Economics 614, Advanced Macro, Spring 2025 Midterm examination

Before you begin, read *all* of the questions. Possibly, the exam is too long for you to answer all of the questions, so start by answering the questions you are surest about. After you have written down answers to those, move on to questions you are less sure about.

1) Consider a real business cycle model where a representative agent (a household) saves by holding capital. The real wage in a period *t* is W_t. The real income the agent receives in a period from holding K_t units of capital in that period is $(1+r_t)K_t$. The agent acts to maximize:

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\theta} C_t^{1-\theta} - \frac{1}{1+\lambda} L_t^{1+\lambda} \right] \quad \textit{where } 0 < \beta < 1$$

where C_t is his consumption and L_t is his hours of labor in period *t*. His wealth (capital) in period *t+1* will be:

$$K_{t+1} = W_t L_t + (1+r_t)K_t - C_t$$

a) Write down the Bellman equation for the agent's problem. **2 pts**

$$V_t = \max_{C,L} \left\{ \frac{1}{1-\theta} C_t^{1-\theta} - \frac{1}{1+\lambda} L_t^{1+\lambda} + \beta E_t V_{t+1} \right\}$$

(You don't need to write it, but constraint is

$$K_{t+1} = W_t L_t + (1+r_t) K_t - C_t$$

Note $\frac{\partial K_{t+1}}{\partial C_t} = -1$, $\frac{\partial K_{t+1}}{\partial L_t} = W_t$

b) Derive an equation that gives the quantity of labor the household chooses to supply at time *t*, L_t^S, as a function of the real wage W_t and consumption C_t. **4 pts**

$$\frac{\partial V_t}{\partial C_t} = \frac{1-\theta}{1-\theta} C_t^{-\theta} + \beta \frac{\partial E_t V_{t+1}}{\partial K_{t+1}} (-1) = 0$$

$$\frac{\partial V_t}{\partial L_t} = -\frac{1+\lambda}{1+\lambda} L_t^{\lambda} + \beta \frac{\partial E_t V_{t+1}}{\partial K_{t+1}} W_t = 0$$

so $C_t^{-\theta} = \beta \frac{\partial E_t V_{t+1}}{\partial K_{t+1}}$, $L_t^{\lambda} \frac{1}{W_t} = \beta \frac{\partial E_t V_{t+1}}{\partial K_{t+1}}$

so $L_t^{\lambda} \frac{1}{W_t} = C_t^{-\theta}$

$$L_t^S = W_t^{\frac{1}{\lambda}} C_t^{-\frac{\theta}{\lambda}}$$

c) What is the elasticity of labor supply? **4 pts**

Take logs!

$$l_t^s = \frac{1}{\lambda} w_t - \frac{\theta}{\lambda} c_t$$

$$\frac{\partial l_t^s}{\partial w_t} = \frac{1}{\lambda}$$

elasticity

d) Using the Bellman equation and the equation that describes evolution of wealth, derive C_t as a function of the real wage and the agent's beliefs about C_{t+1} as of period *t*. Do not use any approximations. **4 pts.**

Start from $C_t^{-\theta} = \beta \frac{\partial E_t V_{t+1}}{\partial K_{t+1}}$. Because E[ ] is linear,

$\frac{\partial E_t V_{t+1}}{\partial K_{t+1}} = E_t\left[\frac{\partial V_{t+1}}{\partial K_{t+1}}\right]$. So what is $\frac{\partial V_{t+1}}{\partial K_{t+1}}$? By "B-S condition" (envelope theorem), it is effect on lifetime utility, looking forward from t+1, of ∂K_{t+1} holding fixed K_{t+2} & L_{t+1}. From $K_{t+2} = W_{t+1} L_{t+1} + (1+r_{t+1}) K_{t+1} - C_{t+1}$, $\partial C_{t+1} / \partial K_{t+2} = (1+r_{t+1})$.

So effect on lifetime utility is $MUC_{t+1} (1+r_{t+1}) \partial$.

