ANSWERS

Economics 614, Advanced Macro, Spring 20261 Final examination

Look over the entire exam before you begin. Good luck!

1) Consider a model in which a representative-agent household maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\ln C_t + \frac{1}{1-\nu}\left(\frac{M_t}{P_t}\right)^{1-\nu} - \frac{1}{2}\theta L_t^2 \right] \text{ subject to } Z_{t+1} = \frac{P_t}{P_{t+1}}\left[\frac{M_t}{P_t} + (1+i_t)\left(Z_t - \frac{M_t}{P_t} - C_t + (W/P)_t L_t\right)\right]$$

In the utility function, C_t is the household's real consumption in period *t*. M_t is the household's nominal money balance held across period *t*. P_t is the price level. L_t is the quantity of labor supplied by the household. β is close to, but a bit less than, one. And $0 < \nu < 1$.

In the budget constraint, Z_{t+1} is a household's real wealth entering period *(t+1)*. i_t is the nominal interest rate paid on nonmonetary assets held across period *t*. The nominal wage is W_t .

In each period, the household takes as given its wealth entering the period, the price level and the nominal wage. It chooses consumption, labor and its real money balance. Assume "certainty equivalence" holds, so that in the agent's optimization problem you take expected values of future variables to be equivalent to actual known values of future variables. In forming expected values, the household has rational expectations.

a) Using the value function, derive an equation that gives the household's demand for real money balance as a function of consumption and the nominal interest rate. *3 pts.*

See notes and problem set. Answer is: $\left(\frac{M}{P}\right)_t = C_t^{1/\nu}\left(\frac{i_t}{1+i_t}\right)^{-1/\nu}$

b) Using the value function, derive an equation that gives the log of current consumption c_t in terms of $E_t c_{t+1}$ and the real interest rate r_t using all the the usual approximations (and assuming r_t is always "small"). *3 pts.*

Get $C_t^{-1} = \beta C_{t+1}^{e\,-1}(1+i_t)\frac{P_{t+1}}{P_t} = \beta^{-1} C_{t+1}^{e}(1+i_t)(1+\pi_{t+1}^{e})^{-1}$

$\approx \beta^{-1} C_{t+1}^{e}(1+i_t-\pi_{t+1}^{e}) = \beta^{-1} C_{t+1}^{e}(1+r_t)$

Take logs:

$c_t = -\ln\beta + c_{t+1}^{e} - r_t$

usual approximations

c) Suppose that the long-run steady-state (LRSS) rates of growth of consumption is g (this is a fraction, not a percent). Using your answer to b), derive the LRSS real interest rate, denoted $\bar{r}$. 3 pts.

In LRSS, $c_{t+1} = c_t + g$ so from b), $\bar{c}_t - \ln\beta + \bar{c} + g - \bar{r}$

$\bar{r} = -\ln\beta + g$

d) Assume the economy is closed, the capital stock is fixed and there is no investment. Output $Y_t = C_t + G_t$ where G is government purchases of goods and services. The share of G in total output is $\gamma_t = (G/Y)_t$. Assume γ is always small in the same way that the real interest rate is small. Starting from your answer to c), derive an equation that gives the log of output y_t as a function of $E_t y_{t+1}$, the real interest rate r_t, γ and any other relevant variables. *3 pts.*

See notes "New Keynesian LSLM part 1".

e) Now assume the economy is closed, the capital stock is fixed and there is no investment, but there is *no government* so output $Y_t = C_t$. Derive an equation that gives the gap between log output and the log natural rate of output, denoted $(y-\bar{y})_t$, as a function of the gap between the real interest rate and the natural rate of interest $(r-\bar{r})_t$. 3 pts.

Again see "New Keynesian LSLM part 1".

2) Consider an economy where a firm's "desired" price for a period t is (in log terms) $p_{it}^* = p_t + \phi y_t$ and $y_t = m_t - p_t$. (Remember that when output equals the natural rate, $y = 0$.) As in the Taylor model, a firm that can set its price in the current period first observes the realized value of m for the current period, then fixes its price at the same value for the current period and the upcoming period such that $x_{it} = p_{it} = p_{it+1}$. But there is *no staggering*. That is, all firms set their prices in periods t, $t+2$, $t+4$, $t+6$,....No firms set their prices in $t+1$, $t+3$, $t+5$,....

a) Derive an equation that gives x_{it} as a function of m_t and $E_t m_{t+1}$. *5 pts.*

