Negative temperature II
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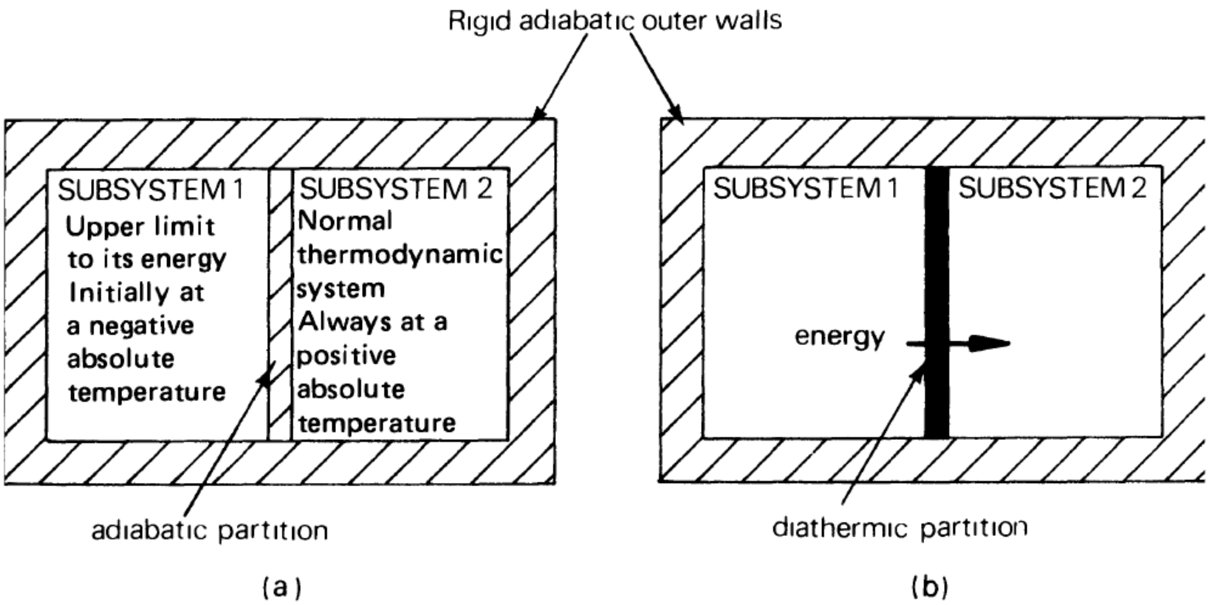

Fig. (a) Subsystem 1 has an upper limit to energy it can have, whereas subsystem 2 is a normal thermodynamic system, which has no upper limit to the energy it can have and is always at a positive absolute temperature ($T_2 > 0$). In the initial state, with the adiabatic partition in place, subsystem 1 starts at a negative absolute temperature ($T_1 < 0$). (b) The adiabatic partition is replaced by a diathermic partition. In the approach to thermal equilibrium, heat is transferred, on average, from subsystem 1, which is initially at a negative absolute temperature, to subsystem 2 which is always at a positive absolute temperature.

$$\frac{1}{T_1} = \left(\frac{\partial S_1}{\partial U_1}\right)_{V_1,N_1}, \qquad \frac{1}{T_2} = \left(\frac{\partial S_2}{\partial U_2}\right)_{V_2,N_2}$$

As the closed composite system as shown in the above figure, approaches thermal equilibrium on average,

$$dS_1 + dS_2 = \left(\frac{\partial S_1}{\partial U_1}\right)_{V_1,N_1} dU_1 + \left(\frac{\partial S_2}{\partial U_2}\right)_{V_2,N_2} dU_2 > 0$$

Since $U_1 + U_2 =$ constant, we get

$$dU_2 = -dU_1$$

Hence we have

$$\begin{aligned} dS_1 + dS_2 &= [-\left(\frac{\partial S_1}{\partial U_1}\right)_{V_1,N_1} + \left(\frac{\partial S_2}{\partial U_2}\right)_{V_2,N_2}]dU_2 \\ &= (\frac{1}{T_2} - \frac{1}{T_1})dU_2 \\ &= \left(\frac{T_1 - T_2}{T_1 T_2}\right)dU_2 > 0 \end{aligned}$$

where

$$\frac{1}{T_1} = \left(\frac{\partial S_1}{\partial U_1}\right)_{V_1,N_1}, \qquad \frac{1}{T_2} = \left(\frac{\partial S_2}{\partial U_2}\right)_{V_2,N_2}$$

Suppose that T_1 and T_2 are positive temperature. Since $T_1 - T_2 > 0$, dU_2 must be positive. The heat transfers from the hot source (T_1) to the cold source (T_2).

Suppose that the sign of T_1 and T_2 is arbitrary (positive or negative). We have

$$\begin{aligned} dS_1 + dS_2 &= [-\left(\frac{\partial S_1}{\partial U_1}\right)_{V_1,N_1} + \left(\frac{\partial S_2}{\partial U_2}\right)_{V_2,N_2}]dU_2 \\ &= (-\frac{1}{T_1} + \frac{1}{T_2})dU_2 > 0 \\ &= k_B(\beta_1' - \beta_2')dU_2 > 0 \end{aligned}$$

where $\beta' = -\beta = -\frac{1}{k_B T}$

If $\beta_1' > \beta_2'$, dU_2 becomes positive. So the heat transfers from the hot source (T_1) to the cold source (T_2).

We use

$$\boxed{\beta' = -\beta = -\frac{1}{k_B T}}$$

$\beta' = -\dfrac{1}{k_B T}$ (K^{-1})	T (K)
0	$- -\infty$
0.0033	-300
0.01	-100
0.1	-10
1	-1
100	-0.01
300	-0.003
500	-0.002
1000	-0.001
∞	-0
$-\infty$	+0
-1000	0.001
-100	0.01
-1	1
-0.1	10
-0.01	100
-0.003	300
-0.001	1000
0	∞

2. Kittel's comment

We note that negative temperatures correspond to higher energies than positive temperatures. When a positive- and a negative-temperature system are brought into thermal contact, heat will flow from the negative temperatures to the positive temperatures. Thus we say that negative temperatures are hotter than positive temperatures. The temperature scale from cold to hot runs

T = +0 K, …, +300 K, …, $\pm\infty$ K , .., -300 K, …, (-0) K.

Note particularly that, when a body at -300 K is brought into contact with an identical body at 300 K, the final temperature is not 0 K, but is $\pm\infty$ K , the two signs corresponding actually to the same temperature. In many respects $-1/T$ is a more instructive measure of temperature than is T. Increasing values of $-1/T$ correspond to the body's becoming hotter and hotter.

$$-\frac{1}{T} = -\infty \text{ (cold)}, \ldots, -\frac{1}{300}, \ldots, 0, \ldots, +\frac{1}{300}, \quad , \infty \text{ (hot)}$$

When the temperature is negative, the population in the upper energy state is larger than the population in the ground state.