So $$C_t^{-\theta} = \beta E_t\left[C_{t+1}^{-\theta}(1+r_{t+1})\right]$$

note r_{t+1} is inside E[ ]

$$C_t = \beta^{-1/\theta}\left(E_t\left[C_{t+1}^{-\theta}(1+r_{t+1})\right]\right)^{-1/\theta}$$

(derivative of expected value = expected value of derivative)

e) Now consider the nonstochastic long-run steady state of this economy. Assume there is no growth in productivity or population, so that in the nonstochastic LRSS the real wage and real return to holding capital are fixed. Again, do not use any approximations.

i) What is the LRSS value of r in the economy? **4 pts**

In nonstochastics LRSS with no growth, consumption is fixed.

$$\bar{c} = \beta^{-\frac{1}{\theta}}\left(\bar{c}^{-\theta}(1+\bar{r})\right)^{-\frac{1}{\theta}}$$

$$\bar{c} = \beta^{-\frac{1}{\theta}}\,\bar{c}\,(1+\bar{r})^{-\frac{1}{\theta}}$$

$$1 = \beta^{-\frac{1}{\theta}}(1+\bar{r})^{-\frac{1}{\theta}}$$

$$\Rightarrow \beta^{\frac{1}{\theta}} = (1+\bar{r})^{-\frac{1}{\theta}}$$

$$\beta^{-1} = (1+\bar{r})$$

$$\bar{r} = \beta^{-1} - 1 = \frac{1}{\beta} - 1$$

Note: since $0 < \beta < 1$, $\frac{1}{\beta} > 1$, so $\bar{r} > 0$.

ii) Let δ denote the depreciation rate in the economy. In the LRSS, what is the marginal product of capital in the economy? **4 pts**

$$\bar{r} = MPK - \delta$$

so $\frac{1}{\beta} - 1 = MPK - \delta$

$$MPK = \frac{1}{\beta} - 1 + \delta$$

Note $MPK > 0$, which it must be.

2) Consider two economies that can be described by Romer's baseline RBC model, where the deviation from trend in the log of government purchases of output is $\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \epsilon_{G,t}$. In economy I, $\rho_G = 0.1$. In economy II, $\rho_G = 0.5$. Otherwise the economies are identical.

Think about the immediate response of output to a government spending shock (a positive realization of ϵ_t), that is, the response within the period that the shock hits. Suppose a positive shock of exactly the same size hits both economies.

10 pts total.

Note that the immediate change in G (that is, the value of $\tilde{G}$ within the period that the shock hits) is the SAME in both economies.

a) Is the immediate increase in output bigger in economy I, in economy II, or the same size in both economies? Explain.

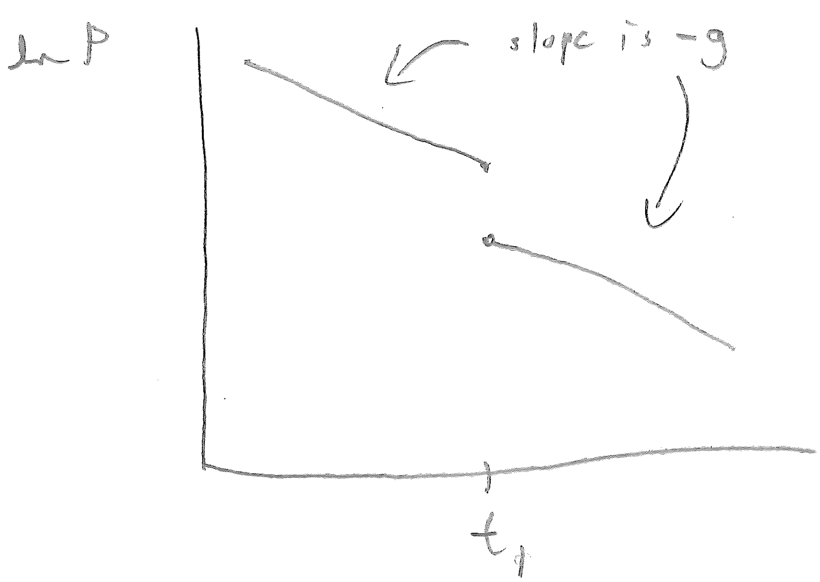
See the textbook p. 216, which says that the effect of $\tilde{G}$ on employment (a_{LG}) is smaller if ρ_G is smaller. Since a shock to G does not affect productivity, and has no immediate effect on the capital stock, this means the effect on output is smaller too. Reason: see discussion on p. 214 about ρ_A and transfer logic to G. See also notes, "RBC model:finishing up the textbook model."

