Econ 614 Hanes Problem set on imperfect capital markets and investment.

This is based on problem 10.4 in Romer chapter 10 (5th edition).

Consider the model of section 10.2 with some changes. There is no cost to an investor of verifying output. Thus, the agreement between the investor and the entrepreneur can take the form of a share contract that promises the investor a share *s* of the realized output *X*. But there is another problem to worry about. After the output is realized, it is possible for the entrepreneur to grab some of it and run away, leaving the investor with the rest. The amount the entrepreneur can grab is a fraction *(1-f)* of what output turns out to be (*X*), that is, he can grab (1-*f*)*X*, leaving the investor with *fX*.

An entrepreneur will grab and run if, after *X* is realized, the amount he would receive under his agreement wirth the investor is less than (1-*f*)*X*. Potential investors know this. "Thus the entrepreneur can only credibly promise to repay fraction *f* of the project's output." That is, there can be no agreement in which the entrepreneur promises the investor a share $s > f$; in any agreement, it must be true that $s \le f$.

1) Assume that the expected payoff from a potential project exceeds the return to the safe alternative investment, that is $\gamma > 1 + r$. What is the condition or conditions that determine whether the project will be undertaken? That is, write down an equation or equations ("equation" can mean $\le$ *or* $\ge$ *or* $=$) that give(s) conditions on the parameters that must be satisfied in order for the project to happen. *For the investor, the expected return to his share of the project must be greater than or equal to* $(1+r)(1-W)$. *Given that investors compete for projects, the expected return to the investor's share must be just equal to* $(1+r)(1-W)$. *So figure out this vlaue of s and compare it with f. If the required value of s is less than or equal to f, the deal will work for the investor.*

$$E[sX] = (1+r)(1-W)$$

$$s\gamma = \frac{(1+r)}{\gamma}(1-W)$$

$$s = \frac{(1+r)}{\gamma}(1-W)$$ If this is $\le f$, then ok. So condition

is $$\frac{(1+r)}{\gamma}(1-W) \le f.$$

Then, check that the deal that works for the investor will also work for the entreprenuer. That means, make sure that the deal will work for the investor gives the entrepreneur and expected return greater than or equal to $(1+r)W$.

$$E[(1-s)X] \ge (1+r)W$$

$$(1-s)\gamma \ge (1+r)W$$ From above, $s = \frac{(1+r)}{\gamma}(1-W)$

$$\left(1 - \frac{(1+r)}{\gamma}(1-W)\right)\gamma \ge (1+r)W$$

$$\gamma - (1+r) + (1+r)W \ge (1+r)W \rightarrow \gamma > (1+r)$$ which we assumed anyway.

So only the first condition matters.

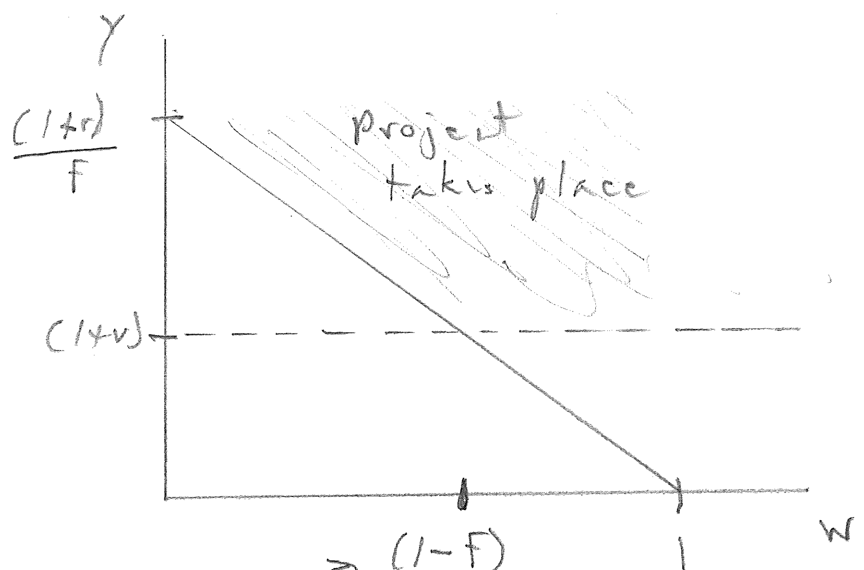
2) Using your answer to a), make a graph with W on the horizontal axis and γ on the vertical axis. For some combinations of W and γ, the project will be undertaken. For other combinations of W and γ, the project will not be undertaken. On your graph, shade in the area that contains the combinations of W and γ under which the project will be undertaken.

