Problem sets

Problem set 1

Write your answers in the indicated spaces. Use a calculator to get your final answers, but I want you to ***write out the algebraic formulas*** you use to get the answers.

1) A bond that promises a payment of $1,000,000 exactly one year from today can be bought for $900,100 today.

a) What is the yield to maturity on this bond? _____

b) Given the answer you got for part a), what price would you expect to receive today if you sold a bond that promises a payment of $50,000 exactly one year from today?

_____

2) A bond that promises a payment of $5,000,000 exactly nine years from today can be bought for $1,000,000 today. What is the yield to maturity on this bond?

_____

3) Suppose the yield to maturity for bonds that pay off one year from today is 5%, and the yield to maturity for bonds that pay off two years from today is 10%. What will be the market price of a bond that makes *two* payments: a payment of $100 one year from today, and a payment of $1,000 two years from today?

_____

Problem set 2

Write your answers in the indicated spaces.

1) Consider a zero-coupon bond that promises a payment of $1,000,000 exactly one year from today.

a) If the current market yield for one-year, zero-coupon bonds is 6%, what is the current price of the bond?

_____

b) Suppose that tomorrow the market yield for one-year, zero-coupon bonds rises to 10%. What will the price of the bond be tomorrow?

_____

c) What is the percent change in the price of the bond from today to tomorrow? _____

2) Consider a zero-coupon bond that promises a payment of $1,000,000 exactly five years from today.

a) If the current market yield for five-year, zero-coupon bonds is 6%, what is the current price of the bond?

_____

b) Suppose that tomorrow the market yield for five-year, zero-coupon bonds rises to 10%. What will the price of the five-year bond be tomorrow?

_____

c) What is the percent change in the price of the bond from today to tomorrow? _____

3) Compare your answers in parts one and two. Would you have lost more money if you bought one-year bonds and sold them tomorrow, or if you bought five-year bonds and sold them tomorrow?

Problem set 3

The following table describes peoples' beliefs, as of today, about the path of overnight interest rates over the next five years.

Path	A	B	C
Probability	1/3	1/3	1/3
Year	Overnight interest rates		
1	5	5	5
2	7	4	6
3	7	4	7
4	7	4	6
5	7	4	5

1) What is the expected value of the average overnight rate over the next 5 years? _____

2) Consider a zero-coupon bond that will pay off $1,000 at the end of five years. Suppose that the price of the bond equates the bond's yield to maturity to the expected value of the average overnight rate over the next five years.
What is the bond's yield to maturity? _____

What is the price of the bond? _____

3) Suppose you don't buy the bond. Instead, you take the money it would cost to buy the bond and make overnight loans, rolling over the interest and principal into more overnight loans, for five years. What is the probability that, at the end of five years, this will give you

a) *more* than $1,000 (the payoff from the bond) _____

b) *less* than $1,000 (the payoff from the bond) _____

4) Suppose you do buy the bond. What will happen to the price of the bond *tomorrow* if:

a) it becomes *certain* that overnight rates will follow path A. _____

b) it becomes *certain* that overnight rates will follow path B _____

Problem set 4

1) Suppose the "expectations hypothesis" is completely correct, that is all people care about is the expected values of returns of their investment. The IBM corporation has issued bonds in the past. There is an IBM bond that promises to pay the bearer $300 one year from now. IBM is in trouble. There is a probability of fifty percent (one half) that it will be bankrupt within the year, in which case it will not pay the IOU on that bond. Calculate today's market price of the bond assuming today's yield on one-year zero-coupon U.S. Treasury bonds is 50 percent. Show your calculations below.

$_____

2) Again suppose the "expectations hypothesis" is correct. The city of Binghamton has issued bonds. Financial market participants believe there is a chance Binghamton will default (totally default) on its bonds, due to population loss, disappearance of the tax base and opioid abuse. What is the perceived probability that Binghamton will default if:
- today's yield on one-year zero-coupon Treasury bonds is 50 percent
- today's market price of a one-year zero-coupon Binghamton bond promising to pay $200 in one year is $100.

Show your calculations below.

