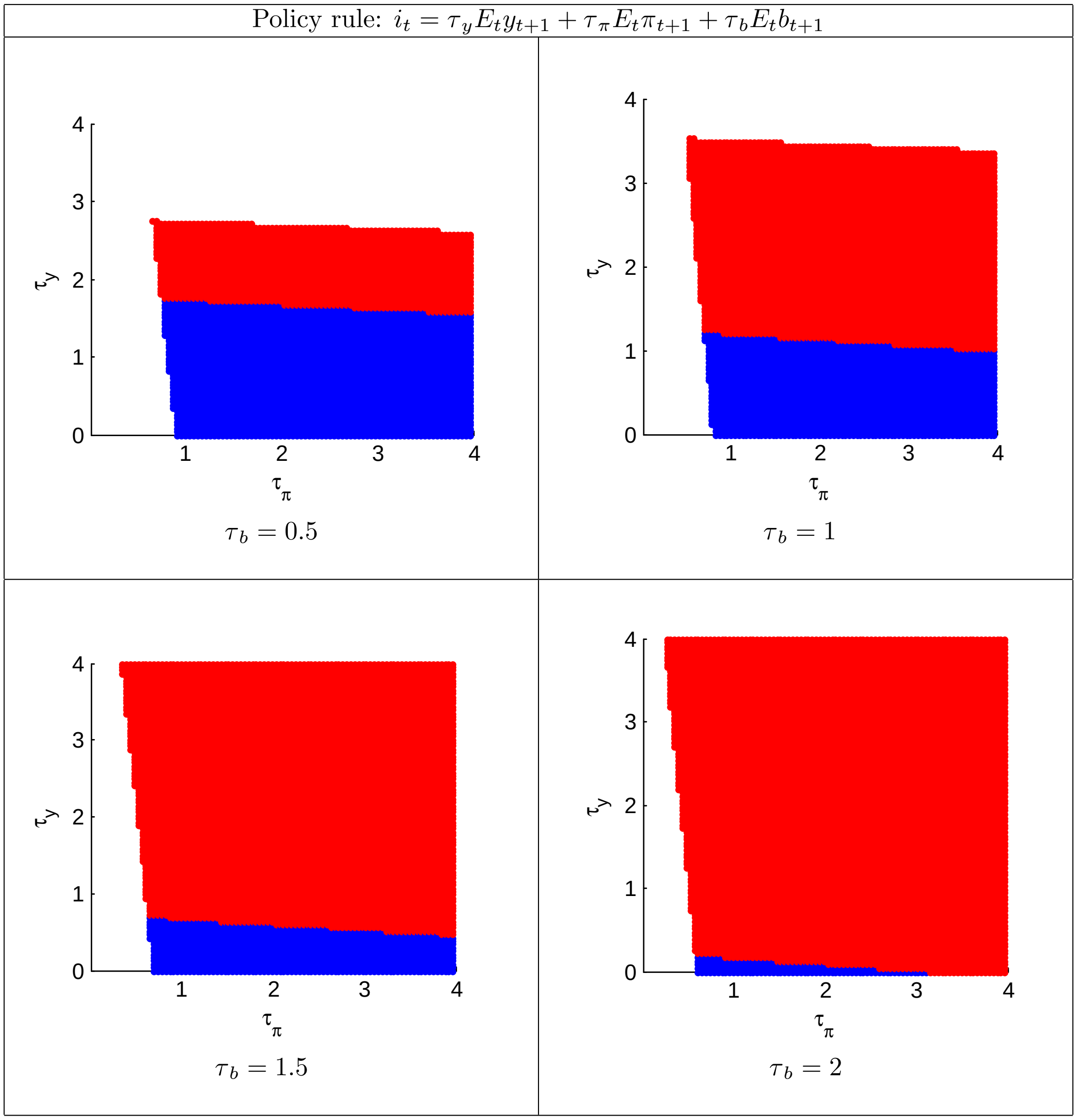

Figure 7: E-stability and Determinacy results under the forward-looking rule with credit targeting.Blue: determinate and E-stable. Red: determinate and E-unstable. White: indeterminate, E-unstable