Physics on bar magnet

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Bar magnets are frequenctly used in our everyday's life. What is a bar magnet? What is the magnetic field distribution around and inside the bar magnet? Such questions are addressed in this note. Electric charges can be isolated as positive and negative ones, whereas magnetic poles cannot be isolated as single monopole. No matter how many times a parmanent magnet is cut, each piece always has a north pole and a south pole, forming the magnetic dipole moment (simply magnetic moment). The magnetic moment of electrons consists of orbital magnetic moment and spin matgnetic moment.

It is well known that a loop current leads to the magnetic dipole moment along the axis of the loop current. The distribution of the magnetic field due to the loop current is essentially the same as that of the electric field due to the electric dipole moment. Without any concept of the single magnetic monopole, the magnetic phenomena can be explained in terms of the magnetic dipole moment arising from the loop current. In general, the magnetic moment due to the current is decribed by $I\ A$, where I is the current and A is the area of the loop.

In this note, we discuss the magnetic field lines around and inside the bar magnet in terms of the magnetic dipole moments (loop currents). The magnetic field lines of the bar magnet are also visualized using the calculations with Mathematica 7.0. This note is written based on many textbooks,[1-9] including standard textbooks on the electricity and magnetism, and textbooks on the introductory physics.

1. Magnetic field lines of bar magnet

A magnetic field $\boldsymbol{B}$ surrounds any materials with permanent magnetism. The magnetic field is a vector field, similar to the electric field $\boldsymbol{E}$. The direction of $\boldsymbol{B}$ at any location is the direction in which the north pole of a compass needle points. Figure 1 shows the pattern of magnetic field lines produced by a bar magnet. The direction of $\boldsymbol{B}$ at any location is monitored by a small comapss needle.

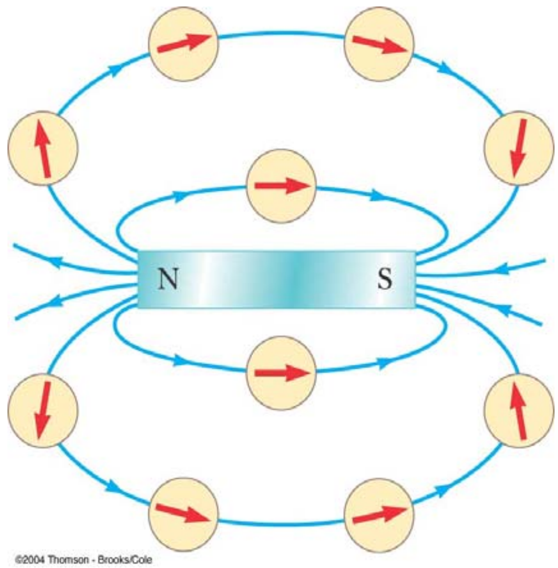


Fig.1 Magnetic field lines of a bar magnet. A small compass can be used to trace the magnetic field lines of a bar magnet.
From Fig.22.1 of Reference 9.

The compass needle (Fig.2) is itself a permanent magnet with north pole and south pole. An ordinary magnetic compass is nothing but a magnet which can rotate around a pivot. The north pole of a bar magnet attracts the south pole of a compass needle and repel the north pole of the compass needle. In response to the Earth's magnetic field, the compass will point toward the geographic North Pole of the Earth because it is in fact a magnetic south pole. The magnetic field lines of the Earth enter the Earth near the geographic North Pole.

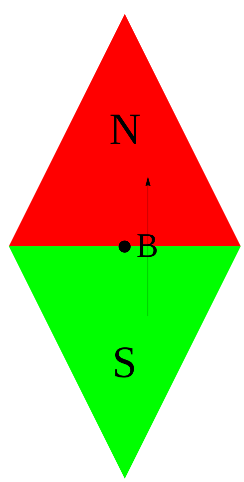


Fig.2 A compass needle is one of bar magnets. It has a north pole and a south pole. The magnetic field ***B*** is directed from the S-pole to the N-pole. It can rotate around a pivot (denoted by a dot) in the cenetr of mass.

The magnetic field lines of a bar magnet can be visualized using small iron filings sprinkled onto a smooth surface. For example, a sheet of glass is placed on top of the bar magnet. When iron filings are droped onto the sheet, they align with the magnetic field in their vicinity, giving a good representation of the overall field produced by the bar magnet (see Fig.3).

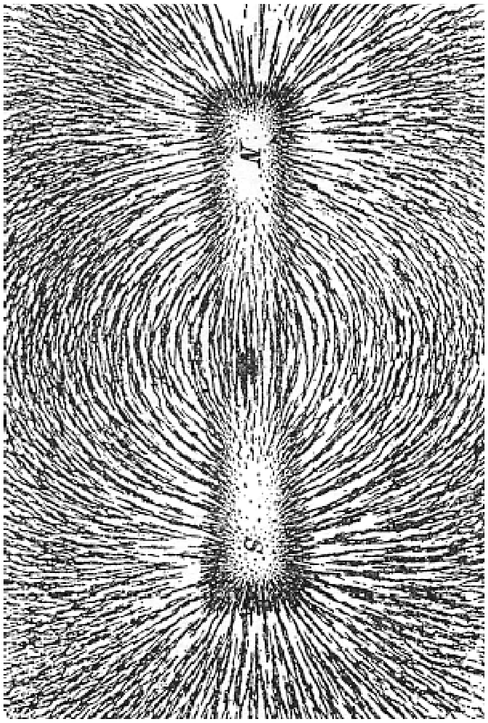


Fig.3 Magnetic field lines of a bar magnet made visible by iron filings sprinkles on a sheet of paper placed over the magnet. The distribution of the filings suggets the pattern of lines of the magnetic field ***B***.
http://upload.wikimedia.org/wikipedia/commons/5/57/Magnet0873.png

Figures 4(a) and (b) shows the lines of ***B*** of a bar magnet in detail. Note that the lines of ***B*** pass through the bar magnet, forming a closed loop. From the clustering of field lines outside the magnet near its end, we infer that the magnetic field has its greatest magnitude there. These ends are called the poles of the magnet, with the designations north and south given to the poles from which the lines respectively emerge and enter. The magnetic field emanates from the N (north) pole of the magnet and enters at the S (south) pole.

In summary, the nature of the magnetic field lines due to the bar magnet is as follows.

(1) The end of the magnet from which the magnetic field lines emerges is the north pole of the magnet, and the end into which the magnetic field lines enter is the south pole.

(2) The magnetic field lines form closed loop. The magnetic field lines do not begin or end anywhere in the way that the electric field lines begin and end on positive and negative

charges. As usual, the magnetic field lines never intersect, or should they begin or end on anything but sources and sinks.

(3) Isolated magnetic poles or magnetic monopoles, do not exist.

(4) Since there is no such “magnetic charge” that act as source or sink of magnetic field lines, the magnetic field lines of any kind of magnetic field must always form closed loop.

