Greenhouse effect

Masatsugu Sei Suzuki
Department of Physics
(Date: September 21, 2018)

1. Solar constant (p_0)

The solar constant is a flux density measuring mean solar electromagnetic radiation (solar irradiance) per unit area. It is measured on a surface perpendicular to the rays, one astronomical unit (AU) from the Sun (roughly the distance from the Sun to the Earth). It is measured by satellite as being 1361 W/m² at solar minimum and approximately 0.1% greater (roughly 1362 W/m²) at solar maximum.

Solar constant is the energy flux from the Sun to the Earth, energy per unit area per unit time. The luminosity of the Sun (which is the total energy per unit time from the Sun) is given by

$$L_{sun} = 4\pi R_{sun}{}^2 \sigma_B T_{sun}{}^4 = 4\pi d^2 p_0 = 3.828 \times 10^{26} \text{ W.}$$

So we have

$$p_0 = \frac{4\pi R_{sun}{}^2 \sigma_B T_{sun}{}^4}{d^2} = =1361.17 \text{ W/m}^2$$

The luminosity of the Earth is

$$L_{earth} = 4\pi R_E{}^2 \sigma_B T_E{}^4$$

which is equal to $\pi R_E{}^2 p_0$;

$$L_{earth} = \pi R_E{}^2 p_0 = 4\pi R_E{}^2 \sigma_B T_E{}^4$$

or

$$J_0 = \frac{p_0}{4} = \sigma_B T_E{}^4$$

The real temperature of the Earth is determined from the relation

$$(1-r)(\pi R_E{}^2 p_0) = 4\pi R_E{}^2 \sigma_B T_0^4$$

or

$$(1-r)\frac{p_0}{4} = \sigma_B T_0^{\ 4}$$

where r is the reflection constant. When $r = 0.3$, we have $T_0 = 255$ K.

The solar constant is not really constant. There is a small variation, typically less than 1W/m^2, due to the change of the solar output over an 11-year solar cycle, as well as variations on cosmological time scales. The solar constant is defined as the amount of solar radiation received outside the Earth's atmosphere at the Earth's mean distance from the Sun. The Earth's orbit is slightly eccentric, which leads to a variation of solar irradiance from 1412W/m^2 in early January to 1321W/ m^2 in early July.

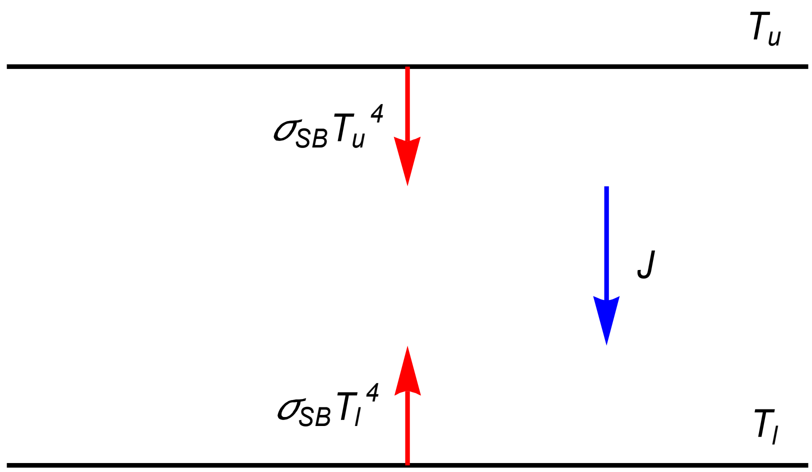
---

2. Example:
6-12
Ralph Baierlein; Thermal Physics

12. *Radiation shield: black surfaces.* Two large plane surfaces, at fixed temperatures T_{hot} and T_{cold}, face each other; a narrow evacuated gap separates the two black surfaces.