b) Is the immediate decrease in the real wage bigger in economy I, in economy II, or the same size in both economies? Explain.

In economy II, since the effect on employment is smaller, the decrease in the real wage must be smaller, because there will be a smaller decrease in the marginal product of labor.

3) Consider an economy in which the money supply m is fixed and demand for real money balances, in logs, is: $(m-p)_t^D = y_t - \lambda i_t$ where i is the log of output. The natural rate of output grows at a constant rate g. Suppose that at time t_1 the natural rate of interest unexpectedly decreases from a higher value $\overline{r}_0$ to a higher value $\overline{r}_1$. Consider the path of the price level that will allow output to remain at the natural rate throughout. Plot the log of this price level on a graph with time on the horizontal axis and the log of the price level on the vertical axis. Clearly mark the point in time t_1.

10 pts. *Before and after t_1, real money balances must be growing at the same rate as output. Money supply is fixed, so that means the price level must be falling at the rate g. Before t_1, the nominal interest rate must be $i_0 = \overline{r}_0 + E\pi = \overline{r}_0 - g$. After t_1 the nominal interest rate must be $i_1 = \overline{r}_1 + E\pi = \overline{r}_1 - g < i_0$. So the nominal interest rate decreases. So demand for real money balances increases. So supply of real money balance must increase suddenly at time t_1. So the price level must suddenly fall at time t_1.*



4) Recall King and Rebelo's version of a real business cycle model in "Resuscitating Real Business Cycles" with "variable capacity utilization." Simplifying that model a bit, the production function is:

$Y_t = (Z_t K_t)^\alpha L_t^{1-\alpha}$ where Z is capacity utilization.

Capital depreciation δ increases with Z, and depreciation affects next period's capital stock:

$$K_{t+1} = K_t + Y_t - C_t - \delta(Z_t)K_t$$

The real interest rate is $r_t = \partial Y / \partial K - \delta(Z_t)$.

The representative agent's "felicity" (single-period utility) function can be written generally as $u_C(C_t) + u_{(1-l)}(1-l_t)$.

Assume specifically that $\delta_t = \frac{1}{2} Z_t^2$.

a) Write down a Bellman equation for the social planner. **2 pts**

$$V_t = \max_{c, l, z} \left\{ u_C(C_t) + u_{(1-l)}(1-l_t) + \beta E_t V_{t+1} \right\}$$

and $$K_{t+1} = K_t + (Z_t K_t)^\alpha L_t^{1-\alpha} - C_t - \tfrac{1}{2} Z_t^2 K_t$$

b) Z is a choice variable for the social planner. Using the Bellman equation, the production function and the equation above that gives K_{t+1} as a function of K_t and other stuff, write down the first-order condition that defines the optimal value of Z_t in terms of K_t and L_t. In the equation, let $\hat{Z}$ denote the optimal value of Z. You do *not* have to solve the equation for $\hat{Z}$. **4 pts.**

$$\frac{\partial V_t}{\partial Z_t} = \beta E_t \left[\frac{\partial V_{t+1}}{\partial K_{t+1}} \frac{\partial K_{t+1}}{\partial Z_t} \right] = \beta E_t \frac{\partial V_{t+1}}{\partial K_{t+1}} \left(\alpha Z_t^{\alpha-1} K_t^\alpha L_t^{1-\alpha} - Z_t K_t \right)$$

setting $\frac{\partial V_t}{\partial Z_t} = 0$,

$$\alpha \hat{Z}_t^{\alpha-1} K_t^\alpha L_t^{1-\alpha} - \hat{Z}_t K_t = 0$$ (defines $\hat{Z}$

which implies

$$\hat{Z}^2 = \alpha \hat{Z}^\alpha (K/L)^{\alpha-1} \qquad \hat{Z}^{2-\alpha} = \alpha K^{\alpha-1} L^{1-\alpha}$$

$$\hat{Z} = (\alpha K^{\alpha-1} L^{1-\alpha})^{\frac{1}{2-\alpha}}$$ etc.

