Entropy Change of the Universe
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((**Example-1**))

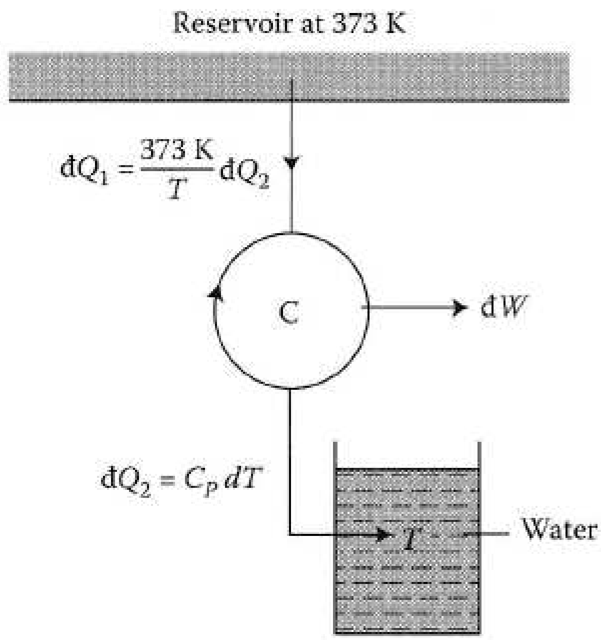


Fig. A beaker of water may be heated reversibly by operating a Carnot engine between it and a reservoir at a higher temperature.

A. Rex, Finn's Thermal Physics (CRC Press)

((**Carnot cycle**))

Suppose that a Carnot engine is operated between the reservoir at 100°C and the water as shown above figure. If the operating cycle of the engine is small, so that the heat dQ_2 rejected by the engine during one cycle causes only an infinitesimally small change dT in the temperature T of the water, then T does not change significantly during one cycle and the required operating conditions for a Carnot engine of operating between a pair of reservoirs exist.

For one Carnot cycle,

$$\frac{dQ_1}{373} = \frac{dQ_2}{T}$$

The temperature of the water changes from T to $T + dT$, where

$$dQ_2 = C_P dT$$

Thus we get

$$dQ_1 = \frac{373}{T} dQ_2 = \frac{373}{T} C_P dT \; .$$

The change of the entropy for the reservoir is

$$S_{reserver} = -\int_{293}^{373} \frac{dQ_1}{373} = -\int_{293}^{373} \frac{1}{T} C_P dT = -C_P \ln(\frac{373}{293})$$

The change of the entropy for the water is

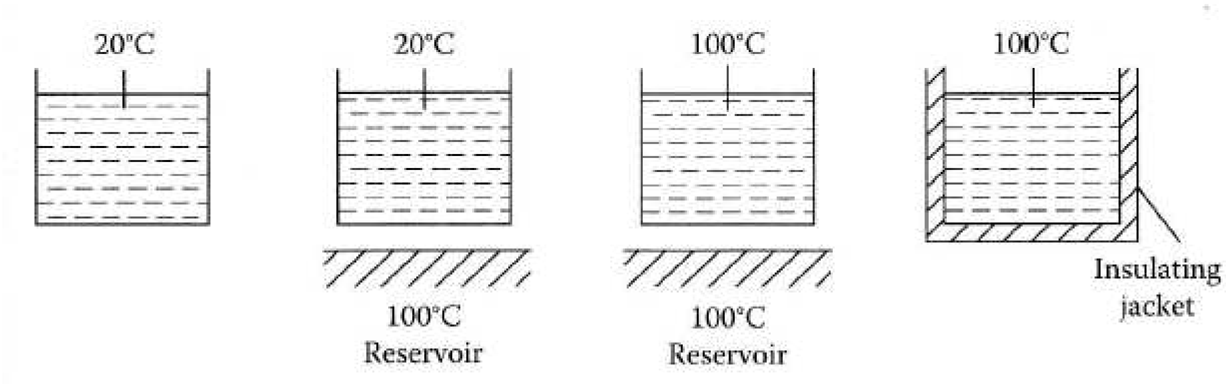
$$S_{water} = \int_{293}^{373} \frac{dQ_2}{T} = \int_{293}^{373} \frac{1}{T} C_P dT = C_P \ln(\frac{373}{293})$$

Then the entropy of the universe is

$$S_{universe} = S_{reserver} + S_{water} = 0$$

as it should be for a reversible process.