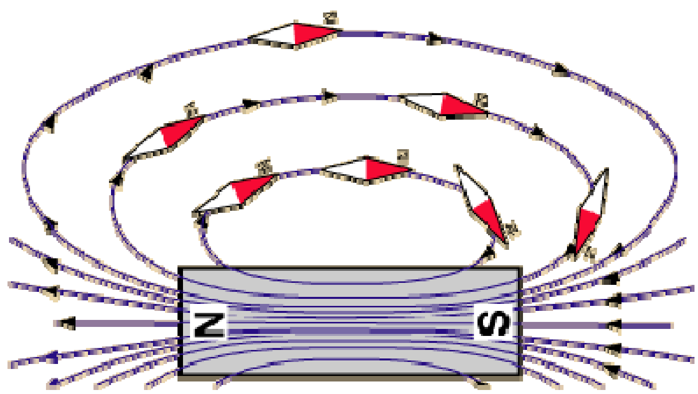


Fig.4(a) Magnetic field lines ***B*** in and around a bar magnet. The magnetic field ***B*** inside the bar magnet is directed from the S pole to the N pole. The direction of B is indicated by the direction of small compass needles. The magnetic field lines form a closed loop.
http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/elemag.html

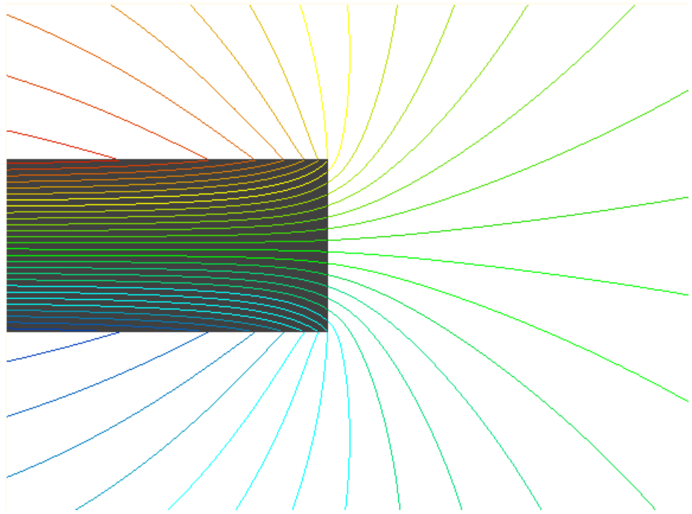


Fig.4(b) The magnetic field lines near and inside a bar magnet.
http://www.coolmagnetman.com/field01.htm

2. Properties of bar magnet

It is impossible to isolate a single monopole, a piece of magnet that is simply and only north pole or south pole. When a bar magnt is broken into half, two new poles appear. Each half has both a north pole and a south pole, just like any other bar magnet. No matter how many times we subdivide the bar magnet, we never obtain an isolated magnetic pole; we always get a dipole (see Fig.5). A single magnetic pole has never been isolated. Magnetic ploes are always found in pairs. This means that the electron itself is the fundamental magnetic dipole moment (or simply magnetic moment). The magnetic moment of electron consists of orbital magnetic moment and spin matgnetic moment.

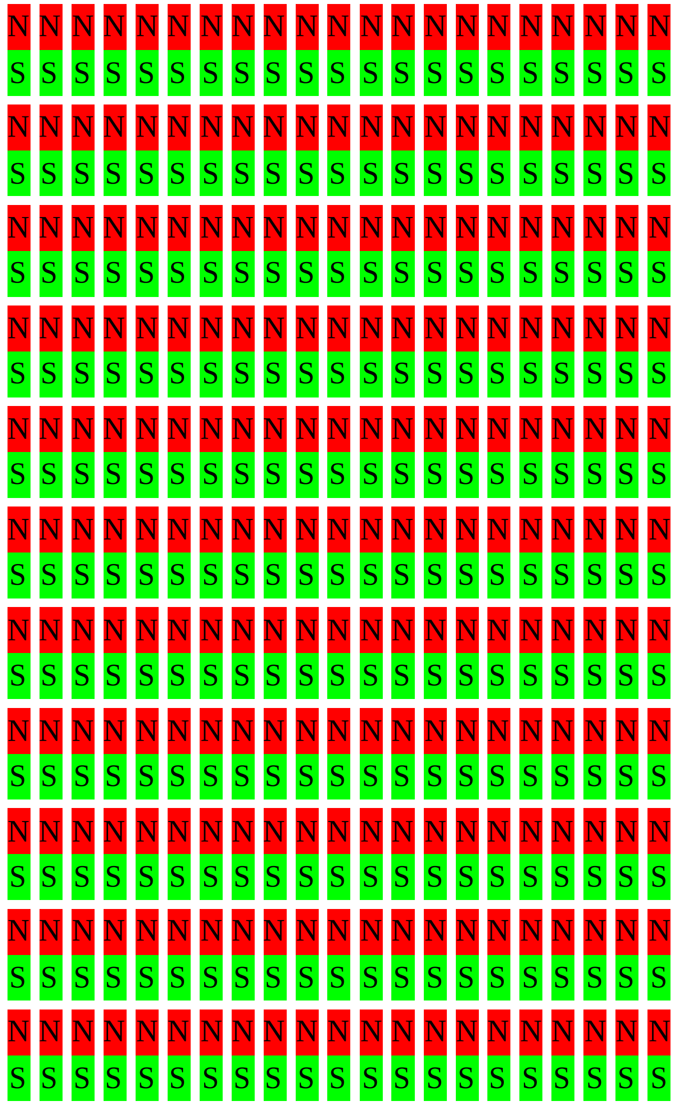


Fig.5 The fragments of a bar magnet always have two poles (north and south poles). A magnet behaves as if it were composed of tiny bipolar units, tiny bar magnets, or magnetic dipoles. If you break a magnet, each fragment becomes a separate magnet, with its own north and south poles.

If the noprth pole of one magnet is brought near the north pole of another magnet, they exert repulsive forces on each other. Two south poles also repel each other, but the north poles of one magnet exert an attractive foprce on the south pole of another magnet (see Fig.6).

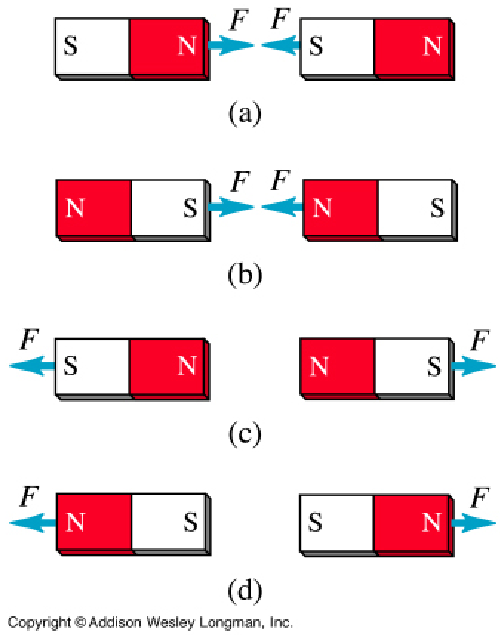


Fig.6 Repulsive interaction between the same types of poles (N poles and N poles, S poles and S poles). Attractive interaction between the different types of poles (N-pole and S-pole).
http://www.physics.sjsu.edu/becker/physics51/mag_field.htm

3. Magnetic field line around a horse-shoe magnet

The horseshoe magnet has north and south poles just like a bar magnet but the magnet is curved so the poles lie in the same plane. The magnetic field lines flow from the N pole to the S pole just like in bar magnet. However, since the poles are located closer together and a more direct path exists for the lines of flux to travel between the poles, the magnetic field is concentrated between the poles (see Figs.7 and 8(a))

If a bar magnet was placed across the end of a horseshoe magnet or if a magnet was formed in the shape of a ring, the magnetic field lines would not even need to enter the air (see Fig.8(b)).