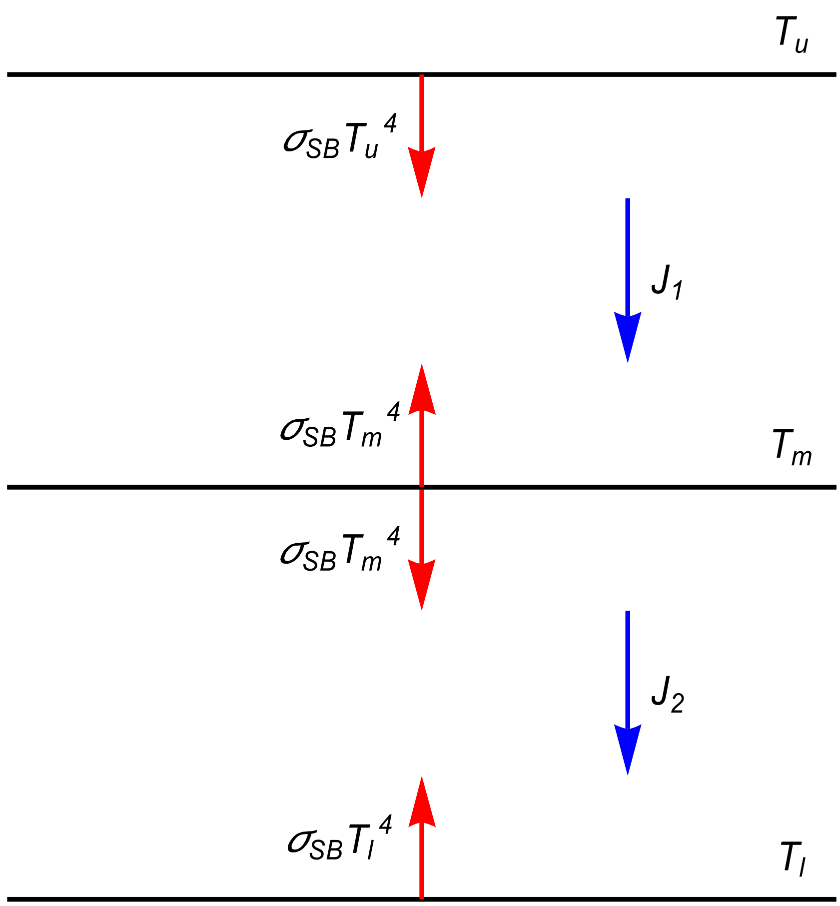
(a) Determine the net radiant energy flux from the hotter to the colder surface (in watts per square meter).
(b) Specify now that a thin metallic sheet, black on both sides, is inserted between the original two surfaces. When the sheet has reached thermal equilibrium, (1) what is the temperature of the metallic sheet and (2) what is the new net radiant energy flux (in terms of T_{hot} and T_{cold})?
(c) If n such black metallic sheets are inserted, what is the net radiant energy flux?

(a)



$J_u = \sigma_{SB}(T_u^{\ 4} - T_l^4)$

(b)



$J_1 = \sigma_{SB}(T_u^{\ 4} - T_m^{\ 4})$, $\qquad J_2 = \sigma_{SB}(T_m^{\ 4} - T_l^4)$

The third black plane is allowed to come to a steady state temperature,

$$J_1 = J_2$$

or

$$\sigma_{SB}(T_u^4 - T_m^4) = \sigma_{SB}(T_m^4 - T_l^4)$$

or

$$T_m = \left(\frac{T_u^4 + T_l^4}{2}\right)^{1/4}$$

Thus we have

$$J_1 = J_2 = \frac{1}{2}\sigma_{SB}(T_u^4 + T_l^4)$$

(c)