$x_t = \frac{1}{2}(p_{it}^*) + \frac{1}{2}(E_t p_{it+1}^*)$

With no staggering, $x_t = p_t = p_{t+1}$

so $x_t = \frac{1}{2}(\phi m_t + (1-\phi)x_t) + \frac{1}{2}(\phi E_t m_{t+1} + (1-\phi)x_t)$

gives $x_t = \frac{1}{2} m_t + \frac{1}{2} E_t m_{t+1}$

b) Suppose m evolves as a random walk: $m_t = m_{t-1} + \epsilon_t$ where ϵ is mean-zero i.i.d. Using this and your answer to a), what are y_t , y_{t+1} , y_{t+2} and y_{t+3} ? *5 pts.*

$E_t m_{t+1} = m_t$ so $x_t = m_t$

$y_t = m_t - x_t = m_t - m_t = 0$

$y_{t+1} = m_t + \varepsilon_{t+1} - m_t = \varepsilon_{t+1}$

$y_{t+2} = m_{t+2} - x_{t+2} = \ldots = 0$

$y_{t+3} = \ldots = \varepsilon_{t+3}$

3) Clarida, Gali and Gertler (1999, "The Science of Monetary Policy: A New Keynesian Perspective") argue that in a New Keynesian model the dynamic inconsistency issue (or "rules versus discretion") is, in one way, different from the issue in an old-fashioned Keynesian model with a Friedman-Phelps expectations-augmented Phillips curve. What is the difference? *10 pts. See notes "Monetary Policy part 2..Clarida, Gali and Gertler" pp. 1, 11 etc.*

4) Consider an economy that can be described by the "Lucas supply function" model. In the past, the country has been subject to many random, unpredictable money-supply shocks. At time t_0 the country unexpectedly adopts a new monetary regime under which there are fewer unpredictable money-supply shocks. The change in regime is immediately communicated to the public, and understood by them. Consider the economy's expectations-augmented Phillips curve: $\pi_t = E_{t-1}\pi_t + \beta y_t$. At time t_0 , does the parameter β increase, decrease, or remain the same? Explain.
10 pts. The change makes β smaller. You could explain this in various ways.

You could say that the coefficient β in the above Phillips curve is 1/b in the LSF model. The value of b depends partly on V_m. Fewer unpredictable money supply shocks means V_m is smaller.

Or you could say that when there are fewer unpredictable money supply shocks, a household will take an observed change in the price of its product to more likely be due to a change in the relative price (which he wants to repsond to) than a change in the price level (which he does not want to respond to). So the household's labor supply and output will respond more strongly to a change in the price level. Thus aggregate output would respond more strongly to the price level in $y_t = (1/\beta)(p_t - E_{t-1}p_t) = (1/\beta)(\pi_t - E_{t-1}\pi_t)$.

5) Consider the following model of a closed economy in which the central bank operates by adjusting the supply of real money balance to keep the real interest rate at a desired level, and the desired level for the real interest rate follows an interest-rate rule.

(i) $y_t = E_t y_{t+1} - r_t$

(ii) $\pi_t = E_t \pi_{t+1} + y_t + u_t$ *where* $u_t = \rho u_{t-1} + \epsilon_t$ and ϵ is mean-zero i.i.d.

(iii) $r_t = \phi \pi_t$

and demand for real money balance is:

iv) $(m-p)_t = y_t - i_t$

y is the output gap and r is the gap between the real interest rate and the natural rate of interest. Expectations are rational. Assume there is a long-run steady state (LRSS) where $y = 0$, $\pi = 0$.

a) Does the monetary policy in this economy satisfy the Taylor principle, that is does it ensure that the assumed LRSS is the *only* possible LRSS? Explain how you know. *5 pts.*

In every LRSS, y = 0, so the question is whether this monetary policy ensures that:

i) a LRSS with $\pi = 0$ is possible
ii) a LRSS with $\pi \neq 0$ is not possible.

i) In LRSS, $u = 0$, $E_t y_{t+1} = 0$, $E_t \pi_{t+1} = \bar{\pi}$. Suppose $\bar{\pi} = 0$.

If $\pi_t = 0$, then $r_t = \phi \pi_t = 0$.

So from $y_t = E_t y_{t+1} - r_t$, $y_t = 0$.

So from $\pi_t = E_t \pi_{t+1} + y_t$, $\pi_t = 0$. It works

ii) Now suppose $\bar{\pi} > 0$ (example).

If $\pi_t = \bar{\pi} > 0$, then $r_t = \phi \pi_t > 0$.

Then $y_t = -r_t < 0$.

Then $\pi_t = \bar{\pi} + y_t < \bar{\pi}$, Not a LRSS.

b) Derive equations that give y_t, π_t, r_t , i_t and $(m-p)_t$ as functions of u_t . *10 pts.*

Derivations of y_t, π_t, r_t are in the notes.