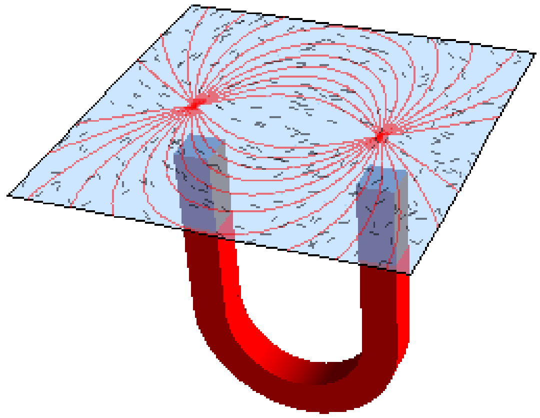


Fig.7 Magnetic field lines around a horse-shoe magnet. Sprinkling iron filings on a sheet of paper and bringing a horse-shoe magnet up close to the underside of the paper, one can see that the filings line up along the lines of force of the magnetic field, thus showing its contours. (Mathematica).
http://demonstrations.wolfram.com/ObservingMagneticFieldsWithIronFilings/

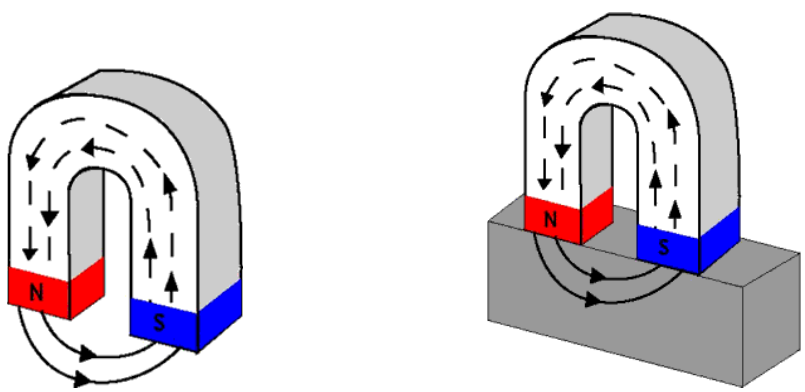


Fig.8 (a) and (b) Magnetic field lines around a hose-shoe magnet with and without a a piece of iron, called a keeper, across its poles. The keeper essentially cancels the bare poles of the horse-shoe magnet via induction.
http://www.ndt-ed.org/EducationResources/CommunityCollege/MagParticle/Physics/MagneticFieldChar.htm

4. Magnetic field of the Earth as a bar magnet

Earth's magnetic field (and the surface magnetic field) is approximately a magnetic dipole, with the magnetic field S pole near the Earth's geographic north pole and the magnetic field N pole near the Earth's geographic south pole (Fig.9). Magnetic fields extend infinitely, though they are less densely grouped further from their source. The Earth's magnetic field, which effectively extends several tens of thousands of kilometers into space, is called the magnetosphere.

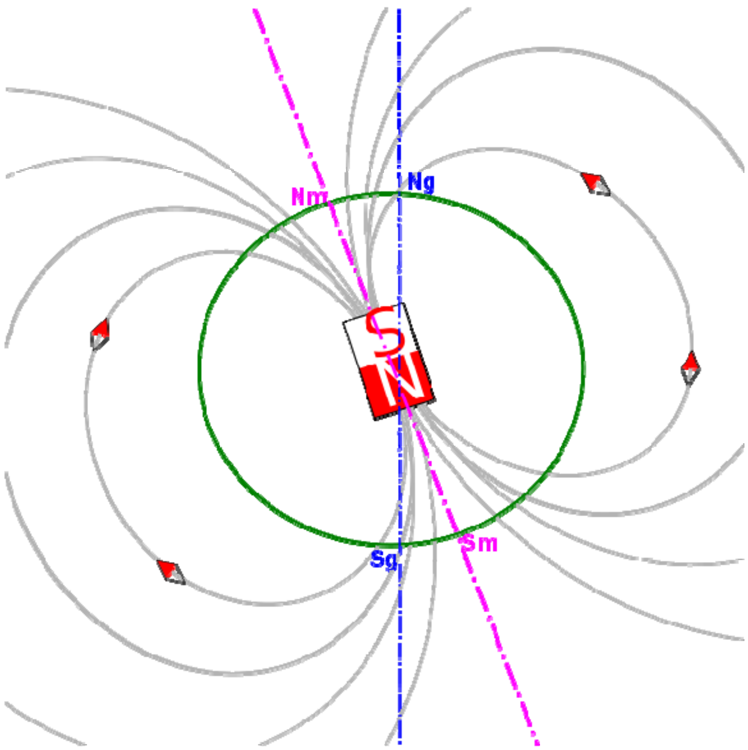


Fig.9 The Earths' magnetic influence resembles that of a tilted bar magnet. A compass needle aligns itself with the field and points roughly toward the noth geographic pole, which is not far from the Earth's south magnetic pole. The field extends thousands of km out into space and is rotationally symmetrical around the axis of the hypothetial bar magnet. The dipole axis makes an angle of 11.5° with Earth's rotational axis.

http://upload.wikimedia.org/wikipedia/commons/2/2b/Geomagnetisme.svg

5. The electric field due to the electric dipole moment

The magnetic force and the electric force are not the same force. The electric field and magnetic field are not the same types of fields. Nevertheless, we find that the magnetic field due to a bar magnet is similar to the electric field produced by an electric dipole. The starting point of electro-statics is the discovery that there are electric charges exerting forces on each other according to the Coulomb's law. In magneto-statics, there is similar situation. The striking difference is the absence of a magnetic charge; that is the magnetic monopole (analogous to the electric charge) does not exist. Magnetic charges occur always only in combination with the opposite magnetic charges, that is in the form of a magnetic dipole. A bar magnet has a magnetic dipole moment and generates a magnetic field ***B***.

First we consider a electric dipole, where the charge q is located at the point A (0, 0, a) and the charge - q is located at the point B (0, 0, -a). Then the electric potential ϕ at the position (x, y, z) can be calculated as

$$\phi = \frac{1}{4\pi\varepsilon_0}\left(\frac{q}{\sqrt{x^2+y^2+(z-a)^2}} + \frac{-q}{\sqrt{x^2+y^2+(z+a)^2}}\right) . \qquad (2)$$

The electric field ***E*** is calculated as

$$\mathbf{E} = -\nabla\phi , \qquad (3)$$

In Fig.10 we show the distribution of the electric field lines, where the positive charge and the negative charge are located at the point A (0, 0, a) and the point B (0, 0, -a), respectively. Note that all the electric field lines emerge from the point A (the positive charge) and enter into the point B (the negative charge).

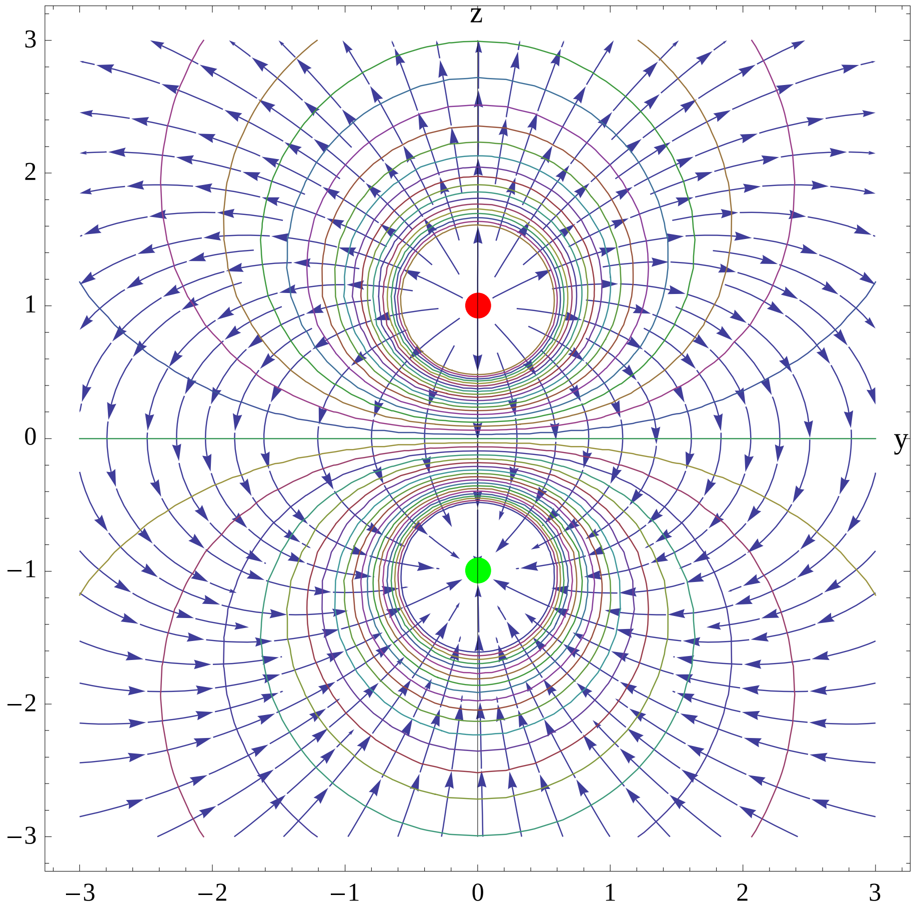


Fig.10 Electric field lines and equipotential lines in the y z plane ($x = 0$) where electric potential ϕ is given by Eq.(2) and the electric field $\boldsymbol{E}$ is given by Eq.(3). This figure is drawn by using the Mathematica 7.0 (StreamPlot). Note that the direction of $\boldsymbol{E}$ along the line connecting between the positive charge q and the negative charge ($-q$) is from the positive charge to the negative charge. This figure is drawn by using the Mathematica 7.0 (StreamPlot). The electric dipole moment $\boldsymbol{p}$ ($p = 2\ aq$) is directed from the negative charge ($-q$) at (0, 0, a) to the positive charge (q) at (0, 0, $-a$).