$$J_1 = \sigma_{SB}(T_u^4 - T_1^4),$$

$$J_2 = \sigma_{SB}(T_1^4 - T_2^4)$$

$$J_3 = \sigma_{SB}(T_2^4 - T_3^4)$$

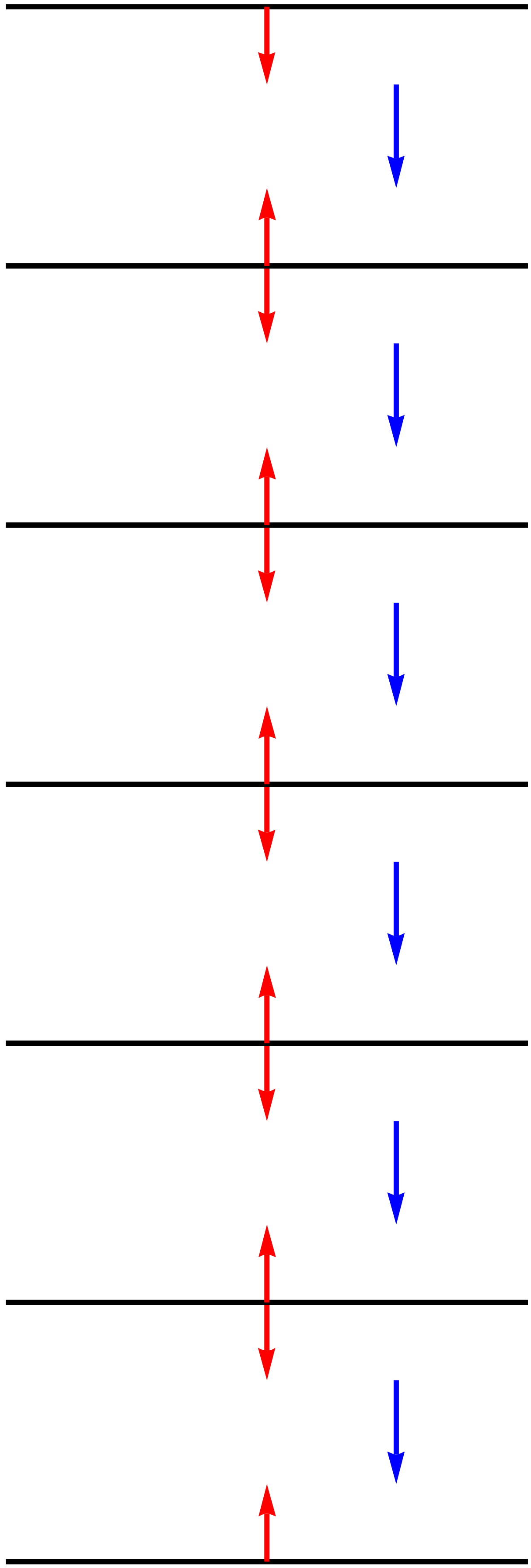
$$\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots\cdots$$

$$J_N = \sigma_{SB}(T_{N-1}^4 - T_N^4)$$

$$J_{N+1} = \sigma_{SB}(T_N^4 - T_l^4)$$

Note that $J_1 = J_2 = \ldots = J_{N+1} = J$ for the steady flow of energy. Addition of both sides of these equations leads to

$$J = \frac{1}{N+1}\sigma_{SB}(T_u^{\ 4} - T_l^{\ 4})$$



3. Example-II

6-13 Ralph Baierlein

13. *Radiation shield: reflective surfaces*. The context is the same as in problem 12 except that all surfaces are partially reflective and have the same absorptivity a, where $a < 1$.

(a), (b), (c) Repeat parts (a), (b), and (c) of problem 12.

(d) Specify that liquid helium at 4.2 K fills a spherical tank of radius 30 cm and is insulated by a thin evacuated shell that contains 60 layers of aluminized plastic, all sheets slightly separated from one another. Take the absorptivity of the aluminum coating to be $a = 0.02$. The outer surface is at room temperature. If the only significant energy influx is through the radiation shield, how long will it take before all the helium has evaporated? [A value for the energy required to evaporate one atom of ^{4}He (the latent heat of vaporization, L_{vap}) is provided by table 12.2, and the number density N/V can be extracted from table 5.2.]

See the solution of Homework.

