Faraday rotation measurement using a lock-in amplifier

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Abstract:

This experiment is designed to measure the Verdet constant *v* through Faraday effect rotation of a polarized laser beam as it passes through different mediums, flint Glass and water, parallel to the magnetic field *B*. As the *B* varies, the plane of polarization rotates and the transmitted beam intensity is observed. The angle through which it rotates is proportional to *B* and the proportionality constant is the Verdet constant times the optical path length.

The optical rotation of the polarized light can be understood circular birefringence, the existence of different indices of refraction for the left-circularly and right-circularly polarized light components. The linearly polarized light is equivalent to a combination of the right- and left circularly polarized components. Each component is affected differently by the applied magnetic field and traverse the system with a different velocity, since the refractive index is different for the two components. The end result consists of left- and right-circular components that are out of phase and whose superposition, upon emerging from the Faraday rotation, is linearly polarized light with its plane of polarization rotated relative to its original orientation.

Michael Faraday, FRS (22 September 1791 – 25 August 1867) was an English chemist and physicist (or *natural philosopher*, in the terminology of the time) who contributed to the fields of electromagnetism and electrochemistry. Faraday studied the magnetic field around a conductor carrying a DC electric current. While conducting these studies, Faraday established the basis for the electromagnetic field concept in physics, subsequently enlarged upon by James Maxwell. He similarly discovered electromagnetic induction, diamagnetism, and laws of electrolysis. He established that magnetism could affect rays of light and that there was an underlying relationship between the two phenomena. His inventions of electromagnetic rotary devices formed the foundation of electric motor technology, and it was largely due to his efforts that electricity became viable for use in technology.

http://en.wikipedia.org/wiki/Michael_Faraday

Émile Verdet (1824–1866) was a French physicist. He worked in magnetism and optics, editing the works of Augustin-Jean Fresnel. Verdet did much to champion the early theory of the conservation of energy in France through his editorial supervision of the *Annales de chimie et de physique*. The Verdet constant is named for him.

http://en.wikipedia.org/wiki/%C3%89mile_Verdet

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1. Introduction

In 1845, Michael Faraday found the diamagnetism in a flint glass contained with PbO. When it was suspended between two magnetic poles of the magnet, the PbO glass was aligned along a direction perpendicular to the magnetic field direction. In 1845, Michael Faraday also discovered that when a block of glass is subjected to a strong magnetic field, it becomes optically active. When plane-polarized light is sent through glass in a direction parallel to the applied magnetic field, the plane of vibration is rotated. Since Faraday's early discovery, the phenomenon has been observed in many solids, liquids, and gases. The amount of rotation observed for any given substance is found by experiment to be proportional to the field strength B and to the distance the light travels through the medium.

Faraday rotation is a principle that relates a change in the plane of polarization of light as it passes through a material with an external magnetic field present. The Verdet constant provides the linear coefficient relating the polarization change to the magnetic field value and is a constant of the material. The relation is shown as

$$\theta_m = vBd ,$$

where θ_m is the angle of rotation of rotation (in radians), B is the magnetic field of propagation (in Tesla), and d is the length of the path (in m) where the light and magnetic field interact. v is the Verdet constant for the material. This empirical proportionality constant (in units of radians per T per m) varies with wavelength and temperature and is tabulated for various materials.

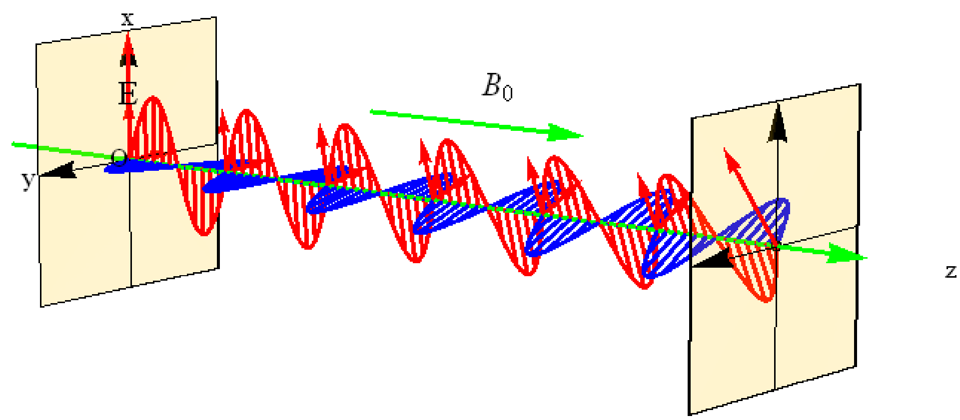

Fig.1 Faraday effect. The rotation of the polarization vector (***E***) in the presence of magnetic field (***B***$_0$) parallel to the propagation direction (the z axis). The angle of rotation of the plane of polarization of a light wave for a transparent material of length d in a magnetic field B is given by: $\theta_m = vBd$.

2. Simple theory

We introduce the two circular polarized electric fields propagating in the z direction,

$$\begin{aligned}\mathbf{E}_+(z,t) &= \mathrm{Re}[E_0(\hat{x} + i\hat{y})\exp[i(kz - \omega t)\\ &= E_0[\cos(kz - \omega t)\hat{x} - \sin(kz - \omega t)\hat{y}]\end{aligned}$$

$$\begin{aligned}\mathbf{E}_-(z,t) &= \mathrm{Re}[E_0(\hat{x} - i\hat{y})\exp[i(kz - \omega t)\\ &= E_0[\cos(kz - \omega t)\hat{x} + \sin(kz - \omega t)\hat{y}]\end{aligned}$$

It is considered to be right-hand (+), clockwise circularly polarized if viewed by the receiver, and to be left-hand (-), counter-clockwise circularly polarized if viewed by the receiver. We write the incident electric field as the superposition

$$\begin{aligned}\mathbf{E}(z=0,t) &= 2\,\mathrm{Re}[E_0\hat{x}\exp[i(k\cdot 0 - \omega t)\\ &= 2E_0\hat{x}\cos(k\cdot 0 - \omega t)\end{aligned}$$

which is a linearly polarized electric field with the polarization along the x axis. Now we assume that the electric fields ***E***$_\pm$ propagate with indices of refraction $n_\pm$ and attenuation constant $\beta_\pm$, respectively. Then the wavenumber $k_\pm$ is given by

$$k_{\pm} = \frac{\omega}{c} n_{\pm} = \frac{2\pi}{\lambda} n_{\pm} .$$

where λ is the vacuum wavelength of the light. The incident electric field propagates through a sample of length L to give the emergent field

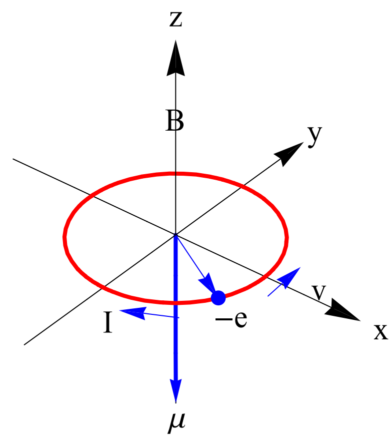

Fig.2 The electron rotates in counter clockwise. Since the electron charge (-*e*, *e*>) is negative, the orbital current flows in clockwise. So the magnetic moment μ is anti-parallel to the z axis. In this case (quantum mechanically) the frequency is equal to $\nu + \nu_L$. The electric field rotates in a clock-wise if viewed from the receiver (E_+, or E_r)