In the limit as the separation distance ($2a$) goes to zero while q becomes infinite, in such a way that $p = q(2a)$ remains constant, all terms in ϕ except the term linear in a vanish. In this limit an electric dipole moment is formed. This dipole has no net charge, no extent in space, and is completely characterized by its dipole moment, which is the limit of $q(2a)$ as a goes to zero. We use the symbol $\boldsymbol{p}$ to represent the vector form the electric dipole moment, and write

$$\mathbf{p} = q(2a)\hat{z}\ ,$$

where $\hat{z}$ is the unit vector along the *z* axis. Then the electric potential of the electric dipole moment ***p*** is obtained as

$$\phi_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{\mathbf{p}\cdot\mathbf{r}}{r^3}$$

where $p = 2aq$ and ***p*** is the vector directed along the positive *z* axis. The electric field produced by the electric dipole moment is given by

$$\mathbf{E}_{dipole} = -\nabla\phi_{dipole} = \frac{p}{4\pi\varepsilon_0}(\frac{3zx}{r^5},\frac{3yz}{r^5},\frac{2z^2-x^2-y^2}{r^5})$$

The on-axis electric field of an electric dipole when $|z|>>2a$ is

$$\mathbf{E}_{dipole} = \frac{1}{4\pi\varepsilon_0}\frac{2\mathbf{p}}{z^3}\,. \qquad (4)$$

Figure 11 shows the electric field lines due to the electric dipole moment.

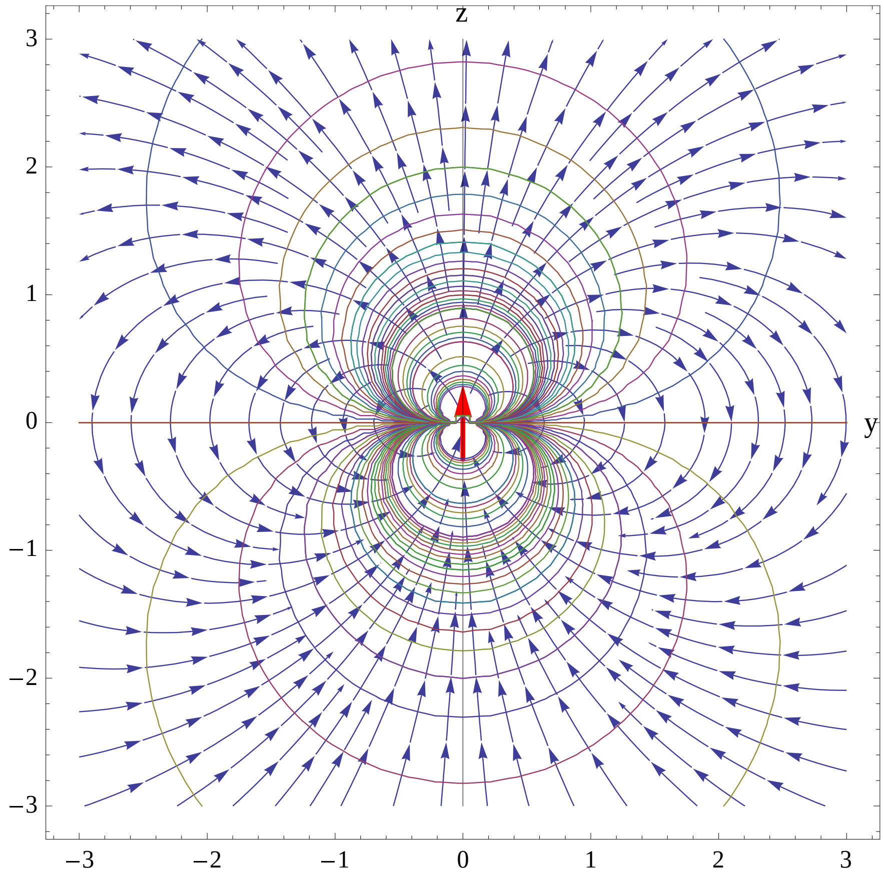


Fig.11 Electric field lines and equipotential lines for the electric dipole moment along the z axis (denoted by a red arrow) in the *y-z* plane ($x = 0$). This figure is drawn by using the Mathematica 7.0 (StreamPlot).

6. Magnetic field lines around a current loop

We consider the magnetic field lines around a loop current. The magnetic field ***B*** at the point (x, y, z) can be calculated using the vector potential ***A***. The magnetic field ***B*** is calculated as $\mathbf{B} = \nabla \times \mathbf{A}$. The results for the expression of ***B*** is presented below.

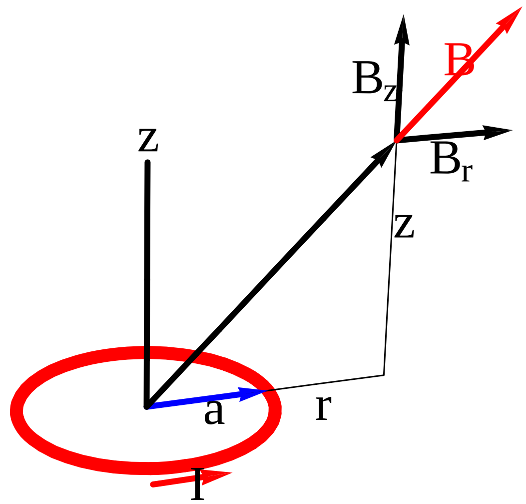


Fig.12 Magnetic field ***B*** due to a current loop (radius a). the current *I* flows along the ring. The magnetic field *B* has a $\boldsymbol{B}_r$ component and a $\boldsymbol{B}_z$ component. Note that Bz and Br can be expressed in terms of the elliptic integrals.

B_z: magnetic field component which is aligned along the *z* axis (the coil axis).
B_r: magnetic field component in a radial direction.
I: current in the wire
a: radius of the current loop
z distance, on axis, from the center of the current loop to the field measurement point.
r radial distance from the axis of the current loop to the field measurement point.

$$B_z(r,z) = \frac{B_0}{\pi\sqrt{Q}}[(\frac{1-\alpha^2-\beta^2}{Q-4\alpha})EllipticE[k] + EllipticK[k]]$$

$$B_r(r,z) = \frac{B_0\gamma}{\pi\sqrt{Q}}[(\frac{1+\alpha^2+\beta^2}{Q-4\alpha})EllipticE[k] - EllipticK[k]] \qquad (5)$$

where

$$\alpha = \frac{r}{a}, \qquad \beta = \frac{z}{a}, \qquad \gamma = \frac{z}{r},$$

$$Q = (1+\alpha)^2 + \beta^2,$$

$$k = \frac{4\alpha}{Q}\ .$$

Note that the definition of the Elliptic integral in the *Mathematica* is a little different from the standard definition.

$$EllipticE[k] = \int_0^{\pi/2} (1 - k\sin^2\theta)^{1/2} d\theta \quad . \tag{6}$$

$$EllipticK[k] = \int_0^{\pi/2} (1 - k\sin^2\theta)^{-1/2} d\theta \tag{7}$$

EllipticE[k]: the complete elliptic integral of the first kind.
EllipticK[k]: the complete elliptic integral of the second kind

where

$$B_0 = \frac{\mu_0 I}{2a}\ .$$

In the origin, we have

$B_r = 0$, and $B_z = B_0$.

which agrees with the result derived from the Bio Savart law. In Figs.13(a) and 13(b), we show the distribution of the magnetic field due to the current loop with radius *a*. The axis of the current loop is along the *z* axis.