---

4. Radiative transfer

A vacuum or Dewar flask (sometimes called thermo flask) consists of a cylindrical container surrounded by a thin evacuated jacket. The entire surface of the evacuated jacket with a material such as silver, which is opaque and has a high reflectivity. The reflectivity is r, and the temperatures inside and outside the flask are T_a and T_b, respectively. What is the energy flux (the rate of radiative energy transfer per unit area)?

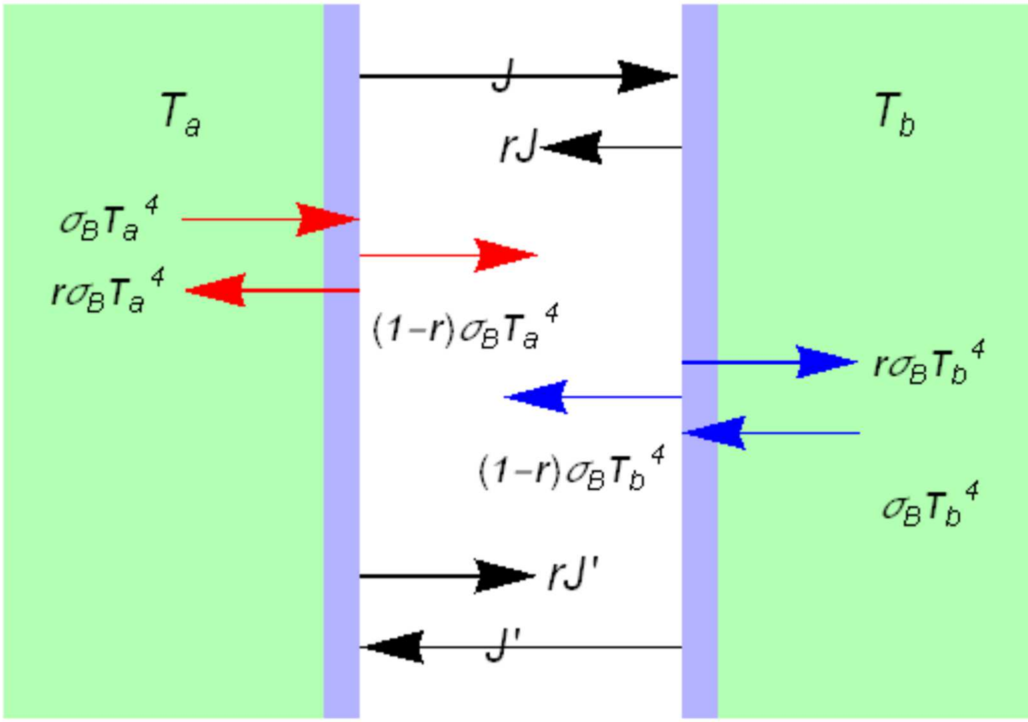
Since the separation between the walls is much less than the diameter of the cylinder, they can be treated as plane parallel surfaces. By the Kirchhoff and Stefan-Boltzmann law, the inner surface radiates at a rate $(1-r)\sigma_B T_a^4$ per square meter, and the outer one at $(1-r)\sigma_B T_b^4$. Each surface reflects a fraction r of the radiation falling on it. Let the total ingoing flux be J and the total outgoing flux be J'. Then

$$J + rJ' = (1-r)\sigma_B T_a^4, \qquad J' + rJ = (1-r)\sigma_B T_b^4$$

The net ingoing flux is thus

$$J - J' = \frac{1-r}{1+r}\sigma_B (T_a^4 - T_b^4)$$

If $r = 0.98$, the net flux is reduced to about 1% of the value it would have if the surfaces were black bodies.



REFERENCES

M.D. Sturge, Statistical and Thermal Physics: Fundamentals and Applications (A K Peters, 2003).