Look again at $\frac{(1+r)}{\gamma}(1-w) \leq F$

If you rearrange this, you get an inequality with γ alone on the LHS and W on the RHS. This inequality defines the values of γ that satisfy the inequality for each possible value of W. The combinations of γ and W that satisfy this ineqality, along with the assumption that $\gamma \geq (1+r)$, are the values under which the project will be undertaken.

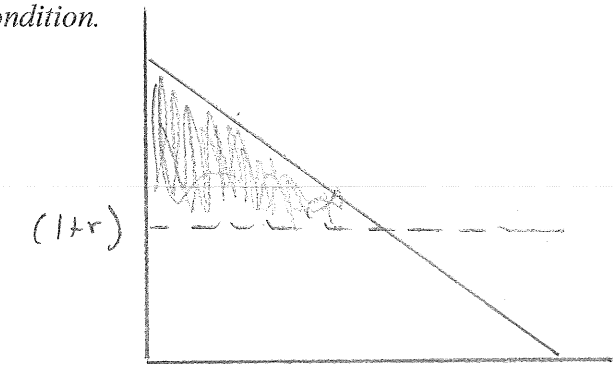
$$\gamma \geq \frac{(1+r)}{F} - \frac{(1+r)}{F} W$$

Look at $\gamma = \frac{(1+r)}{F} - \frac{(1+r)}{F} W$. This is equation of a line with a vertical intercept at $\frac{(1+r)}{F}$ & slope $-\frac{(1+r)}{F}$

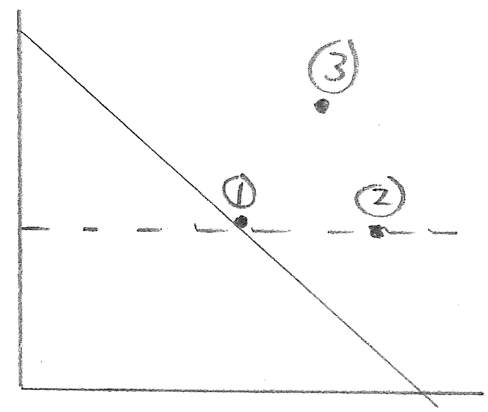


this is value of W at which $\frac{(1+r)}{F} - \frac{(1+r)}{F} W = 1+r$

3) "Inefficiency" occurs if $\gamma > 1 + r$ but the project is not undertaken. According to your graph, can such inefficiency occur? Under what circumstances does it occur? *Yes. Inefficiency happens if combinations of W and γ are in the shaded area below. This is the area where $\gamma > 1 + r$, but γ is not big enogh to satisfy the other condition.*



4) Consider a combination of *W* and γ under which the project *will* be undertaken. Then, there is a decrease in the entreprenuer's wealth. Will that cause the project to *not* be undertaken?
It depends. If the initial point is 1), then even a tiny decrease in W will stop the project. If the initial point is 2) or 3), you need a big decrease in W to stop the project.



5) Consider a combination of *W* and γ under which the project *will* be undertaken. Then, there is a decrease in *f*. Will that cause the project to *not* be undertaken?
The graph below shows what happens to the lines when f decreases from a larger value f_1 to a smaller value f_2. So the answer is again, it depends. Consider point 1) versus point 2).