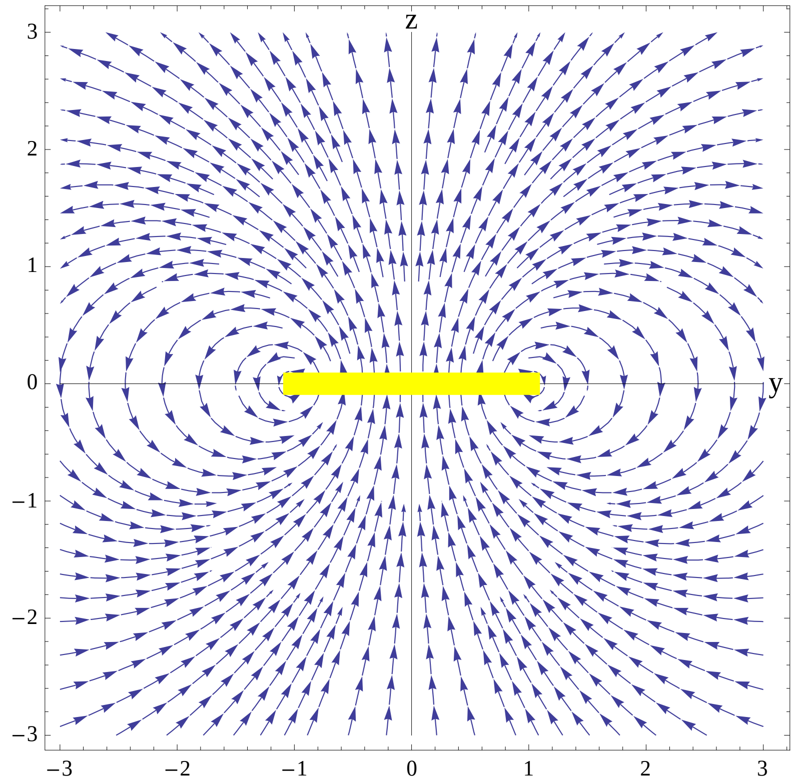


Fig.13(a) Magnetic field lines ***B*** (in the *y*-*z* plane) surrounding a current loop with a radius *a*. The axis of the current loop is the *z* axis. The yellow line indicates a part of current loop. At large distances from the circle, this field becomes purely dipole-like. This figure is drawn by using the Mathematica 7.0 (StreamPlot).

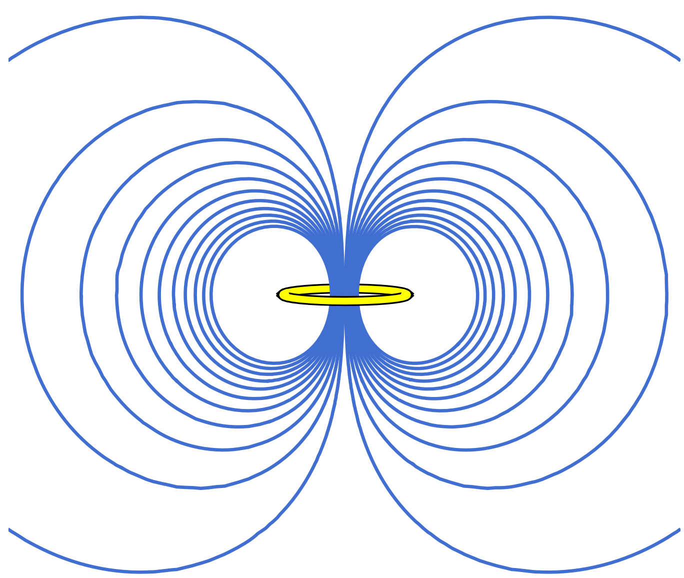


Fig.13(b) Magnetic field distribution around a closed loop current denoted by the yellow ring. "Magnetic Field of a Current Loop" from The Wolfram Demonstrations Project\[ParagraphSeparator] http://demonstrations.wolfram.com/MagneticFieldOfACurrentLoop/

7. Magjnetic field line due to magnetic dipole moment

What is the magnetic dipole moment (or simply, magnetic moment) due to a current loop with radius *a*? The magnetic field *B* (at the origin) due to the current loop is directed along the *z* axis and is given by

$$B_z = \frac{\mu_0 I}{2a}. \tag{8}$$

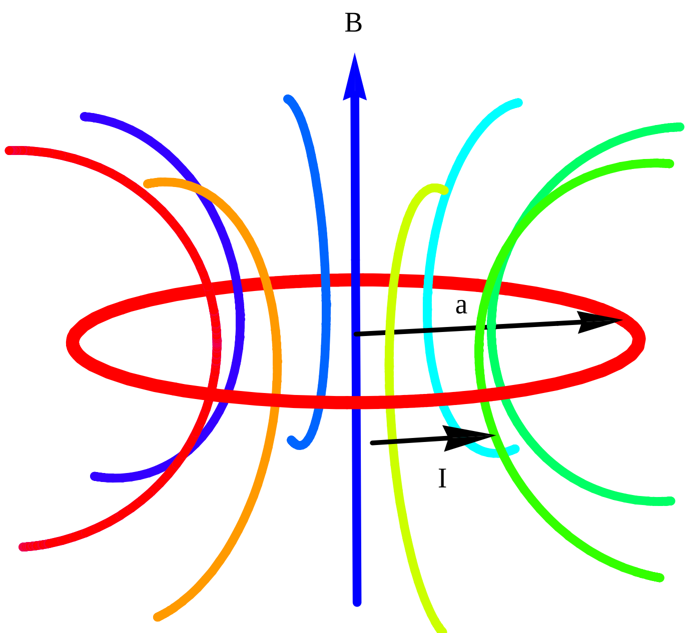


Fig.14 Magnetic field lines B due to a current path with a radius a, carrying the current *I* and a radius a.

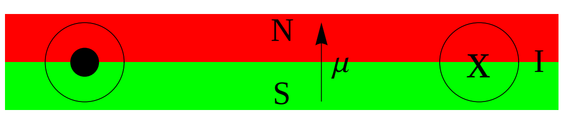


Fig.15 (a) and (b) Magnetic field lines produced by a current loop. The current loop generates a magnetic moment $\boldsymbol{\mu}$ with the north and south poles. The magnetic moment $\boldsymbol{\mu}$ is perpendicular to the loop, in the direction of the right-hand rule. The magnitude of $\boldsymbol{\mu}$ is *IA*, where *A* is the area of the loop.

We consider the distribuition of the magnetic field ***B*** due to the magnetic moment $\boldsymbol{\mu}$ at the origin, whose direction is along the *z* axis. The vector potential ***A*** due to the magnetic dipole moment $\boldsymbol{\mu}$ (along the *z* axis) can be described by

$$\mathbf{A} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{\mu}\times\mathbf{r}}{r^3}, \qquad (9)$$

where ***r*** is the position vector; ***r*** = (*x*, *y*, *z*). The magnetic field ***B*** is obtained as

$$\mathbf{B} = \nabla\times\mathbf{A}, \qquad (10)$$

or

$$\mathbf{B} = \frac{\mu_0\mu}{4\pi}\left(\frac{3zx}{r^5},\frac{3yz}{r^5},\frac{2z^2-x^2-y^2}{r^5}\right). \qquad (11)$$

This form of ***B*** is the same as that of the electric field ***E*** due to the electric dipole moment ***p*** (along the *z* axis), except for the proportinal constants. When *x* = 0 (in the *y*-*z* plane), we have

$$\mathbf{B} = \frac{\mu_0}{4\pi r^5}\mu(0,3yz,(2z^2-y^2)) \qquad (12)$$

When *x* = *y* = 0 and *r* = *z*, the magnetic field ***B*** has only the *z* componenet and is given by

$$B_z = \frac{\mu_0}{2\pi z^3}\mu, \qquad (13)$$

where |*z*|>>*a*. Figure 16 shows the magnetic field lines *B* in the *y*-*z* plane, where the magnetic moment is directed along the *z* axis. This figure is drawn using the StreamPlot of the Mathematica 7.0. The magnetic field lines due to the magnetic moment is very similar to the electric file lines due to the electric field lines.