_____ (probability of default)

Problem set 5

Suppose there are 100 zero-coupon bonds with the same maturity (duration), issued by 100 private corporations. I want you to consider how the average yield on the bonds will change from Monday to Tuesday as described below. (The average yield on these 100 bonds is the number you get by adding up all the 100 yields and dividing by 100).

On Monday, *everyone* knows that 50 of these corporations will be bankrupt before the bonds are due. But *no one* knows which 50 it will be. And everyone knows that no one knows.

On Tuesday, *some* people receive secret information that tells them exactly which 50 corporations will go bankrupt. *Everyone* knows that some people have received this secret information. But *no one* knows who has received the information. (If you received the information you know you received it, but you don't know who else received it.)

From Monday to Tuesday, will the average yield on the 100 bonds rise, fall, or remain the same? Explain.

Problem set 6

Baumol-Tobin Model

For the model presented in class, we assumed that each financial transaction imposes a cost F on the person in the story.

For this problem, make a slightly different assumption. Assume that the cost of a financial transaction is higher for a person with a higher income. Thus, the cost of one financial transaction is fY. The total cost of N financial transactions per year is NfY. With this slightly different assumption,

1) Derive N^* using calculus.

2) Using N^* from 1), write down the person's money demand $(M/P)^D$.

3) Compare your answer to 2) with the money-demand function we derived in class. How is it different?

Problem set 7
Financial intermediation and interest-rate risk

You need a calculator or Excel for this problem set.

Suppose that today the overnight rate ${}_0i$ is 2 percent. People think there is a 1/3 probability ${}_0i$ will remain 2 percent for at least two years; a 1/3 probability ${}_0i$ will be immediately cut to zero percent and remain there for at least two years; and a a 1/3 probability ${}_0i$ will be immediately hiked to 4 percent and remain there for at least two years.

1) What is the price today of a two-year zero-coupon Treasury bond with an IOU of $1,000 assuming the two-year term premium is 1/2 percent?

$_____

2) You decide to borrow overnight to finance a purchase of this Treasury bond and profit from the term premium. Today you will borrow enough money overnight to pay for the Treasury bond. You will keep rolling over your overnight borrowing until the bond matures: every morning you will pay off the principal *and* interest due on the previous day's overnight borrowing with *more* overnight borrowing, day after day. On the day the bond matures, you will take the $1,000 IOU, pay off your accumulated overnight-loan debt, and keep the rest as your profit.

a) If the overnight rate happens to remain 2 percent for at least two years, how much money will you be paying off in accumulated debt on the day the bond matures? (Hint: it is the same as the amount of money you would have at the end of two years *lending* overnight, if you started with with the amount given by your answer to 1).)

Amount you must repay two years from now: $_____

b) What will your profit be? $_____

3) Now consider something that might happen one year from today, that is halfway through your two-year plan. Suppose that at that time the overnight rate is still 2 percent and people still think that, looking forward, there is a 1/3 probability ${}_0i$ will remain 2 percent for at least two years; a 1/3 probabability ${}_0i$ will be immediately cut to zero percent and remain there for at least two years; and a 1/3 probabaility ${}_0i$ will be immediately hiked to 4 percent and remain there for at least two years.

a) What is the market price of your bond at that point in time, assuming the term premium on one-year bonds is 1/4 percent?

$_____

b) Suppose that for some reason people stop lending to you. They demand all of their money back, and you can't borrow overnight any more. You sell the bond to pay back the overnight loans and keep what is left over. How much will be left over for you to keep?

$_____

4) Let's return to the situation described in 3), but with a change. On the day people stop lending to you the overnight rate is still 2 percent, but expectations for the future are different. People think that, looking forward, there is a 1/2 probability ${}_0i$ will remain 2 percent for at least two years, and a 1/2 probabability ${}_0i$ will be immediately hiked to 4 percent and remain there for at least two years. When you sell the bond to pay back the overnight loans, how much is left for you to keep?

$_____

Problem set 8
Financial intermediaries

1) The three paragraphs below describe the situation of three different financial intermediaries.