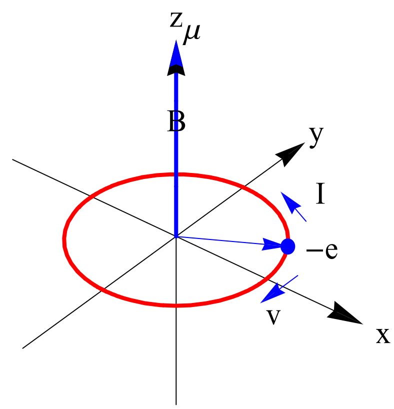

Fig.3 The electron rotates in clockwise. Since the electron charge (-*e*, *e*>) is negative, the orbital current flows in counter clockwise. So the magnetic moment μ is parallel to the *z* axis. In this case (quantum mechanically) the frequency is equal to ν - ν_L. The electric field rotates in a counter clockwise if viewed from the receiver (E_-, or E_l)

$$\mathbf{E}(z=L,t) = \mathrm{Re}[E_0 \exp(-\beta_+ L)(\hat{x}+i\hat{y})\exp[i(\frac{2\pi}{\lambda}n_+ L-\omega t)$$

$$+\mathrm{Re}[E_0 \exp(-\beta_- L)(\hat{x}-i\hat{y})\exp[i(\frac{2\pi}{\lambda}n_- L-\omega t)]$$

In the usual case of equal attenuations for the two polarizations ($\beta_+ = \beta_- = \beta$), this reduces to

$$
\begin{aligned}
\mathbf{E}(z=L,t) &= E_0 \exp(-\beta L)\{\mathrm{Re}[(\hat{x}+i\hat{y})\exp[i(\frac{2\pi}{\lambda}n_+L-\omega t)] \\
&+(\hat{x}-i\hat{y})\exp[i(\frac{2\pi}{\lambda}n\_L-\omega t)]\} \\
&= E_0\exp(-\beta L)\{\cos[\frac{2\pi}{\lambda}n_+L-\omega t]+\cos[\frac{2\pi}{\lambda}n\_L-\omega t]\}\hat{x} \\
&+E_0\exp(-\beta L)\{-\sin[\frac{2\pi}{\lambda}n_+L-\omega t]+\sin[\frac{2\pi}{\lambda}n\_L-\omega t]\}\hat{y} \\
&= 2E_0\exp(-\beta L)\cos[\frac{2\pi L}{\lambda}\frac{(n_- - n_+)}{2}]\hat{x}\cos[\frac{2\pi L}{\lambda}\frac{(n_+ + n_-)}{2}-\omega t] \\
&+2E_0\exp(-\beta L)\sin[\frac{2\pi L}{\lambda}\frac{(n_- - n_+)}{2}]\hat{y}\cos[\frac{2\pi L}{\lambda}\frac{(n_+ + n_-)}{2}-\omega t] \\
&= 2E_0\exp(-\beta L)[\cos\theta\hat{x}+\sin\theta\hat{y}]\cos[\frac{2\pi L}{\lambda}\frac{(n_+ + n_-)}{2}-\omega t]
\end{aligned}
$$

where

$$\theta = \frac{2\pi L}{\lambda}\frac{(n_- - n_+)}{2} = \frac{\phi_- - \phi_+}{2}.$$

Note that the phase change during traversal for the right- and left-circularly polarized light is

$$\phi_+ = \frac{2\pi L}{\lambda}n_+, \qquad \phi_- = \frac{2\pi L}{\lambda}n_-$$

We have the relation,

$$\phi_- - \theta = \phi_+ + \theta, \qquad \text{or} \qquad \theta = \frac{\phi_- - \phi_+}{2}.$$

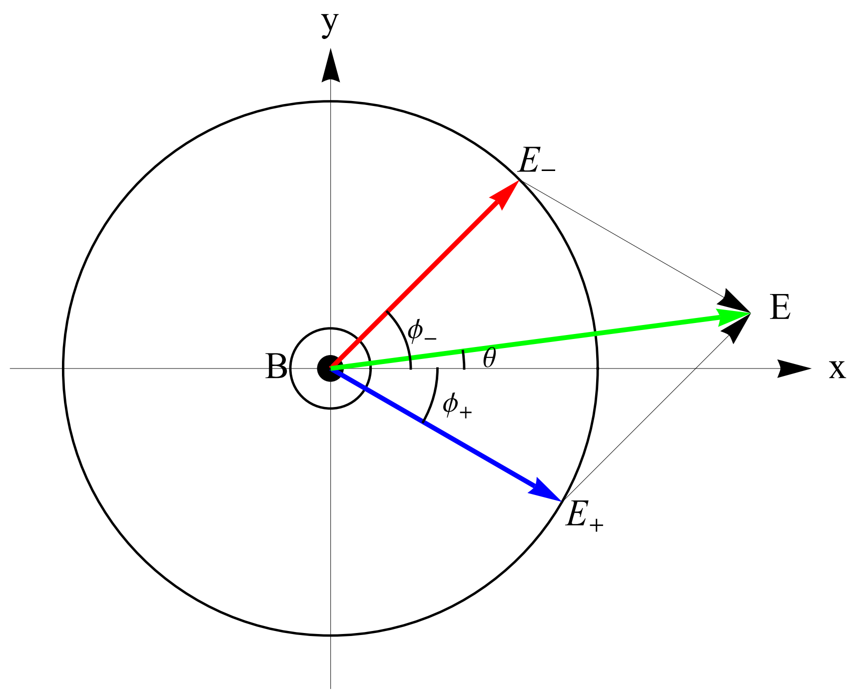

Fig.4 Superposition of left- and right-circularly polarized light into linearly polarized after traversing the sample. $\boldsymbol{E}_+$ (right-circularly polarized light). $\boldsymbol{E}_-$ (left-circularly polarized light), which is viewed from the receiver. $\boldsymbol{E} = \boldsymbol{E}_+ + \boldsymbol{E}_-$.From the geometry, there is a relation that $\phi_- - \theta = \phi_+ + \theta$, or $\phi_- - \phi_+ = 2\theta$.

What is the value of n_+ and n_-? The right and left components of the light appear to rotate with frequencies of $\nu + \nu_L$ and $\nu - \nu_L$, where ν_L is the Larmor frequency.