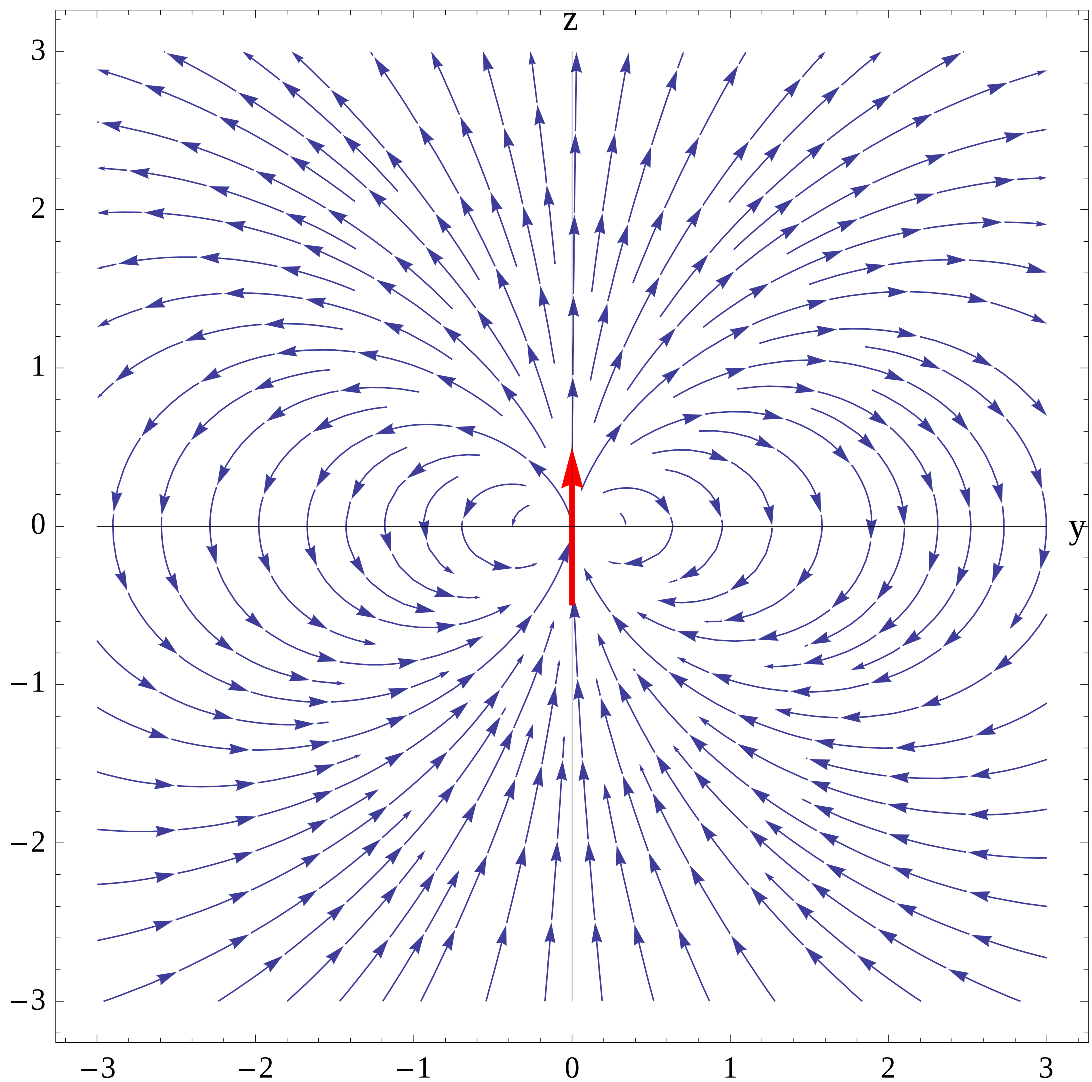


Fig.16 Magnetic field line around a magnetic dipole moment at the origin. The magnetic moment (denoted by red arrow) is located at the origin. The direction of $\boldsymbol{\mu}$ is along the z axis. This figure is drawn by using the Mathematica 7.0 (StreamPlot).

The on-axis magnetic field due to a current loop is

$$B_z = \frac{\mu_0}{2}\frac{Ia^2}{(z^2+a^2)^{3/2}}. \qquad (14)$$

For $|z|>>a$, the on-axis magnetic field is approximated by

$$B_z = \frac{\mu_0}{2}\frac{Ia^2}{z^3}. \qquad (15)$$

Suppose that the on-axis magnetic field concides with the magnetic field due to the magnetic moment μ. Then we get the relation

$$\mu = I(\pi a^2) \quad (16)$$

More generally, the magnetic moment is defined by

$$\mu = IA, \quad (17)$$

where A is the area of the current loop of any shape. Since the direction of $\boldsymbol{\mu}$ is along the z axis, the magnetic field due to the magnetic moment can be rewritten in the vector form,

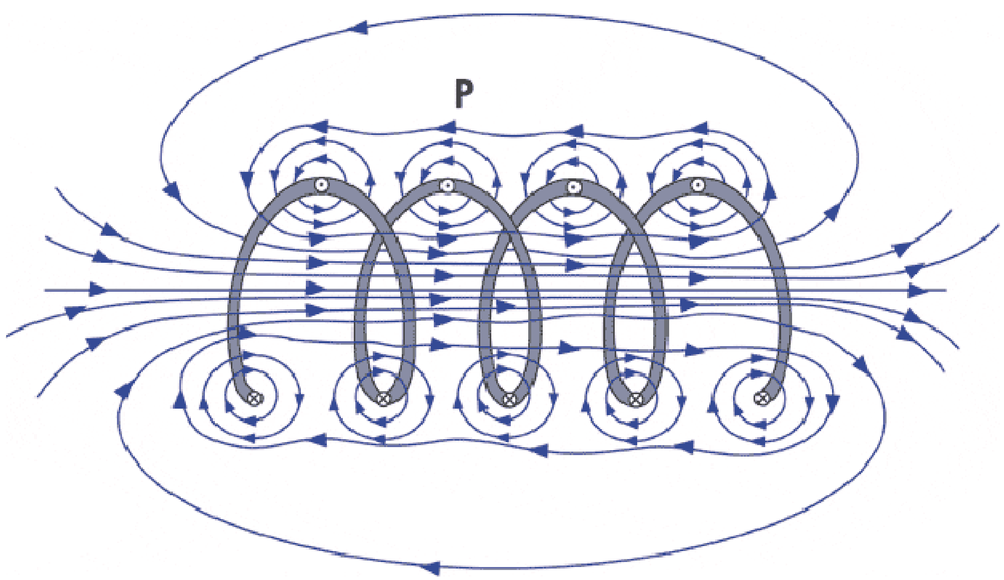
$$\mathbf{B}_{dipole} = \frac{\mu_0}{4\pi}\frac{\boldsymbol{\mu}}{z^3}. \quad (18)$$

If $\boldsymbol{B}_{\mathrm{dipol}}$ (Eq.18) is compared with $\boldsymbol{E}_{\mathrm{dipole}}$ (Eq.4) one can find that the magnetic field of a magnetic dipole has the same basic shape as the electric field of an electric dipole.