((**Example-2**)) **one stage**



We consider the example of heating a beaker of water, with heat capacity C_P from $T_i = 293$ K to $T_f = 373$ K. The net entropy change of the universe is

$$S_{universe} = S_{water} + S_{reserver}$$

The water gains entropy

$$\Delta S_{water} = \int_{293}^{373} \frac{C_P dT}{T} = C_P \ln\left(\frac{373}{293}\right)$$

The reservoir loses entropy

$$\Delta S_{reservoir} = -\frac{1}{373} C_P (373 - 293) = -\frac{80}{373} C_P$$

The entropy change of the universe is

$$\Delta S_{universe} = C_P \ln\left(\frac{373}{293}\right) - \frac{80}{373} C_P = 0.0269286 C_P$$

which is positive.

((**Example-3**)) **2 stages**

If the water is heated in two stages by placing it first on a reservoir at 50°C (=323 K) and, when it has reached that temperature, transferring it to a second reservoir at 100°C (= 373 K) for the final heating?

293 K$\rightarrow$323 K $\rightarrow$373 K

$$\Delta S_{water} = C_P \ln\left(\frac{323}{293}\right) + C_P \ln\left(\frac{373}{323}\right) = C_P \ln\left(\frac{373}{293}\right)$$

$$\Delta S_{reservoir} = -C_P\left(\frac{30}{323} + \frac{50}{373}\right)$$

leading to

$$\Delta S_{universe} = 0.0144783 C_P$$

This is reasonable, because the use of two reservoir is closer to a reversible.

((**Example-4**)) **8 stages**

If the water is heated in eight stages such as

293 K → 303 K →313 K → 323 K → 333 K → 343 K→ 353 K→ 363 K→ 373 K

the entropy change of the universe is

$$\begin{aligned}\Delta S_{universe} &= C_P[\ln\left(\frac{373}{293}\right) - \sum_{n=1}^{8} \frac{80/8}{293+10n}] \\ &= 0.00362285 C_P\end{aligned}$$

((**Example-5**)) **16 stages**

If the water is heated in n = 1 - 16 stages such as

293 +5n (K)

the entropy change of the universe is

$$\begin{aligned}\Delta S_{universe} &= C_P[\ln\left(\frac{373}{293}\right) - \sum_{n=1}^{16} \frac{80/16}{293+5n}] \\ &= 0.00182072 C_P\end{aligned}$$

((**Example-6**)) **80 stages**

If the water is heated in n = 1 - 80 stages such as

293 +n (K)

the entropy change of the universe is

$$\Delta S_{universe} = C_P[\ln\left(\frac{373}{293}\right) - \sum_{n=1}^{80}\frac{80/80}{293+n}]$$
$$= 0.00036563 C_P$$

which tends to zero (reversible).

The second part of $\Delta S_{universe}$ can be written as

$$\int_{293}^{373}\frac{dT}{T} = [\ln T]_{293}^{373} = \ln\left(\frac{373}{293}\right)$$

REFERENCES

A. Rex, Finn's Thermal Physics (CRC Press, 2017)

((**Mathematica**))

Evaluation of the entropy of universe

```
Clear["Global`*"];
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$$h1 = Log\left[\frac{373}{293}\right] - \frac{80}{373};\ h1\ //\ N$$

0.0269286

$$h2 = Log\left[\frac{373}{293}\right] - \left(\frac{30}{323} + \frac{50}{373}\right);\ h2\ //\ N$$

0.0144783

$$h3 = Log\left[\frac{373}{293}\right] - Sum\left[\frac{10}{293 + 10\, n},\ \{n, 1, 8\}\right];$$

h3 // N

0.00362285

$$h4 = Log\left[\frac{373}{293}\right] - Sum\left[\frac{5}{293 + 5\, n},\ \{n, 1, 16\}\right];$$

h4 // N

0.00182072

$$h5 = Log\left[\frac{373}{293}\right] - Sum\left[\frac{1}{293 + n},\ \{n, 1, 80\}\right];$$

h5 // N

0.00036563

$$h6 = Log\left[\frac{373}{293}\right] - Sum\left[\frac{0.5}{293 + 0.5\, n},\ \{n, 1, 160\}\right];$$

h6 // N

0.000182908