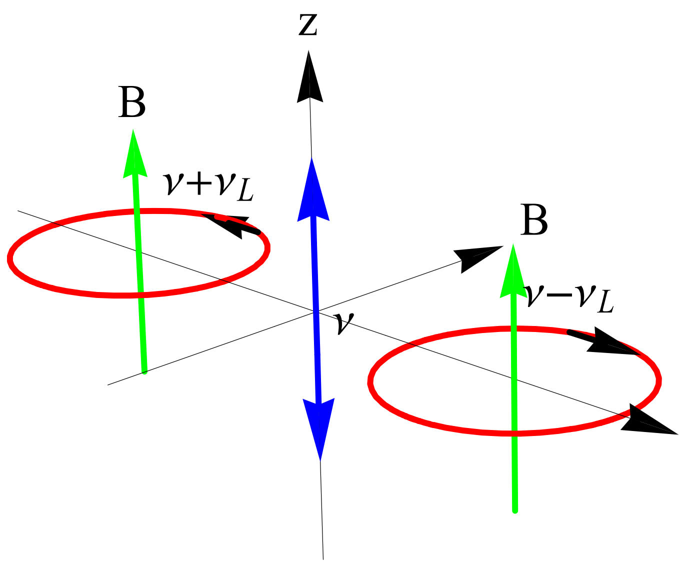

Fig.5 Lorentz model for the normal Zeeman effect. The motion of the electrons are shown by arrows. The magnetic field is applied along the *z* direction. When the electron rotates in counter-clockwise, the direction of the corresponding current is clock-wise. In this case the system has a frequency of $\nu + \nu_L$. When the electron rotates in clockwise, the direction of the corresponding current is counter clock-wise. In this case the system has a frequency of $\nu - \nu_L$. Note that $\nu >> \nu_L$.

Then the index of refraction are given by

$$n_+ = n(\nu + \nu_L), \qquad n_- = n(\nu - \nu_L)$$

Using the Taylor expansion, we have

$$
\begin{aligned}
n_+ - n_- &= n(\nu+\nu_L) - n(\nu-\nu_L) \\
&= [n(\nu) + \frac{dn}{d\nu}\nu_L] - [n(\nu) - \frac{dn}{d\nu}\nu_L] \\
&= 2\frac{dn}{d\nu}\nu_L
\end{aligned}
$$

We note that the Larmor frequency for the electron is given by

$$\omega_L = 2\pi\nu_L = \frac{eB}{2m} > 0 \qquad (e>0, -e \text{ is the charge of electron})$$

Then we have

$$n_+ - n_- = 2\left(\frac{eB}{4\pi m}\right)\frac{dn}{d\nu} = -2\left(\frac{eB}{4\pi m}\right)\frac{\lambda^2}{c}\frac{dn}{d\lambda}$$

where we use the relation $\nu = c/\lambda$, c is the velocity of light, and dm/dl is the rotary dispersion. Note that

$$\frac{dn}{d\nu} = -\frac{\lambda^2}{c}\frac{dn}{d\lambda}$$

Then we have

$$\theta = \frac{2\pi L}{\lambda}\frac{(n_- - n_+)}{2} = \frac{2\pi L}{\lambda}\left(\frac{eB}{4\pi m}\right)\frac{\lambda^2}{c}\frac{dn}{d\lambda} = \frac{e\lambda}{2mc}\frac{dn}{d\lambda}BL$$

The rotation angle θ_{m} is defined by

$$\theta_m = -\theta = vBL$$

since $\theta<0$ in usual way. Here we have

$$v = \frac{e}{2mc}(-\lambda\frac{dn}{d\lambda}) = 293.34\ (-\lambda\frac{dn}{d\lambda}) \qquad [\mathrm{rad/(T\ m)}]$$

or

$$\nu = 1.0083\ \lambda \frac{dn}{d\lambda} \qquad\qquad [\text{min/G cm})]$$

with

$m = 9.10938 \times 10^{-31}$ kg, $e = 1.60218 \times 10^{-19}$ C, and $c = 2.99782 \times 10^{8}$ m.

Note that ν is positive when the direction of the rotation for polarization vector is the same as the direction of current producing the magnetic field B (along the z axis).

((**Cauchy's equation**))

The index of refraction n depends on the wavelength λ, and is described by a Cauchy's equation

$$n = a + \frac{b}{\lambda^2} \qquad\qquad -\lambda \frac{dn}{d\lambda} = 2\frac{b}{\lambda^2} > 0$$

Note that the plot of n vs $1/\lambda^2$ for SF-59 (from the Ref. of glass is shown below.

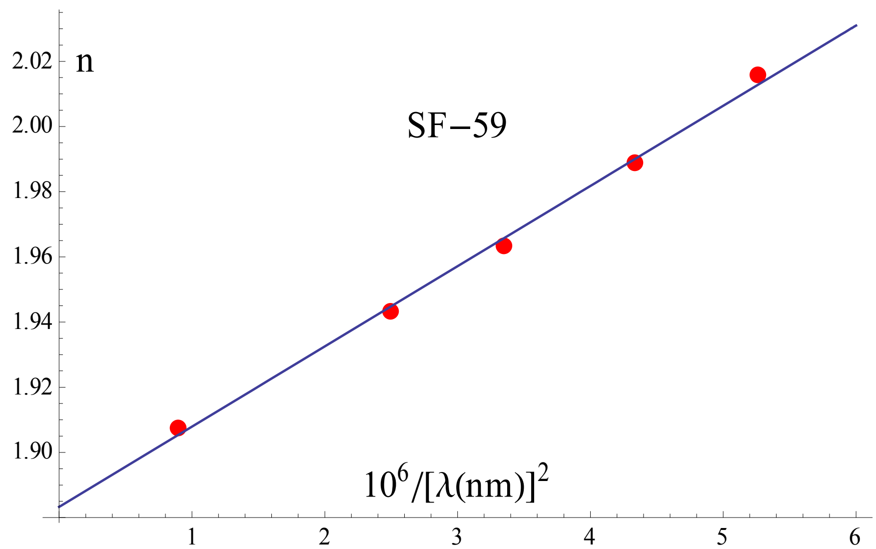

Fig.6 The wavelength dependence of the index of refraction (From the Ref. H. Bach and N. Neuroth).

The straight line is well described by

$$n = (1.8833 \pm 0.003) + (0.0246 \pm 0.0008) \times \frac{10^6}{[\lambda(nm)]^2}$$

We use the least-squares fit of the data to the straight line. The index of refraction for SF-59 glass decreases with increasing λ.