8. Magnetic field in the solenoid coil

A solenoid is a long wire wound in the form of a helix. If the turns are closely spaced, this configuration can produce a reasonably uniform magnetic field throughout the volume enclosed by the solenoid, except close to its ends. Each of the turns can be modeled as a circular current loop, and net magnetic field is the vector sum of the magnetic fields due to all the turns.

If the turns are closely spaced and the solenoid is of finite length, the magnetic field lines are as shown in Fig.17. In this case, the field lines diverge from one end and converge at the opposite end. An inspection of this field distribution exterior to the solenoid shows a similarity to the field of a bar magnet. Hence, one end of the solenoid behaves like the N (north) pole of a magnet while the opposite end behaves like the S (south) pole.


P

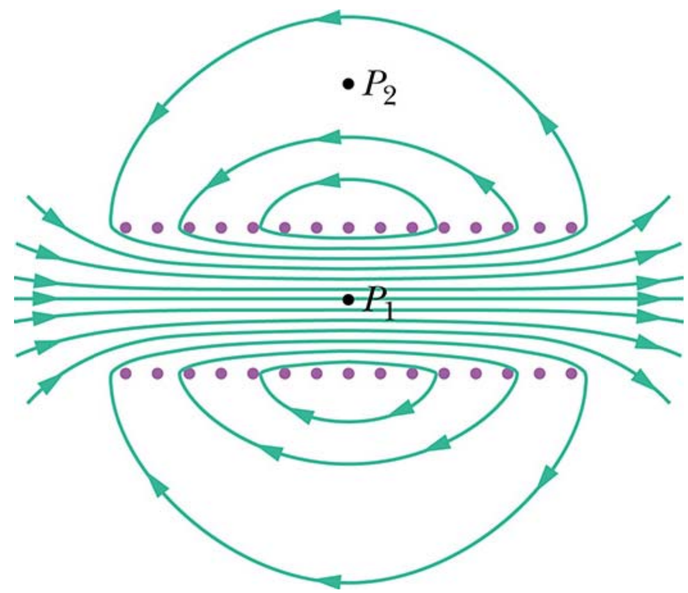

P_2
P_1

Fig.17 Magnetic field lines for a tightly wound solenoid of finite length carrying a steady current. The field in the space enclosed by the solenoid is nearly uniform. Note that the field lines are similar to those of a bar magnet, so that the solenoid effectively has north and south poles.
From Figs.29.18 and 29.19 from Reference 8.

We consider a the solenoid coil carrying current I, where the total number of turns is N. Since each loop current generates a magnetic moment $\boldsymbol{\mu}$ along the z axis, the magnetic field due to the solenoid is equivalent to that for the sytem of N magnetic moments stacked along the z axis. See Fig.18 for $N = 2$ and Fig.19 for $N = 12$.

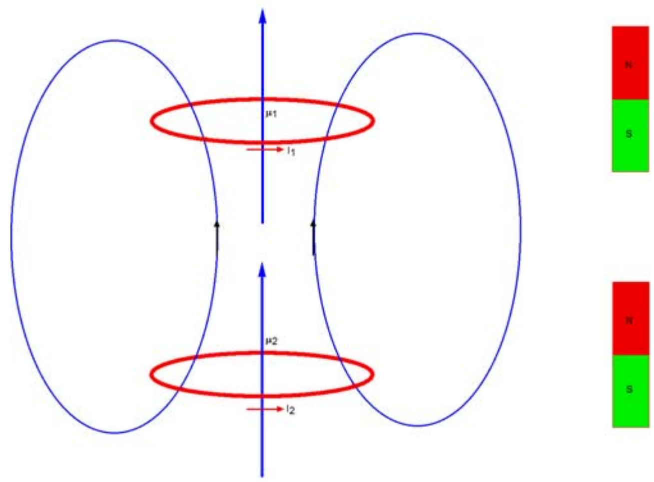


Fig.18 The system of two coils carrying current I. Two bar magnets aligning along the z axis. The magnetic field lines due to the two coil system is equivalent to those of the system of two bar magnets.

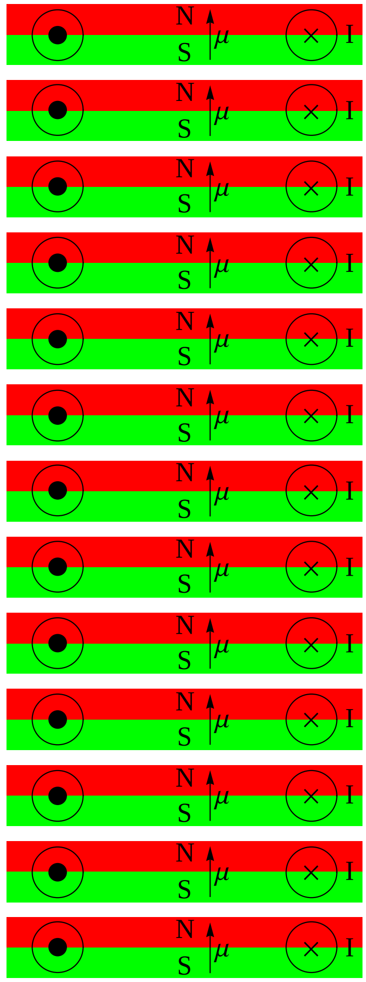


Fig.19 Magnetic field lines from the solenoid carrying current I is equivalents to those from the system (the superposition of the magnetic moment ($\mu = IA$) per turn stacked along the z axis).

The magnetic field B for the system consisting of many loop current layers equally spaced along the z axis (N layers) can be calculated using Eq.(5) as

$$(B_r)_{total} = \sum_{i=0}^{N-1} B_r(z - z_0 i, r)$$
$$(B_z)_{total} = \sum_{i=0}^{N-1} B_z(z - z_0 i, r), \qquad (19)$$

where z_0 is the separation distance between the nearest neighbor current loops, and the notations of this equation is the same as that of Eq.(5). Figure 20 shows the magnetic field lines of the system with N = 7. When N increases, the magnetic field line inside the system becomes homogeneous and parallel to the z axis

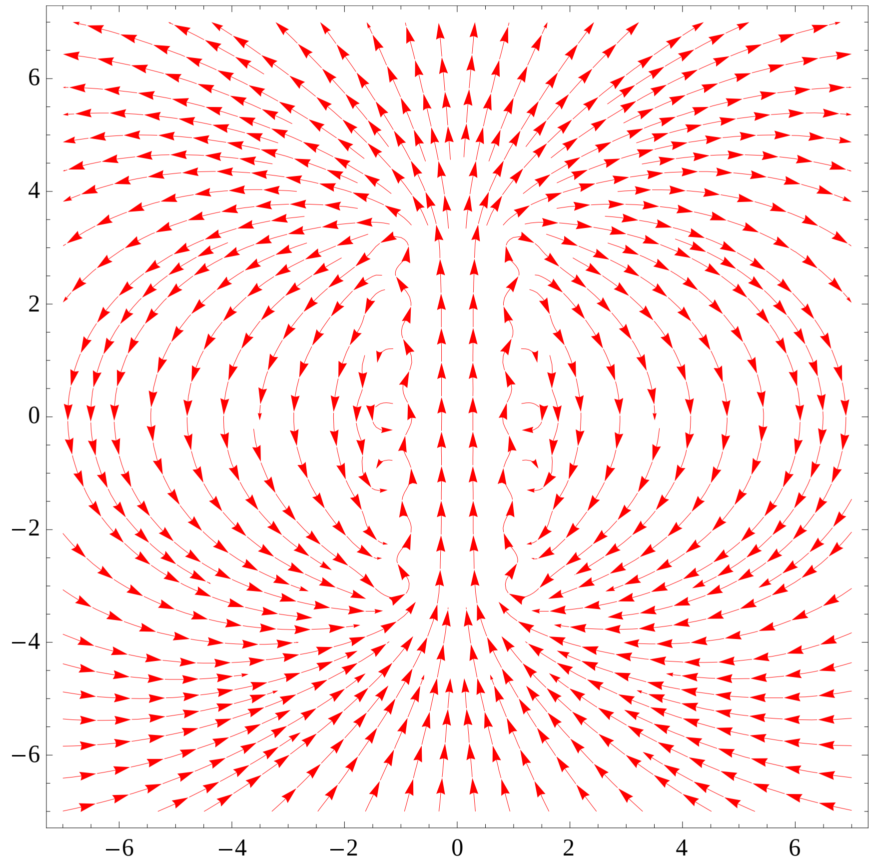


Fig.20 Magnetic field lines of the system of 7 current loops (stacked along the z axis) which are equally spaced. When the coli is wound tightly and there are more loops, the magnetic field inside become larger and more uniform. The magnetic field B forms a closed loop.

