Possibility of Bose-Einstein Condensation in One- and two-dimensional system

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1. 1D Bose-Einstein condensation

$$D(\varepsilon)d\varepsilon = \frac{gL}{2\pi}2dk$$

with $g = 1$.

$$\varepsilon = \frac{\hbar^2 k^2}{2m}, \qquad k = \left(\frac{2m}{\hbar^2}\right)^{1/2}\sqrt{\varepsilon}$$

$$dk = \left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{1}{2\sqrt{\varepsilon}}d\varepsilon$$

The density of states for the 1D system;

$$\begin{aligned} D(\varepsilon)d\varepsilon &= \frac{L}{2\pi}2\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{1}{2\sqrt{\varepsilon}}d\varepsilon \\ &= \frac{L}{2\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{1}{\sqrt{\varepsilon}}d\varepsilon \end{aligned}$$

or

$$D(\varepsilon) = \frac{L}{2\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2}\frac{1}{\sqrt{\varepsilon}}$$

The number $N(T, z)$;

$$
\begin{aligned}
N(T,z) &= \int_0^\infty D(\varepsilon) f_{BE}(\varepsilon) d\varepsilon \\
&= \frac{L}{2\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} \int_0^\infty \frac{\varepsilon^{-1/2}}{e^{\beta(\varepsilon-\mu)}-1} \\
&= \frac{L}{2\pi}\left(\frac{2m}{\hbar^2}\right)^{1/2} \int_0^\infty \frac{\varepsilon^{-1/2}}{\frac{1}{z}e^{\beta\varepsilon}-1}
\end{aligned}
$$

We put $x = \beta\varepsilon$.

$$
N(T,z) = \frac{L}{2\pi}\left(\frac{2m}{\beta\hbar^2}\right)^{1/2} \int_0^\infty \frac{x^{-1/2}}{\frac{1}{z}e^{x}-1} dx
$$

with

$$
\frac{1}{\sqrt{\pi}} \int_0^\infty \frac{x^{-1/2}}{\frac{1}{z}e^{x}-1} dx = Li_{1/2}(z)
$$

Note that

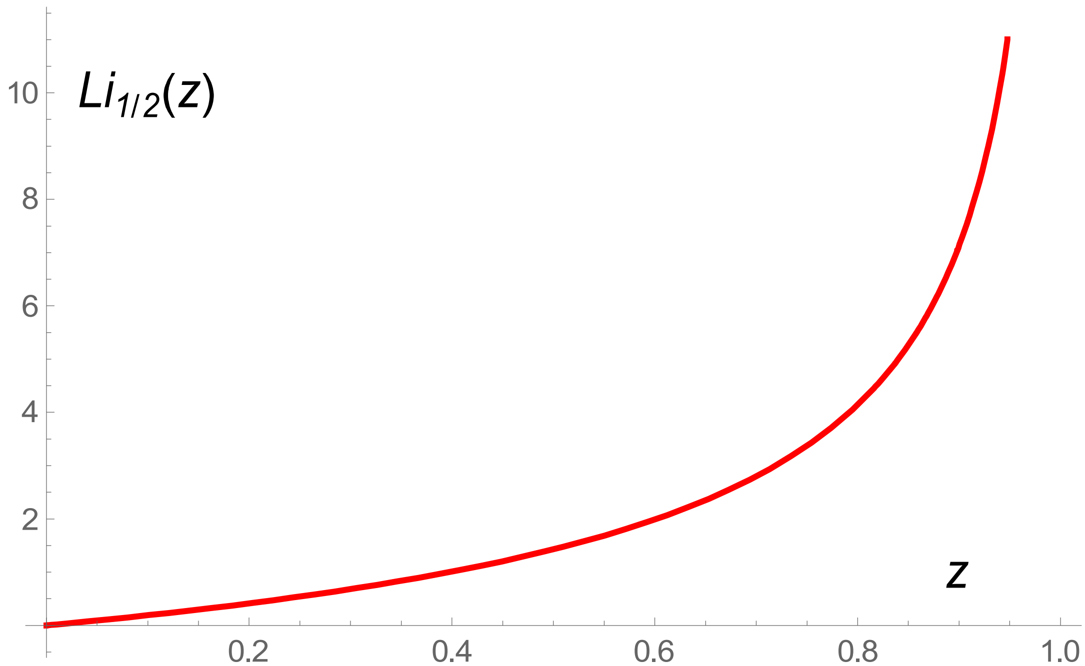
$Li_{1/2}(z)$=PolyLog[1/2,z]

Thus we have

$$
\begin{aligned}
N(T,z) &= \frac{L}{2\pi}\left(\frac{2m}{\beta\hbar^2}\right)^{1/2} \sqrt{\pi} Li_{1/2}(z) \\
&= L\left(\frac{mk_B T}{2\pi\hbar^2}\right)^{1/2} Li_{1/2}(z)
\end{aligned}
$$

Suppose that $T = T_E$ at $z = 1$..

$$
\left(\frac{mk_B T_E}{2\pi\hbar^2}\right)^{1/2} = \frac{N(T_E, z=1)}{L} \frac{1}{Li_{1/2}(z=1)} = n_L \frac{1}{Li_{1/2}(z=1)}
$$



$Li_{1/2}(z)$
10
8
6
4
2
z
0.2
0.4
0.6
0.8
1.0