((Calculation of the Verdet constant for SF-59))

$$\theta_m = 2934(-\lambda \frac{dn}{d\lambda}) = \frac{293.34 \times 2 \times 0.024611 \times 10^6}{(\lambda[nm])^2} = \frac{1.4.442 \times 10^7}{(\lambda[nm])^2}.$$

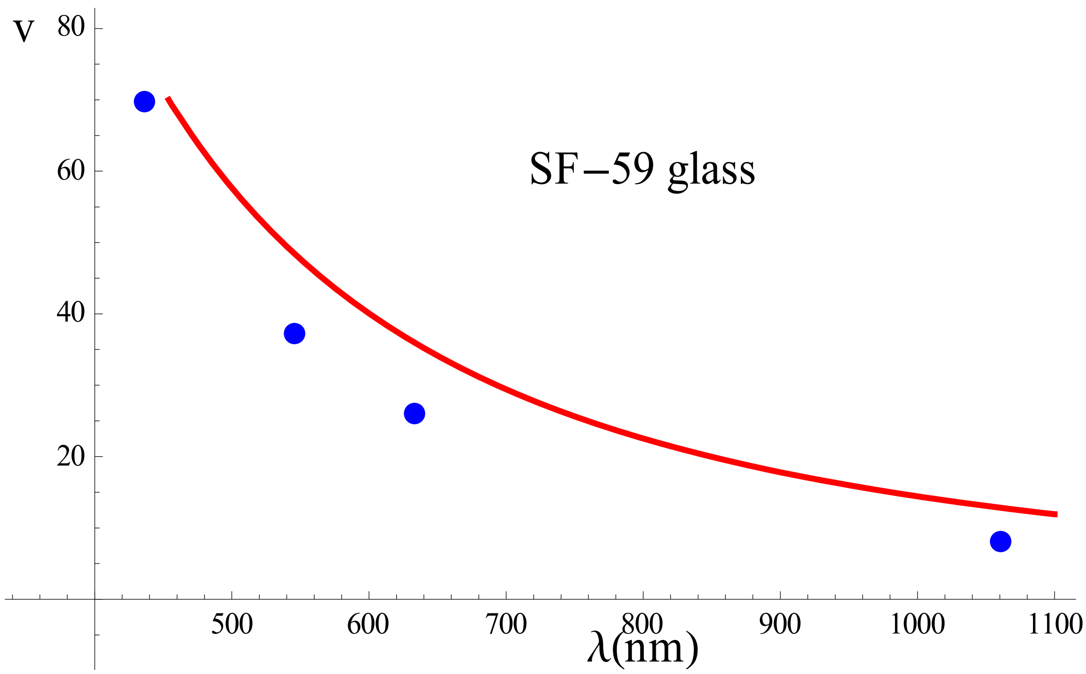

Fig.7 Verdet constant v (rad/(T m)) vs the wavelength λ (nm) for SF-59 glass. Experimental data (blue solid circles). The calculation curve (red line) using the Cauchy's equation.

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3. Schematic diagram for the DC method

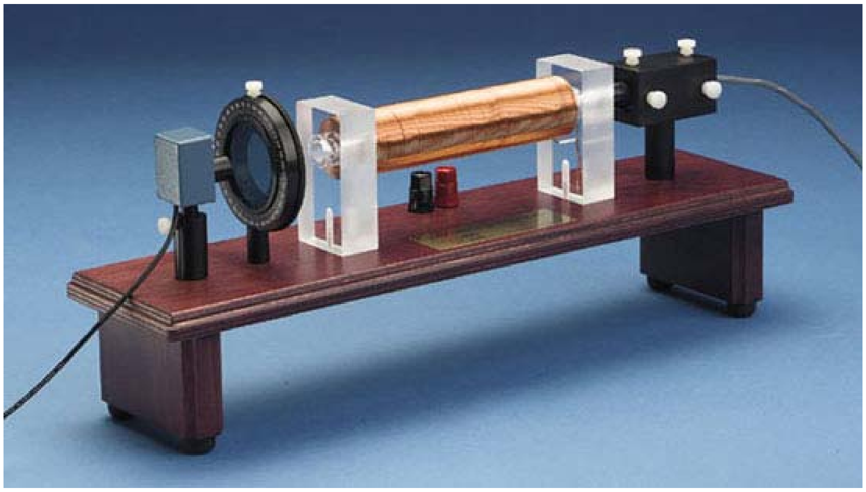

Fig.8 Apparatus of Faraday rotation (TeachSpin). FR1-A apparatus.

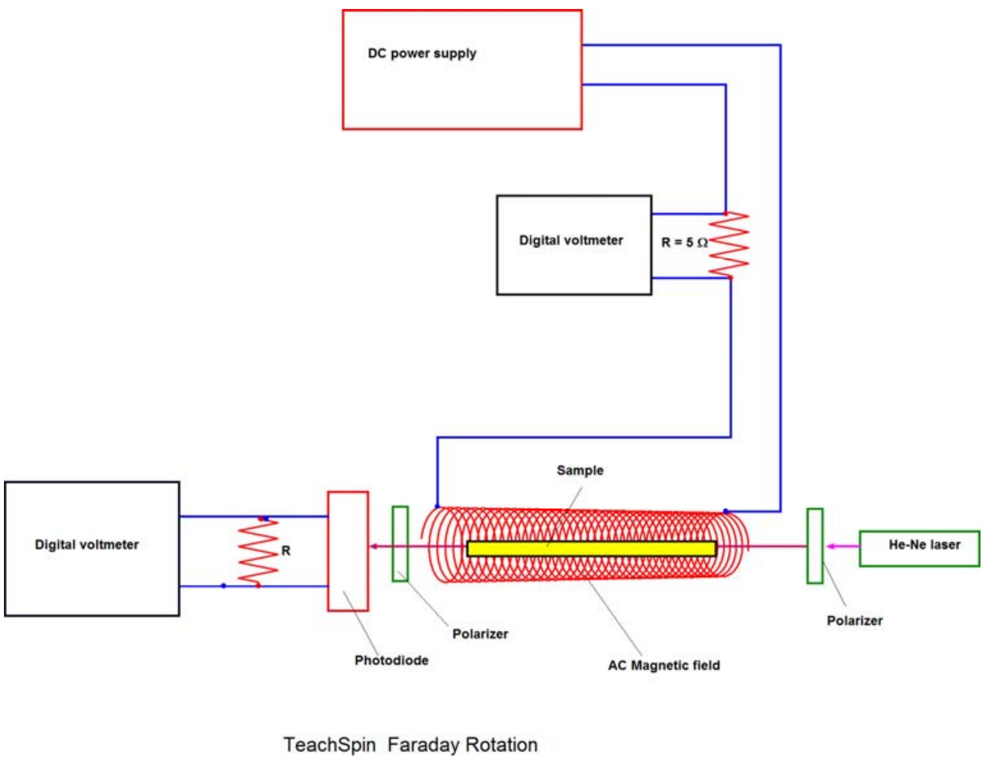

Fig.9 Schematic diagram of DC Faraday rotation method. He-Ne laser (3 mW, 650 nm). Schott SF-59 glass is used as a sample. The DC magnetic field is generated by a DC power supply. The DC current flowing in the solenoid is monitored by a DC voltage (digital volt meter) across a resistance R.

4. Maulus's law

An electric field component parallel to the polarization direction is passed (transmitted) by a polarizing sheet. A component perpendicular to it is absorbed.

