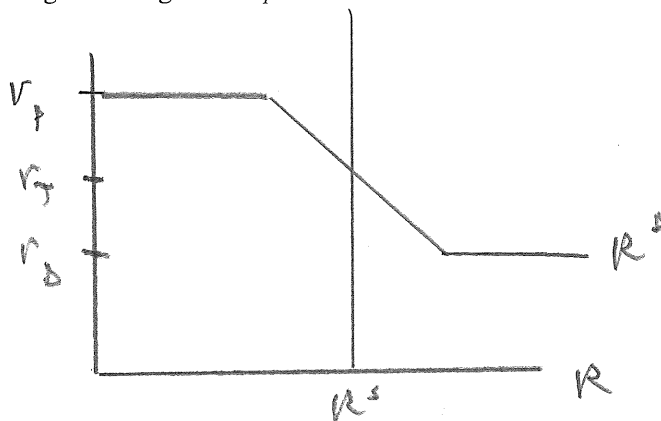


Problem set 9
Reserve demand

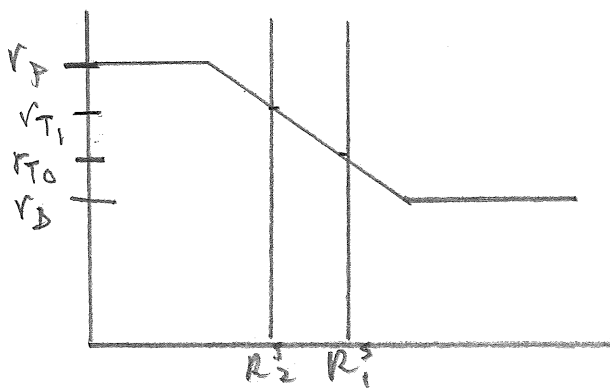
1) Consider the demand for reserves and determination of the market overnight interest rate in an economy where the central bank pays an interest rate r_D on reserve balances and charges an interest rate r_P for emergency loans to cover overdrafts. r_D is lower than the central bank's target overnight rate r_T . r_P is higher than the central bank's target overnight rate r_T .

a) Draw a graph that shows reserve demand and the reserve supply that will cause the market overnight rate r to hit the central bank's target.



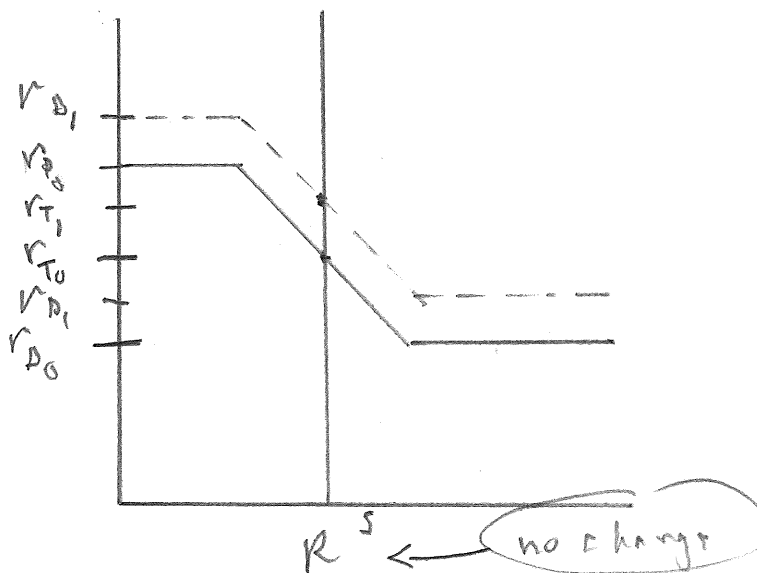
b) Suppose the central bank's policy committee raises the target overnight rate r_T while making no change to r_D and r_P . (It raises r_T only a little, so that it is still between r_D and r_P .)

Draw a graph that describes this event, and what is likely to happen to reserve supply.



c) Now suppose that the central bank always adjusts r_D and r_P when it changes r_T : r_P is always equal to r_T plus one percent; r_D is always equal to r_T minus one percent.

Draw a graph that describes this event, and what is likely to happen to reserve supply.



2) Consider a bank that has total funds F to divide between its reserve account at the central bank and overnight lending. The bank receives an interest rate r on overnight lending. If the bank puts a sum R in its reserve account, it has $(F-R)$ left to lend out overnight, giving earnings of $(F-R)r$.

The central bank does *not* pay interest on reserves. After the end of the day, the central bank clears payments between banks, adding a net sum P to the bank's reserve account, where P can be a negative number. That leaves $R+P$ in the bank's reserve account. From the bank's point of view, P is a random variable, uniformly distributed between a minimum value (the smallest possible net payment into the bank's reserve account) of -10 , and a maximum value (the largest possible payment into the bank's reserve account) of $+10$.

$$\bar{P} = 10, \quad -\bar{P} = -10$$

The reserve requirement is 5. If the balance in the bank's reserve account falls below 5 after clearing, the bank must take an emergency loan from the central bank to cover the shortfall. The central bank charges an interest rate r_p for emergency loans to cover overdrafts. $K = 5$

a) What is the smallest quantity of reserves that the bank will choose to hold if the market overnight rate r is equal to zero? 15 (Even if $P = -\bar{P}$, $R+P \geq 5$)

b) What is the largest quantity of reserves that the bank will choose to hold if the market interest rate r is as high as the central bank's emergency lending rate r_p ? -5 (Even if $P = \bar{P}$, $R+P \leq 5$)

c) Given a value of R somewhere between the values in a) and b), what is the probability that a bank will run a shortfall in its reserve account? Check: a higher value of R should make this probability *smaller*.

prob $R+P < K$ is

$$\text{Prob } P < -R+K = \frac{F\{-R+K\}}{10 - (-10)} = \frac{-R+5 - (-10)}{20} = \frac{15}{20} - \frac{R}{20} = \frac{3}{4} - \frac{R}{20}$$

d) Assuming a bank runs a shortfall in its reserve account, what is the expected value of the amount that the bank will have to borrow from the central bank?

If shortfall, must borrow $K - (R+P)$. Shortfall if $P < -R+K$.

What is $E[K - (R+P) / P < -R+K]$?

$$= K - R - E[P / P < -R+K]$$

$$= 5 - R - \frac{1}{2}(-R+5 + (-10))$$

$$= 5 - R - \frac{1}{2}(-R-5) = 5 - R + \frac{1}{2}R + \frac{1}{2}5$$

$$= 7\frac{1}{2} - \frac{1}{2}R$$