In conclusion, we find that the distribution of the magnetic field $\boldsymbol{B}$ for the solenoid is the same as that for the bar magnet.

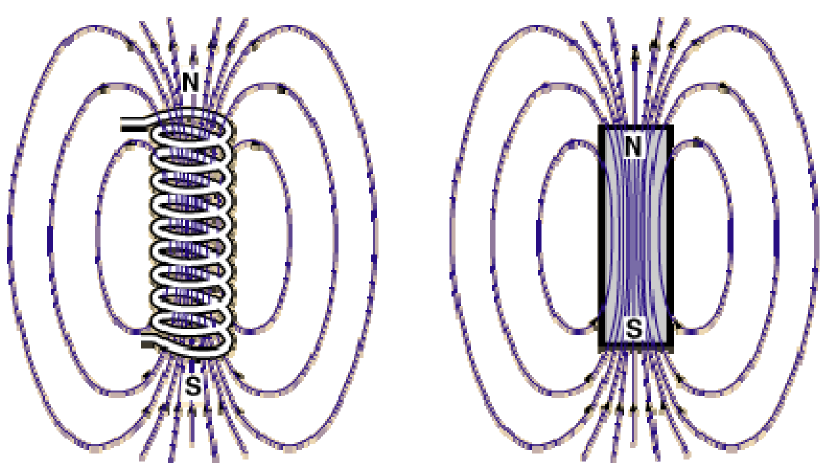


Fig.21 Magnetic field lines for the solenoid and bar magnet
http://hyperphysics.phy-astr.gsu.edu/hbase/magnetic/elemag.html

Consider a small loop of area d**a** with a current *I* flowing around it. The magnetic dipole moment $d\mu_z$ along the z axis is defined by

$$d\mu_z = AIndx \quad . \tag{20}$$

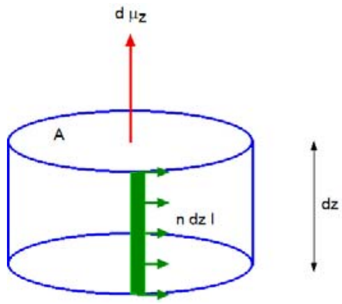


Fig.22 Model for the calculation of total magnetic momemt.

where *n* is the number of turns per unit length. The total magnetic moment is

$$(\mu_z)_{tot} = AIn\int_0^L dx = AInL = AI\frac{N}{L}L = ANI \quad . \tag{21}$$

The magnetization *M* is the magnetic moments per unit volume,

$$M = \frac{(\mu_z)_{tot}}{V} = \frac{ANI}{AL} = nI \quad . \tag{22}$$

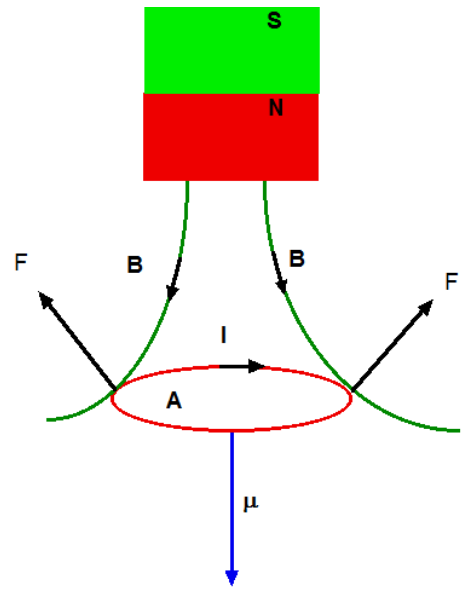
The magnetic field is

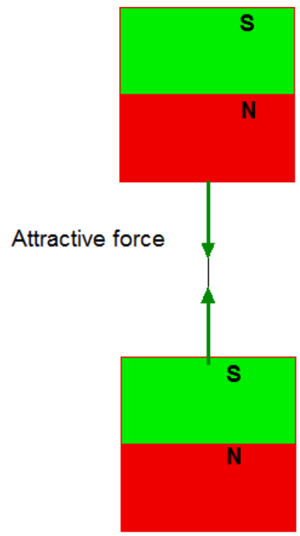
$$B = \mu_0 nI = \mu_0 M \quad . \tag{23}$$

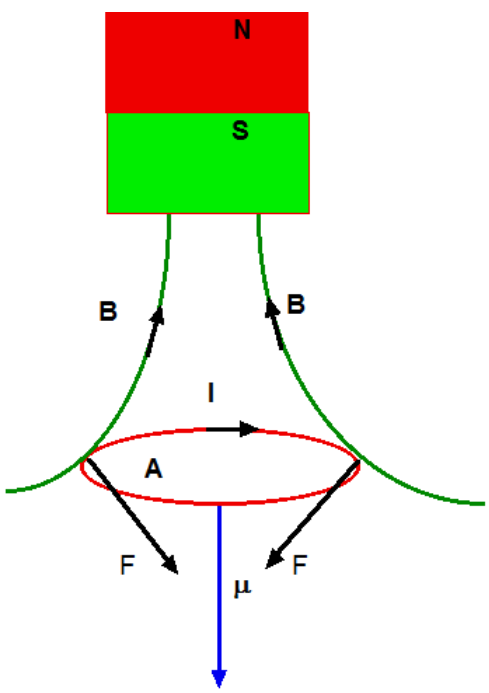
9. Interaction between bar magnets

Two bar magnets stick together when opposite poles are brought together (north-south), and repel when the same poles are brought together (north-north, south-south). The magnetic field of a small bar magnet is equivalent to a small current loop, so two magnets stacked end-to-end vertically are equivalent to two current loops stacked:

Another way to see this attraction is to consider the $\mathbf{F}_{ext} = i\mathbf{L} \times \mathbf{B}$ force acting on the current in loop 1 in the presence of the *non-uniform* field of loop 2.







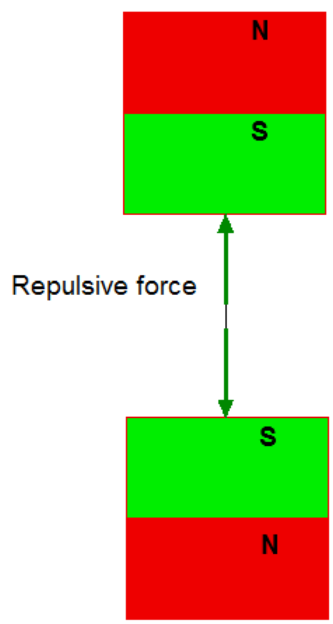


Fig.23 Origin of the attractive force between the different types of poles (north and south poles) and repulsive force between the same types of poles (the north and north poles, south and south poles). When $\boldsymbol{\mu}$ of the current loop and *non-uniform* magnetic field $\boldsymbol{B}$ from the bar magnet are parallel to each other, the interaction between the loop and the bar magnet is attractive. When $\boldsymbol{\mu}$ of the current loop and $\boldsymbol{B}$ of the bar magnet are anti-parallel to each other, the interaction between the loop and the bar magnet is repulsive.

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