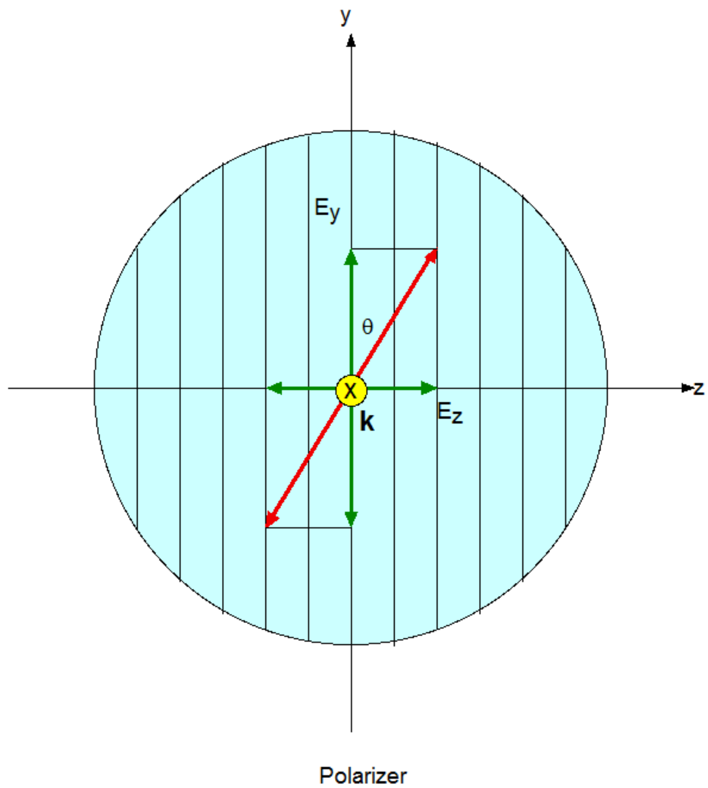

Fig.10 Maulus's law: $I = I_0 \cos^2(\theta)$.

The electric field along the direction of the polarizing sheet is given by

$$E_y = E\cos\theta$$

Then the intensity I of the polarized light with the polarization vector parallel to the y axis is given by

$$I = I_0 \cos^2\theta \quad \text{(Malus' law)}$$

where

$$I = S_{avg} = \frac{E_{rms}{}^2}{c\mu_0} = I_0 \cos^2\theta$$

Suppose that the light passes through the first polarizer and enters into the solenoid. The light goes out from the solenoid and passes through the analyzer (the second polarizer). In the polarizer, the direction of the electric field $\boldsymbol{E}_{\mathrm{i}}$ of the incident light is the same as that of the polarization vector $\boldsymbol{P}_{\mathrm{i}}$.

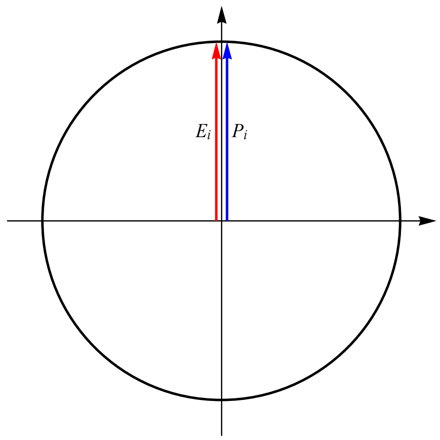

Fig.11 The direction of the electric field E_{i} is the same as that of the polarizing vector ($\boldsymbol{P}_{\mathrm{i}}$) for the first polarizer

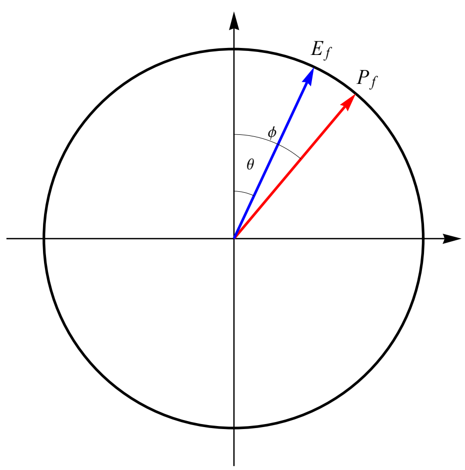

Fig.12 The direction of the electric field E_{f} is different from that of the polarizing vector ($\boldsymbol{P}_{\mathrm{f}}$) for analyzer

Suppose that there is no magnetic field. In this case, the light intensity measured by the photo-diode detector is given by

$$I = I_0 \cos^2(\phi) = \frac{1}{2} I_0 [1 + \cos(2\phi)]$$

according to the Maulus's. The output voltage from the photo-diode is given by

$$V_{DC} = kI = \frac{1}{2}kI_0[1+\cos(2\phi)]$$

where k is a constant.

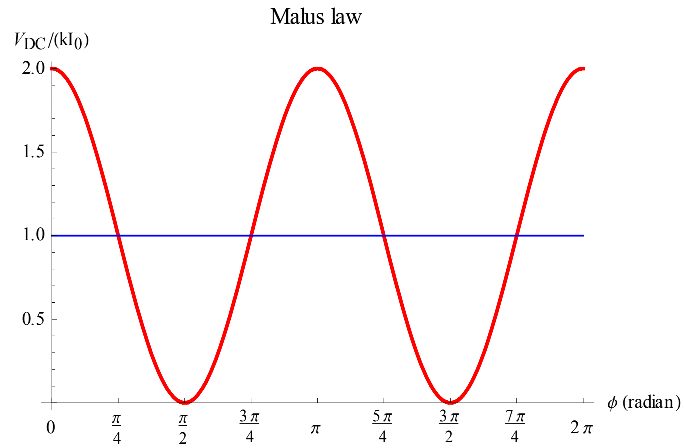

Fig.13 Malus's law. V_{DC}/kI_0 vs the rotation angle ϕ.

5. Verdet constant *v*

When the DC magnetic field B is applied along the z axis, the rotation angle of the Faraday effect is given by

$$\theta = vdB = vdB$$

where v is a Verdet constant and d is the length of the system along the z axis. Then we have

$$\begin{aligned} V_{DC} &= I_0\cos^2(\phi-\theta) = \frac{1}{2}kI_0[1+\cos(2(\phi-\theta)] \\ &= \frac{1}{2}kI_0[1+\cos(2\phi)\cos(2\theta)+\sin(2\phi)\sin(2\theta)] \\ &\approx \frac{1}{2}kI_0[1+\cos(2\phi)+\sin(2\phi)2\theta] \end{aligned}$$

We choose $\phi = \frac{\pi}{4}$; $\sin(2\phi)$=1 and $\cos(2\phi) = 0$. Then we get

$$\begin{aligned} V_{DC} &= \frac{1}{2}kI_0(1+2\theta) = kI_0 + kI_0\theta \\ &= kI_0 + kI_0(vdB) \end{aligned}$$

6. Experimental procedure

1. We use Schott SF-59 glass as sample. Put the sample inside the solenoid. In the absence of an external magnetic field, measure V_{DC} as a function of the rotation angle of polarizer, ϕ. Confirm that the Malus's law is valid. Determine the maximum value of V_{DC}: $V_{max} = 2\, kI_0$.
2. Apply the magnetic field B along the z axis (axis of the solenoid). The magnetic field B (TeachSpin) is related to the flowing current I_0 through

$$B = 111.0 I_0\,(\mathrm{A}) \qquad (\mathrm{Oe})$$

Then the magnitude of B is related to the voltage across the resistance $R = 10\ \Omega$ as

$$B = 111.0\frac{V_0}{R} \qquad (\mathrm{Oe})$$

Note that 1 T = 10^4 Oe.