National Bank of Hazard. This bank has taken $4 billion in checkable deposits. (That is, it has borrowed $4 billion in the form of checkable deposits.) It has also borrowed $1 billion through overnight loans. Its has $1 billion in cash in its vault, $2 billion in short-term U.S. Treasury bills, and has lent $5 billion in long-term loans to businesses.

Lizard Brothers Investment Bank. This investment bank has borrowed $2 billion by issuing long-term bonds, and $3 billion through overnight loans. Its only assets are $6 billion in long-term U.S. Treasury bonds.

Edward Bear Investment Bank. This investment bank has borrowed $2 billion from other banks in long-term loans, and $3 billion through overnight loans. Its only assets are $4 billion in Treasury bills.

a) For each institution, fill out the table below.

	Capital	Reserves	Secondary reserves
Natl. Bank of Hazard	$_____	$_____	$_____
Lizard Brothers	$_____	$_____	$_____
Edward Bear	$_____	$_____	$_____

b) One of these institutions is *insolvent.* Which one?

b) One of these institutions is solvent and its assets are perfectly liquid, but it is subject to interest-rate risk. Which one?

2) Consider a financial intermediary which has been borrowing overnight to fund purchases of relatively illiquid bonds. It has been borrowing from two lenders: Warren Buffett and Scrooge McDuck. Each has been lending the intermediary $D. Each morning, each must choose whether to roll over his loans to the intermediary - that is, to lend $D for another day - or to withdraw the funds from the intermediary. If both Buffett and McDuck to roll over their loans, the intermediary will stay in business and pay them both the overnight interest rate *i*. That is, each will receive $(1+i)D. If either or both of the lenders withdraws, the intermediary will have liquidate its bondholdings at low prices. If just one withdraws, the lender who withdraws will get his $D back with no interest; the lender who rolls over his loans will get nothing. If *both* lenders withdraw, each will get half of his $D back, with no interest.

a) Draw a set of 4 boxes that describes this situation, as we did in class.

b) Which of the 4 boxes above is a possible equilibrum? Explain.

Problem set 9
Reserve demand

1) Consider the demand for reserves and determination of the market overnight interest rate in an economy where the central bank pays an interest rate r_D on reserve balances and charges an interest rate r_P for emergency loans to cover overdrafts. r_D is *lower* than the central bank's target overnight rate r_T. r_P is *higher* than the central bank's target overnight rate r_T.

a) Draw a graph that shows reserve demand and the reserve supply that will cause the market overnight rate r to hit the central bank's target.

b) Suppose the central bank's policy commitee raises the target overnight rate r_T while making *no* change to r_D or to r_P. (It raises r_T only a little, so that it is still between r_D and r_P.)

Draw a graph that describes this event, and what is likely to happen to reserve supply.

c) Now suppose that the central bank always adjusts r_D and r_P when it changes r_T : r_P is always equal to r_T *plus* one percent; r_D is always equal to r_T *minus* one percent.

Draw a graph that describes this event, and what is likely to happen to reserve supply.

2) Consider a bank that has total funds F to divide between its reserve account at the central bank and overnight lending. The bank receives an interest rate r on overnight lending. If the bank puts a sum R in its reserve account, it has $(F-R)$ left to lend out overnight, giving earnings of $(F-R)r$.

The central bank does *not* pay interest on reserves. After the end of the day, the central bank clears payments between banks, adding a net sum P to the bank's reserve account, where P can be a negative number. That leaves $R+P$ in the bank's reserve account. From the bank's point of view, P is a random variable, uniformly distributed between a minimum value (the smallest possible net payment into the bank's reserve account) of -10, and a maximum value (the largest possible payment into the bank's reserve account) of $+10$.

The reserve requirement is 5. If the balance in the bank's reserve account falls below 5 after clearing, the bank must take an emergency loan from the central bank to cover the shortfall. The central bank charges an interest rate r_p for emergency loans to cover overdrafts.

a) What is the smallest quantity of reserves that the bank will choose to hold if the market overnight rate r is equal to zero?

b) What is the largest quantity of reserves that the bank will choose to hold if the market interest rate r is as high as the central bank's emergency lending rate r_p?

c) Given a value of R somewhere between the values in a) and b), what is the probability that a bank will have a shortfall in its reserve account? Check: a higher value of R should make this probability *smaller*.

d) Assuming a bank runs an overdraft in its reserve account, what is the expected value of the amount that the bank will have to borrow from the central bank?