---

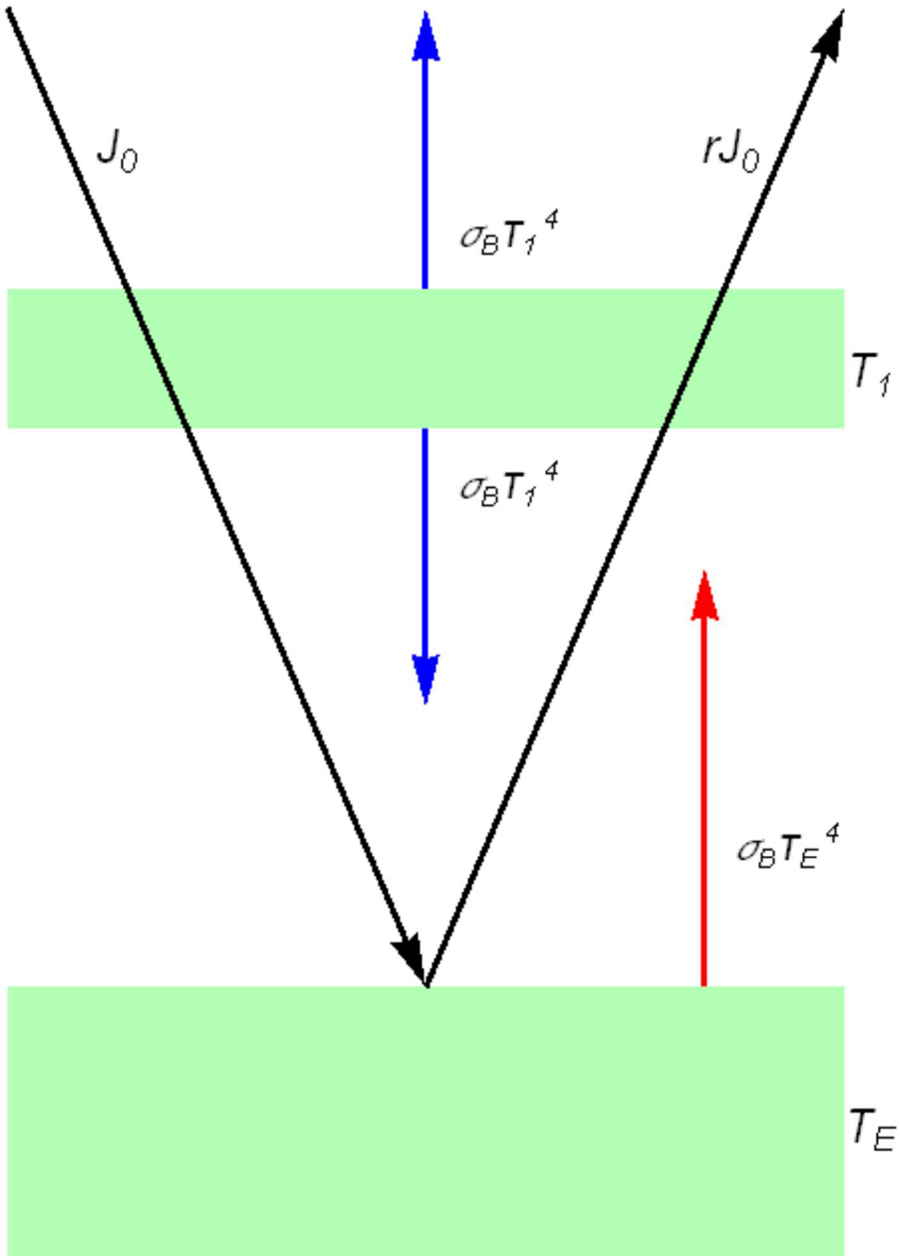
5. Greenhouse effect

Schroeder: Thermal Physics p.307

The equilibrium temperature of 255 K applies (roughly) to the atmosphere, while the surface below is heated both by the incoming sunlight and by the atmospheric "blanket." If we model the atmosphere as a single layer that is transparent to visible light but opaque to infrared, we get the situation in **Fig.** as shown below. Equilibrium requires that the energy of the incident sunlight (minus what is reflected) be equal to the energy emitted upward by the atmosphere, which in turn is equal to the energy radiated downward by the atmosphere. Therefore, the earth's surface receives twice as much energy (in the simplified model) as it would from sunlight alone. This mechanism raises the surface temperature by a factor of $2^{1/4}$, to 303 K. This is a bit high, but then, the atmosphere is not just a single perfectly oparaque layer. By the way, this mechanism is called the greenhouse effect, even though most greenhouses depend primarily on a different mechanism (namely, limiting convective cooling).