3. At $B = 0$, find the rotation angle ϕ at which $V_{DC} = kI_0$. In fact, when $2\phi = \pi/2$ (or $\phi = \pi/4$): the angle between the analyzer and polarizer directions is 45º (or 315º), we have

$$V_{DC} = kI_0 + kI_0(vdB)\,.$$

4. Measure the DC voltage V_{DC} as a function of B. Show that V_{DC} is propotional to B as shown in Fig.

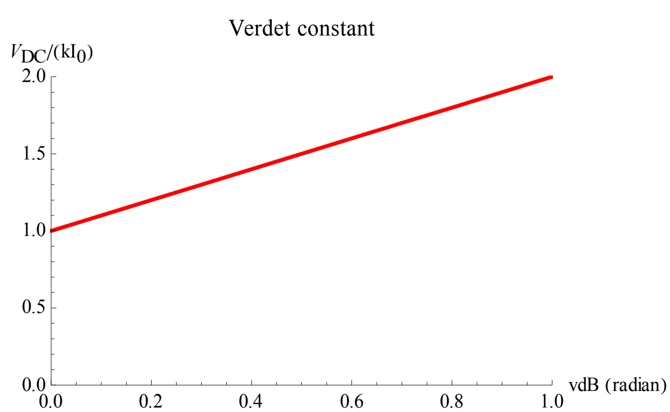

Fig.14 V_{DC}/kI_0 vs vdB, showing the straight line with a slope 1 and a y-intercept 1. d is the length of the system and B is the external magnetic field along the z axis.

7. Schematic diagram of AC method

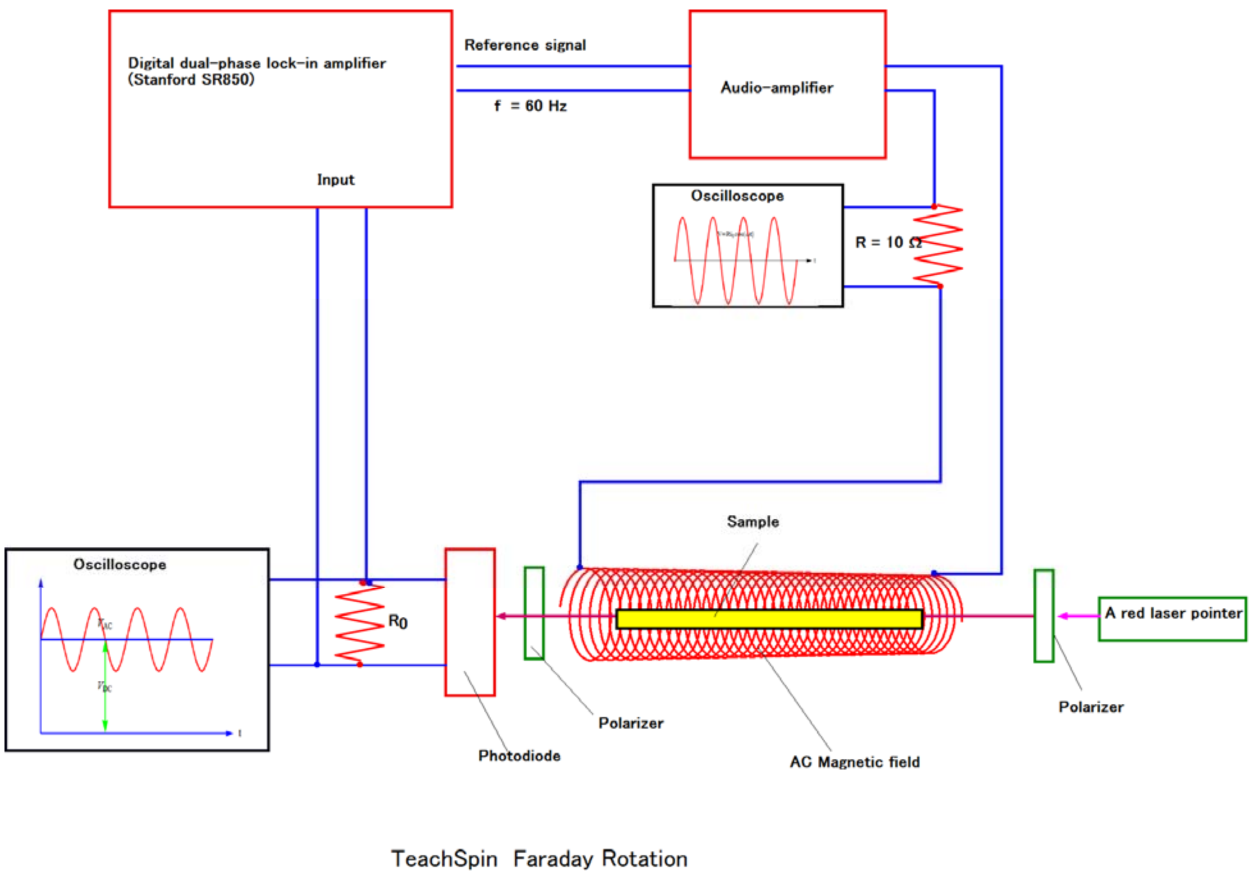

Fig.15 Schematic diagram of the AC Faraday method using a lock-in amplifier (digital, dual-phase Stanford SR850). A red laser pointer (3 mW, λ = 650 nm). The small oscillation on top of the DC signal in the oscilloscope (photo-diode output) is due to the Faraday effect. The audio amplifier is used to amplify the reference signal from the lock-in amplifier. The frequency of the AC magnetic field is the same as that of the reference signal. Schott SF-59 glass is used as a sample. R_0 = 1 k, 3 k, and 10 kΩ. We use a power audio amplifier (TeachSpin).

