Reserve demand model example

$K = 0$

$r_D = 0$

$\bar{p} = 10$

Probability of shortfall

$$\text{prob}(R + P < 0) = \text{prob}(P < -R) = \frac{-R - (-10)}{10 - (-10)} = \frac{1}{2} - \frac{1}{20} R$$

Expected value of shortfall conditional on shortfall occurring

Amount of shortfall: $0 - (R + P) = -R - P$

$$E[-R - P \mid P < -R] = -R - E[P \mid P < -R]$$

$$= -R - \tfrac{1}{2}(-R + (-10)) = 5 - \tfrac{1}{2} R$$

$$E[\pi] = r(F-R) - \left(\tfrac{1}{2} - \tfrac{1}{20}R\right) r_p \left(5 - \tfrac{1}{2}R\right)$$

$$= r(F-R) - r_p\left[\tfrac{1}{2}\cdot 5 - \tfrac{1}{2}\tfrac{1}{2}R - \tfrac{1}{20}R\,5 + \tfrac{1}{20}R\,\tfrac{1}{2}R\right]$$

$$= r(F-R) - r_p\left[\tfrac{5}{2} - \tfrac{1}{2}R + \frac{1}{40}R^2\right]$$

$$\frac{\partial E[\pi]}{\partial R} = -r - r_p\left[-\tfrac{1}{2} + \frac{1}{20}R\right]$$

To find R^D, set $\frac{\partial E[\pi]}{\partial R} = 0$, solve for R.

$$0 = \ldots\ldots$$

$$R^D = 10 - 20\frac{1}{r_p}r$$

General answer was $R^D = K + \bar{P}\left(\frac{r_p - 2r + r_D}{r_p - r_D}\right)$

Here, $K = 0 \quad r_D = 0 \quad \bar{P} = 10$

$$\Rightarrow R^D = 0 + 10\left(\frac{r_p - 2r + 0}{r_p - 0}\right)$$

$$= 0 + 10\left(\frac{r_p}{r_p} - \frac{2r}{r_p}\right)$$

$$= 10 - 20\frac{1}{r_p}r$$