Problem set 10
More about reserve demand

Suppose the Fed does *not* pay interest on reserves. There is no reserve requirement. The Fed charges an interest rate for emergency loans
r_p to cover overdrafts. This interest rate is equal to *2.* That is, $r_p = 2$. All banks in the country are identical. Each bank has *$100* to divide between its reserve account and overnight lending. At 5 pm each bank will choose how much to leave in its reserve account.Between 5 and 6 pm. the Fed will clear payments between banks, adding a net sum P to each bank's reserve account. P can be a postive or negative number. That leaves $R+P$ in the bank's reserve account at 6 pm. A bank will have overdrawn its reserve account if the balance after clearing, at 6 pm, falls below zero..A bank that overdraws its reserve account must take an emergency loan from the Fed to cover the overdraft, to bring its reserve account up to a zero balance. From a bank's point of view, P is a random variable, uniformly distributed between a minimum value (the smallest possible net payment into the bank's reserve account) of *-2*, and a maximum value (the largest possible payment into the bank's reserve account) of *+2*. The market overnight rate is denoted *r.*

1) Using the information given above, write an expression that gives the probability that a bank will run an overdraft in its reserve account, for any given value of *R.,* assuming *r* is greater than zero but less than 2.

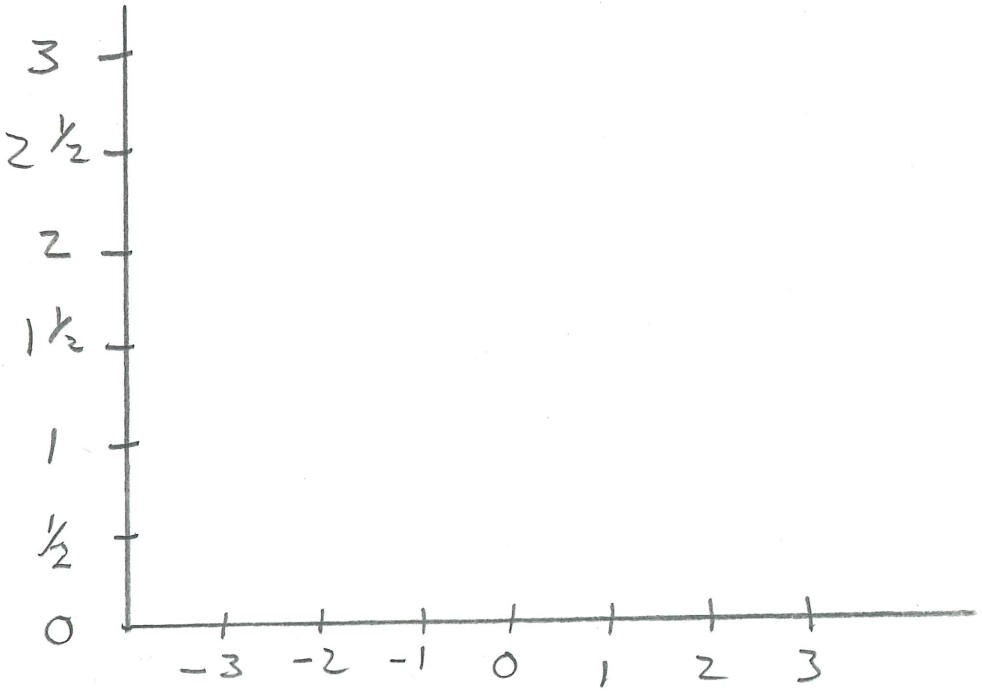
2) *Assuming* a bank runs an overdraft in its reserve account, what is the expected value of the amount that the bank will have to borrow from the Fed, for any given value of *R*?

3) Using your answers to a) and b), write an expression that gives, for any value of *R,* the expected value of the bank's profit. Remember $r_p = 2$!