The light intensity measured by the detector is given by

$$I = I_0 \cos^2[\phi - \theta(t)]$$

Suppose that the AC magnetic field is applied along the *z* axis.

$$B(t) = B_0 \cos(\omega t + \varphi_0)$$

Then the rotation angle is given by

$$\theta(t) = vdB(t) = vdB_0 \cos(\omega t + \varphi_0) = \theta_m \cos(\omega t + \varphi_0)$$

where *v* is a Verdet constant and *d* is the distance. Then we have

$$I = I_0 \cos^2[\phi - \theta_m \cos(\omega t + \varphi_0)] = \frac{I_0}{2}\{1 + \cos[2\phi - 2\theta_m \cos(\omega t + \varphi_0)]\}$$
$$= \frac{I_0}{2}\{1 + \cos(2\phi)\cos[2\theta_m \cos(\omega t + \varphi_0)] + \sin(2\phi)\sin[2\theta_m \cos(\omega t + \varphi_0)]\}$$

Since

$$2\theta_m << 1$$

I is approximated by

$$I \approx \frac{I_0}{2}[1 + \cos(2\phi) + 2\theta_m \sin(2\phi)\cos(\omega t + \varphi_0)]$$

The output voltage of the detector is proportional to *I*, as

$$V = kI = \frac{kI_0}{2}[1 + \cos(2\phi) + 2\theta_m \sin(2\phi)\cos(\omega t + \varphi_0)]$$

The output signal consists of the DC voltage and the AC voltage

$$V_{AC} = kI_0\theta_m \sin(2\phi)\cos(\omega t + \varphi_0)$$

and

$$V_{DC} = \frac{kI_0}{2}[1 + \cos(2\phi)]$$

Using the lock-in amplifier, we get the AC amplitude (root-mean square value)

$$\left(\Delta V_{AC}\right)_{rms} \approx \frac{kI_0\theta_m}{\sqrt{2}}\sin(2\phi)$$

Then the ratio $\Delta V_{AC}/V_{DC}$ is obtained as

$$\frac{\left(\Delta V_{AC}\right)_{rms}}{V_{DC}} = \frac{\frac{1}{\sqrt{2}}kI_0\theta_m\sin(2\phi)}{\frac{kI_0}{2}[1+\cos(2\phi)]} = \sqrt{2}\theta_m\frac{\sin(2\phi)}{1+\cos(2\phi)} = \sqrt{2}\theta_m\tan\phi$$

When we use $\phi = \pi/4$ for the experiment, we have

$$\frac{\left(\Delta V_{AC}\right)_{rms}}{V_{DC}} = \sqrt{2}\theta_m, \qquad \text{or} \qquad \frac{\left(\Delta V_{AC}\right)_{peak}}{V_{DC}} = 2\theta_m$$

8. The possible use of second harmonics for AC method

We discuss the possibility of using the second harmonics signal to determine the Verdet constant. As we show above, the output intensity of analyzer is given by

$$\begin{aligned}
I &= I_0\cos^2[\phi-\theta_m\cos(\omega t)] = \frac{I_0}{2}\{1+\cos[2\phi-2\theta_m\cos(\omega t)]\} \\
&= \frac{I_0}{2}[1-(\theta_m{}^2-1)\cos(2\phi)] + \frac{I_0}{2}[\theta_m\cos(2\phi-\omega t)-\theta_m\cos(2\phi+\omega t) \\
&\quad -\frac{1}{2}\theta_m{}^3\cos(2\phi-\omega t)+\frac{1}{2}\theta_m{}^3\cos(2\phi+\omega t)] \\
&\quad +\frac{I_0}{4}\theta_m{}^2[\cos(2\phi-2\omega t)+\cos(2\phi+2\omega t)] \\
&\quad +\frac{I_0}{12}\theta_m{}^3[\cos(2\phi-3\omega t)-\cos(2\phi+3\omega t)]
\end{aligned}$$

(a) When $\phi = \pi/4$, we have

$$\begin{aligned}
I &= \frac{I_0}{2}[1-\theta_m(\theta_m{}^2-2)\sin(\omega t)]+\frac{1}{3}\theta_m{}^3\sin(3\omega t)] \\
&\approx \frac{I_0}{2}[1+2\theta_m\sin(\omega t)]
\end{aligned}$$

which means no second harmonics signal in I.

(b) When $\phi = \pi/3$

$$I = \frac{I_0}{12}[3 + 3\theta_m^{\ 2} - 3\sqrt{3}\theta_m(-2 + \theta_m^{\ 2})\sin(\omega t) - 3\theta_m^{\ 2}\cos(2\omega t) + \sqrt{3}\theta_m^{\ 3}\cos(3\omega t)]$$

$$\approx \frac{I_0}{12}[3 + 3\theta_m^{\ 2} + 6\sqrt{3}\theta_m\sin(\omega t) - 3\theta_m^{\ 2}\cos(2\omega t)]$$

In this case we have both the first harmonics and the second harmonics signals. The first harmonics which is in phase with the reference signal, is described by

$$I_{1in} = \frac{I_0}{12}k6\sqrt{3}\theta_m .$$

The second harmonics (out-of phase) is described by

$$I_{2out} \approx \frac{I_0}{12}k3\theta_m^{\ 2},$$

where k is a constant. Then the ratio is obtained as

$$\frac{I_{2out}}{I_{1in}} = \frac{3}{6\sqrt{3}}\theta_m = \frac{\sqrt{3}}{6}\theta_m .$$

This implies the simplest way to determine the value of θ_m. We do not need any information of the DC component of the intensity. We do not have to take into account of the factor $\sqrt{2}$ for the root mean square value of the lock-in amplifier (DC output).

9. Experimental Procedure (Conventional AC method)

1. We use the low frequency as a reference signal; typically $f \approx 60$ Hz. The output signal from the photodiode (voltage across the resistance R_0) is examined by using the oscilloscope. We find that the signal voltage consists of the AC component and DC component. The AC component (the small oscillation on top of the DC component) is due to the Faraday effect. The phase of the AC component signal is the same as that of the AC magnetic field. The rotation angle of the polarization vector is proportional to the AC magnetic field and length of path. Using the lock-in amplifier (in-phase, first harmonics, we use the digital dual-phase lock-in amplifier, Stanford SR850), measure the DC amplitude of V_{AC} signal (the output of the photodiode detector). We note that the DC

output voltage of the lock-in amplifier is equal to the root-mean square value of the real AC amplitude [$(\Delta V_{AC})_{\rm rms}$].

((Note))

When the lock-in technique is applied, care must be taken to calibrate the signal, because lock-in amplifiers generally detect only the root-mean-square signal of the operating frequency. For a sinusoidal modulation, this would introduce a factor of $\sqrt{2}$ between the lock-in amplifier output and the peak amplitude of the signal, and a different factor for non-sinusoidal modulation. In the case of extremely nonlinear systems, it may in fact be advantageous to use a higher harmonic for the reference frequency, because of frequency-doubling that takes place in a nonlinear medium.

2. Next find the maximum value of $(\Delta V_{AC})_{\rm rms}$ by rotating the analyzing polarization. In fact, when $2\phi = \pi/2$ (or $\phi = \pi/4$): the angle between the analyzer and polarizer directions is 45° (or 315°), we have

$$\left(\Delta V_{AC}\right)_{rms} = \frac{kI_0\theta_m}{\sqrt{2}}\sin(2\phi) = \frac{kI_0\theta_m}{\sqrt{2}}. \qquad (1)$$

3. Using the oscilloscope, measure the DC voltage. When $2\phi = \pi/2$, we have

$$V_{DC} = \frac{kI_0}{2}. \qquad (2)$$

4. Calculate the ratio between the AC and DC voltages,

$$\frac{\left(\Delta V_{AC}\right)_{rms}}{V_{DC}} = \sqrt{2}\theta_m = \sqrt{2}vdB_0,$$

or

$$\frac{\left(\Delta V_{AC}\right)_{peak}}{V_{DC}} = 2\theta_m = 2vdB_0$$

5. The magnetic field B_0 (TeachSpin) is related to the flowing current i through

$$B = 111.0i\,(\mathrm{A}) \qquad (\mathrm{Oe})$$

((Note)) 1 T = 10^4 Oe.