((**Problem**))

Suppose that the concentration of infrared-absorbing gases in earth's atmosphere were to double, effectively creating a second "blanket" to warm the surface. Estimate the equilibrium surface temperature of the earth that would result from this catastrophe.



((**Energy conservation**))

On the surface of the Earth:

$$J_0 - rJ_0 + \sigma_B T_1^4 - \sigma_B {T_E}^4 = 0 \qquad (1)$$

At the atmosphere (1)

$$2\sigma_B T_1^4 = \sigma_B {T_E}^4 \qquad (3)$$

At the top of the atmosphere (1)

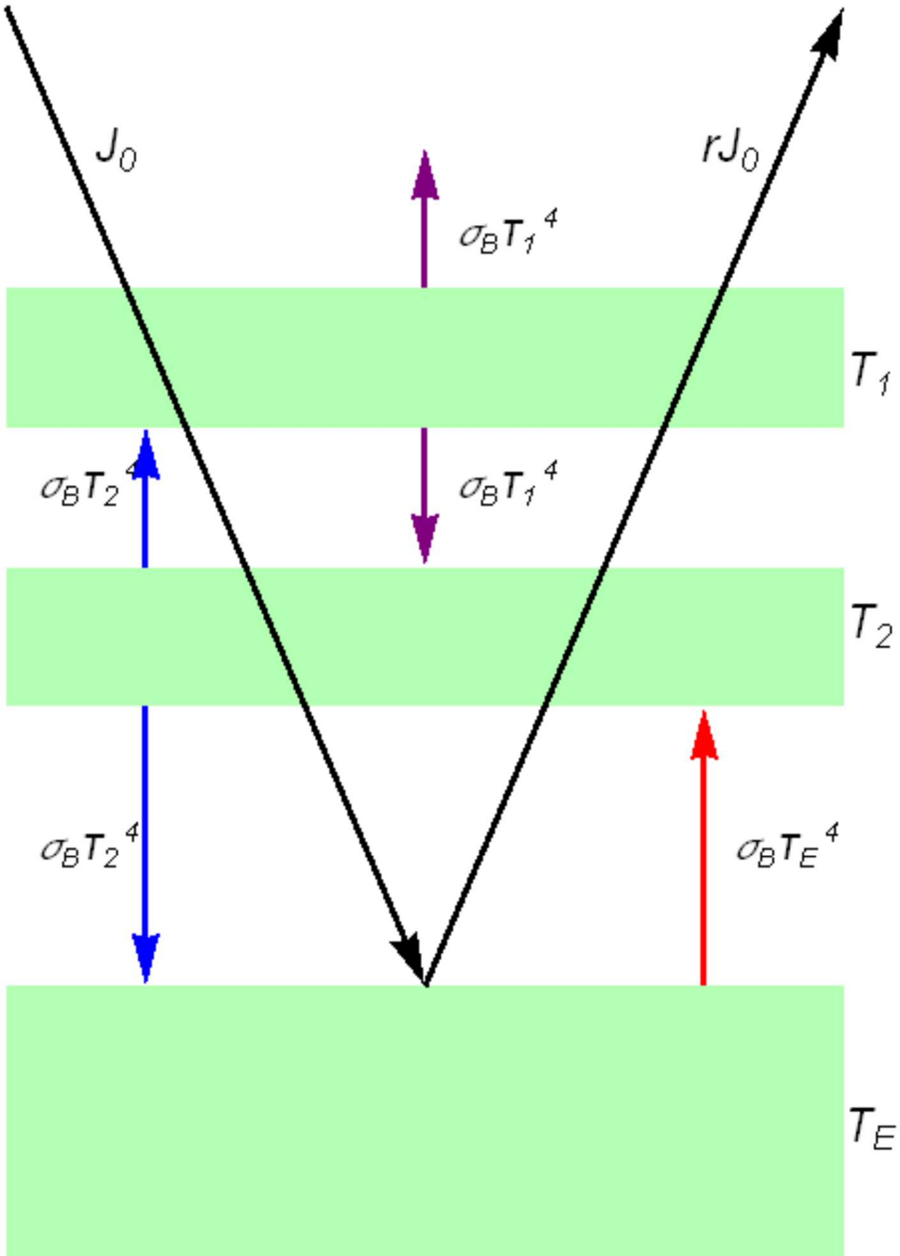
$$J_0 - rJ_0 = \sigma_B T_1^4 \qquad (4)$$