4) Using your answer to 3) and calculus and algebra, find the reserve balance R^D that a bank would choose to leave in its reserve account at 5 pm, as a function of r.

5) Suppose the target overnight rate is 1/2. What is the reserve supply per bank that will cause the market overnight rate to hit the target?

6) On the graph below, draw a bank's reserve demand curve, and the reserve supply per bank that will cause the market overnight rate to hit the target. Be precise; notice the numbers on the axes.



Problem set 11
An analogue to the reserve demand model

Suppose you are planning a party. Let N denote the number of people who will come to the party. You are not sure what N will be - you are not sure how many people will come to the party - but you have a probability distribution for N. This distribution is *uniform*. The largest possible value of N (the most people who will possibly come) is 200. The smallest possible value of N (fewest people who will possibly come) is zero.

1) Let Z denote a number between zero and 200. What is the probability that the number of people who come to the party N is less than or equal to Z?

2) What is the probability that N turns out to be *more* than Z?

3) *Assuming* more than Z people come to the party, what is the expected value of N (that is, on the condition that $N>Z$)? (This is the same as the expected value of N assuming Z or more people come to the party.)

4) Each guest will want to drink exactly one case of beer. (You yourself will drink no beer.) You can buy beer the day before the party at a store where beer is cheap. You don't want to buy too much beer because your mother is coming to visit the morning after the party and you don't want her to know you keep beer in your apartment. Let B denote the number of cases you buy the day before the party. Assuming B is between zero and 200, what is the probability you will run out of beer at the party?

5) If you run out of beer at the party, you will have to run to an expensive beer store next to your apartment building and buy the cases you lack. Given B (between zero and 200), what is the expected value of the number of cases you will have to buy from the expensive beer store *assuming* you run out of beer at the party (that is, *on the condition* that you run out of beer)?

6) Suppose that the price of a case of beer at the cheap beer store is P. The price of a case at the expensive store next to your aparment building is $\hat{P}$. ($\hat{P}$ is greater than P.) Write a mathematical expression that gives the expected value of the total cost of beer that you buy, that is the total cost of beer from the cheap store plus the beer you have to buy from the expensive store, given B. (B still denotes the number of cases you buy at the cheap store, between zero and 200).

7) When you are at the cheap beer store, you will buy the amount of beer that minimizes the expected value of the total cost of beer as you defined it in part 6). Using your answer to 6), figure out the number of cases you will buy at the cheap beer store (that is the optimal value of B).

Problem set 12
Inflation targeting

Suppose Fed policymakers follow an inflation targeting strategy. The target inflation rate π^T is 2 percent. Fed policymakers know, for sure, that the natural rate of interest r^* is one percent and the natural rate of unemployment (or NAIRU) u^* is five percent.

1) Suppose expected inflation π^e is 2 percent. Given the way the Fed will react to the situation,

a) What is the unemployment rate likely to be equal to? _____

b) What is the target fed funds rate likely to be equal to? _____

2) Suppose expected inflation π^e is 4 percent. Given the way the Fed will react to the situation,

a) Is the unemployment rate likely to be less than, greater than or equal to 5 percent?_____

b) Is the target fed funds rate likely to be less than, greater than or equal to 3 percent?_____

3) Suppose expected inflation π^e is 0 percent. Given the way the Fed will react to the situation,

a) Is the unemployment rate likely to be less than, greater than or equal to 5 percent?_____

b) Is the target fed funds rate likely to be less than, greater than or equal to 3 percent?_____

4) Suppose expected inflation π^e is equal to 2 percent. State whether each of the following pieces of incoming news is likely to cause the FOMC to raise, lower or not change the target fed funds rate.

a) Congress will raise taxes next year, without changing spending. _____

b) Congress will raise spending next year, without changing taxes. _____

c) The stock market is crashing. _____

5) State whether each of the events listed in 4) is likely to be associated with a steepening, flattening, or unchanged yield curve, *assuming* that each economic development is expected to be temporary. (For example, the tax hike in a) is expected to be reversed eventually.)

a) _____ b) _____ c) _____