6. Find the root-mean square amplitude of the AC current flowing the solenoid from the other oscilloscope (see the schematic diagram). The AC current is related to the AC voltage across the resistance R (= 10 Ω) by

$$V_{BR} = Ri_m \cos(\omega t), \qquad \text{or} \qquad (V_{BR})_{rms} = \frac{Ri_m}{\sqrt{2}}$$

Then the amplitude of the AC current is evaluated as

$$i_m = \frac{\sqrt{2}(V_{BR})_{rms}}{R}$$

7. We have the relation (TeachSpin)

$$B_0 = 111.0 i_m$$

8. Then we get the ratio

$$\begin{aligned}
\frac{(\Delta V_{AC})_{rms}}{V_{DC}} &= \frac{2vdB_0}{\sqrt{2}} \\
&= \sqrt{2}vd[111.0 i_m] \\
&= \sqrt{2}vd[111.0\frac{\sqrt{2}(V_{BR})_{rms}}{R}] \\
&= 2(111.0)\frac{vd}{R}(V_{BR})_{rms}
\end{aligned}$$

Note that the values of $(\Delta V_{AC})_{rms}$ $(V_{BR})_{rms}$ are the RMS values. In other words, these values are digitally monitored using lock-in amplifier and oscilloscope.

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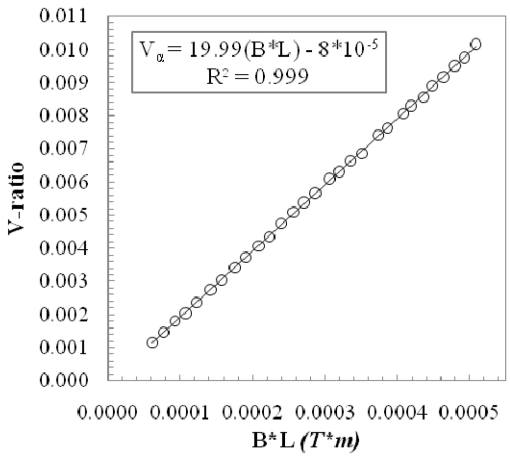
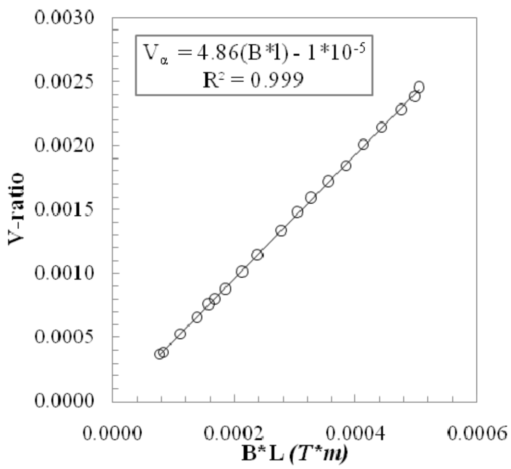
10. Results and Discussion

We measured the Verdet constants for SF-59 glass (diameter 5mm, length 10 cm, TeachSpin) and liquid.

v = 19.99 ± 0.05 rad/(T m) for SF-59 glass

v = 4.86 ± 0.02 rad/(T m) for liquid(distilled water)





REFERENCES

Sidney Malak unpublished paper (Senior Laboratory, Phys.429, Binghamton University, 2011).

J.F. Reichert, Faraday Rotation Instructor's Guide to TechSpin FR1-A.

User manual of Tektronix, TDS 2000-Series, Digital Storage Oscilloscope

A.C. Melissinos and J. Napolitano, Experiments in Modern Physics, second edition (Academic Pressm New York, 2003).

F.L. Pedrotti and P. Bandettini, Am. J. Phys. **58**, 542 (1990).

A. Jain, J. Kumar, F. Zhou, L. Li, and S. Tripathy, Am. J. Phys. **67**, 714 (1999).

V.K. Valev, J. Wouters, and T. Verbiest, Am. J. Phys. **76**, 626 (2008).

R.M. Duffy and R.P. Netterfield, Applied Optics **22**, 1272 (1983).

V.K. Valev, J. Wouters, and T. Verbiest, Eur. J. Phys. **29**, 1099 (2008)

E. Hecht and A. Zajac, Opticsn (Addison_Wesley Publishing Comapny, Reading MA, 1979). p.261-3.

F.A. Jenkins and H.E. White, Fundamentals of Optics, McGraw-Hill Book Company 1957.

H. Bach and N. Neuroth, Springer-Verlag, Berlin, 1998, edited by, The Optical Properties of Optical Glass

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APPENDIX

Sample: SF-59 glass

A-1 Verdet constant

The Faraday effect is chromatic (i.e. it depends on wavelength) and therefore the Verdet constant is quite a strong function of wavelength.

The Verdet constant decreases monotonically with increasing wavelength in nearly all cases. For glasases of the SF type the Verdet constant increases monotonically with the PbO content. A large values of Verdet are measured for the heavy flint glass type SF59. This glass has a extremely large PbO content, namely about 80 % by weight. Because of its large Verdet constant, this glass type is excellently suited as Faraday rotator materials. The symbol v is defined as the Verdet constant. For the SF-59 glass rod sold with the TeachSpin apparatus, the Verdet constant for 650 nm light is

v = 23 rad/T m.

Table

Verdet constant for distilled water

λ (nm)	v (rad/(Tm)
590	3.81
600	3.66
800	2.04
1000	1.28
1250	0.84

A-2 Verdet constant for SF-59 glass

The Verdet constant decreases monotonically with increasing wavelength λ in nearly all cases. For glasses of the SF type the Verdet constant increases monotonically with the PbO content. Very large values if Verdet constant have been measured for the heavy flint glass type SF 59. This glass has an extremely large PbO content namely about 80% by weight. Because of its large Verdet constant, this type of glass is excellently suited as a Faraday rotator material. It is to be mentioned that almost all glasses of these types show diamagnetic. On the other hand, special glasses with a large content of rare-earth ions and large magnetic moments such as Tb^{3+} have been developed, providing large Verdet constants. Since these glasses are paramagnetic, the Verdet constant and consequently the Faraday rotation angle depend on temperature.

Taking advantage of the rotation in opposite directions, yielding a Verdet constant close to zero. This has been almost perfectly achieved for the optical glasses SF L16 and SF L56 at room temperatures. Thus, in these glasses, the plane of polarization of an electromagnetic wave is nearly unaffected by magnetic fields.

A-3 Index of refraction and Verdet constant for SF-59 glass

Table

Wavelength (nm)	Index of refraction	Verdet constant
435.8	2.0156	69.8
480	1.989	
546.1	1.9635	37.2
632.8	1.9432	25.9
1060	1.9074	8.1