From these equations, we get

$$T_1 = [\frac{(1-r)}{\sigma_B} J_0]^{1/4} = [\frac{(1-r)}{4\sigma_B} p_0]^{1/4} = 255 \text{ K}$$

$$T_E = 2^{1/4} T_1 = 1.1892 T_1 = 303.2 \text{ K}$$

6. Greenhouse effect (II)



((**Energy conservation**))

On the surface of the Earth:

$$J_0 - rJ_0 + \sigma_B T_2{}^4 - \sigma_B T_E{}^4 = 0 \qquad (1)$$

At the atmosphere (2):

$$2\sigma_B T_2{}^4 = \sigma_B T_E{}^4 \quad (2)$$

At the atmosphere (1)

$$2\sigma_B T_1^4 = \sigma_B T_2{}^4 \quad (3)$$

At the top of the atmosphere (2)

$$J_0 - rJ_0 = \sigma_B T_1^4 \quad (4)$$

From these equations, we get

$$T_1 = [\frac{(1-r)}{4\sigma_B} p_0]^{1/4} = 255\,\mathrm{K}$$

$$T_E = 3^{1/4} T_1 = 1.31607 T_1 = 335\ \mathrm{K}, \qquad T_E = \left(\frac{3}{2}\right)^{1/4} T_1 = 1.10668 T_1 = 282.2\ \mathrm{K}$$

REFERENCES

M.H.P. Ambaum, Thermal Physics of the Atmosphere (Wiley-Blackwell, 2010).
M.D. Sturge, Statistical and Thermal Physics: Fundamentals and Applications (A K Peters, 2003).

APPENDIX Temperature of the Earth

The radius of the Earth:

$$R_{Earth} = 6.372 \times 10^6\,\mathrm{m}$$

The radius of the Sun:

$$R_{Sun} = 6.9599 \times 10^8\ \mathrm{m}$$

The distance between the Sun and the Earth:

$$d = \mathrm{AU} = 1.49597870 \times 10^{11}\ \mathrm{m}$$

The Stefan Boltzmann constant

$$\sigma_B = 5,670400 \times 10^{-8} \text{ W/m}^2$$

The temperature of the Earth

$$(\sigma_B T_{sun}{}^4)(R_{Sun}{}^2 d\Omega) = (\sigma_B T_E{}^4)(4\pi R_E{}^2)$$

where the solid angle is

$$d^2 \Delta\Omega = \pi R_E{}^2$$

Thus we have

$$(\sigma_B T_{sun}{}^4)(R_{Sun}{}^2 \frac{\pi R_E{}^2}{d^2}) = (\sigma_B T_E{}^4)(4\pi R_E{}^2)$$

or

$$T_E{}^4 = T_{sun}{}^4 (\frac{R_{Sun}{}^2}{4d^2})$$

or

$$T_E = \sqrt{\frac{R_{Sun}}{2d}} T_{sun} = 282.15 \text{ K.}$$