The nominal interest rate $i_t = r_t + E_t[\pi_{t+1}]$. (For some reason several of you thought $i_t = r_t + \pi_t$.)

$E_t[\pi_{t+1}] = \rho\pi_t$.

6) Consider a model where:

(i) $y_t = E_t y_{t+1} - r_t + u_t$ *where* $u_t = \rho u_{t-1} + \epsilon_t$ and ϵ is mean-zero i.i.d.

(ii) $\pi_t = E_t \pi_{t+1} + y_t$

There is a central bank that chooses r_t to minimize: $L = \sum_{\tau=0}^{\infty} \beta^\tau E\left[\frac{1}{2} y_{t+\tau}^2 + \frac{1}{2}\pi_{t+\tau}^2\right]$

Note that the central bank has "discretion" in choosing r_t .

The public's expectations are rational.

At the time the central bank chooses r_t , it knows the true structure of the economy and what the public's expectations are, that is it knows $E_t y_{t+1}$ and $E_t \pi_{t+1}$. It does *not* know what u_t is. But it *does* know the values of macro variables that were realized in the previous period *t-1*, that is it knows π_{t-1} and y_{t-1} as well as r_{t-1} . And of course it knows $E_{t-1} y_t$ and $E_{t-1}\pi_t$.

What will be the values of r_t , y_t and π_t ? Hint: I am not necessarily looking for a complete derivation. Think intuitively about how such a loss-minimizing central bank responds to a shock of this kind, given what this central bank knows.

10 pts. Because the central bank has "discretion," minimizing that loss function amounts to minimizing the loss in the immediate period - the central bank doesn't have to worry about periods after that (see notes on Clarida, Gali, Gertler).

Because the central bank does not know u_t , it does not know exactly what values of y and π will result from a given value of r. Thus, it is wrong to skip the step of choosing r and describing the central bank as choosing y or π directly.

Because it knows the structure of the economy, y_{t-1} and $E_{t-1} y_t$, the central bank knows what u_{t-1} was:

$u_{t-1} = y_{t-1} - E_t y_{t+1} + r_t$

The intuition is that, because u is a demand shock not a supply shock, the central bank might adjust r to fully counteract the effect of u on output if it could (that would stabilize inflation as well). As it is, the central bank might do its best to do that, which means setting r_t to counteract its expected value for u_t , which is ρu_{t-1} . So conjecture that $E_t y_{t+1} = 0$, $E_t \pi_{t+1} = 0$. Intuitively, you might think that the central bank would set r_t to make its expected value for y_t equal to zero, which would mean $r_t = -\rho u_{t-1}$. The result of that would be $y_t = \pi_t = \epsilon_t$. Which would mean that, confirming the conjecture, $E_t y_{t+1} = 0$, $E_t \pi_{t+1} = 0$! conjecture

That would be a good answer. But you could also set up and solve the first-order condtion for choosing r_t :

$$\min_{r_t} L_t = \tfrac{1}{2} E\left[(0 - r_t + u_t)^2 + (0 - r_t + u_t)^2\right]$$

$$= \tfrac{1}{2}\left[(0 - r_t + \rho u_{t-1})^2 + (0 - r_t + \rho u_{t-1})^2 + \sigma_\epsilon^2 + \sigma_\epsilon^2\right]$$

$$= (-r_t + \rho u_{t-1})^2 + \sigma_\epsilon^2$$

$$0 = \frac{\partial L_t}{\partial r_t} = \ldots \text{ gives } r_t = -\rho u_{t-1}.$$

7) Recall the old-Keynesian Friedman-Phelps expectations-augmented Phillips curve $\pi_t = E_{t-1}\pi_t + \kappa y_t$. Assuming rational expectations, this is inconsistent wirh the actual behavior of output and inflation. Explain, using equations. *10 pts.*

In this expectations-augmented Phillips curve + rational expectations, the expected value of the output gap in the upcoming period is always zero. To put it another way, there is no serial correaltion in the output gap. In reality, there is serial correlation in the output gap and people frequently forecast that the output gap will be different from zero.

$$\pi_t = E_{t-1}\pi_t + \kappa y_t$$

$$\pi_t = E_{t-1} + \varepsilon_t$$ ← error in expected value, must be uncorrelated with anything known at time $(t-1)$, including y_{t-1}.

$$y_t = \frac{1}{\kappa}\left(\pi_t - E_{t-1}\pi_t\right) = \frac{1}{\kappa}\varepsilon_t$$

so y_t must be uncorrelated with y_{t-1}.

And $E_t[y_{t+1}] = \frac{1}{\kappa}E_t[\varepsilon_{t+1}] = 0$.