c) Take your answer to b) and rearrange it so that $\hat{Z}^2$ is alone on the left-hand side of the equation: that is, get an equation that looks like $\hat{Z}^2 = \ldots\ldots\ldots$ **4 pts.**

$$\hat{Z}^2 = \alpha \hat{Z}^{\alpha} K^{\alpha-1} L^{1-\alpha}$$

d) Let a denote the marginal product of capital, that is $a = \partial Y / \partial K$. Using the production function, derive an equation that gives a_t (on the left-hand side) as a function of K_t and L_t. **2 pts.**

$$a_t = \alpha Z_t^{\alpha} K_t^{\alpha-1} L_t^{1-\alpha}$$

Note that for $Z = \hat{Z}$, $a_t = \hat{Z}_t^2$

e) Using your answers to c) and d), and the definition of the real interest rate, derive an equation that gives $\hat{Z}_t$ as a function of r_t. **4 pts.**

$$r_t = a_t - \delta_t = a_t - \tfrac{1}{2} Z^2$$

so $a_t = r_t + \tfrac{1}{2} Z^2$

At $\hat{Z}$, $a_t = \hat{Z}_t^2$ so

$$\hat{Z}_t^2 = r_t + \tfrac{1}{2}\hat{Z}^2$$

implies $\tfrac{1}{2}\hat{Z}_t^2 = r_t$, $\hat{Z}_t = (2 r_t)^{1/2}$

See: $r_t \uparrow \rightarrow \hat{Z}_t \uparrow$ as in King & Rebelo

5) Consider an economy similar to the Taylor model. A firm's "desired" log price for a period is $p_{it}^* = p_t + \phi y_t$ where p is the log price level and y is the output gap (deviation of log output from the natural rate). "Aggregate demand" is $y_t = m_t - p_t$. As in the Taylor model, each firm i in the economy must fix its price at the same value for two periods, and each firm can wait until it has observed m_t to set its price $x_{it} = p_{it} = p_{it+1}$. But there is *no staggering*. All firms do their pricesetting in the *same* period. (That is, all firms set prices in periods *t, t+2, t+4,...*; no firms set prices in periods *t+1, t+3, t+5,...*Thus $x_{it} = p_t = p_{t+1}$.

a) Write down x_{it} as a function of m_t, $E_t m_{t+1}$, p_t and $E_t p_{t+1}$. **4 pts.**

$$X_t = \tfrac{1}{2} p_{it}^* + \tfrac{1}{2} E_t p_{it+1}^* = \tfrac{1}{2}(\phi m_t + (1-\phi) p_t)$$
$$+ \tfrac{1}{2}(\phi m_t^e + (1-\phi) p_{t+1}^e)$$

b) Write down x_t as a function of m_t and $E_t m_{t+1}$ alone. **4 pts.**

Because there is no staggering, $X_t = P_t = P_{t+1}^e$

so above gives

$$X_t = \tfrac{1}{2}(m_t + m_{t+1}^e)$$

c) Now suppose that m evolves as a random walk: $m_t = m_{t-1} + \epsilon_t$ where ϵ is mean-zero i.i.d. Using this fact and your answer to b), what are y_t, y_{t+1} and y_{t+3} ? **4 pts.**

From random walk, $m_{t+1}^e = m_t$ so $X_t = \tfrac{1}{2}[m_t + m_t] = m_t$

$$Y_t = m_t - P_t = m_t - X_t = m_t - m_t = 0$$

$$Y_{t+1} = m_{t+1} - P_{t+1} = m_{t+1} - X_t = m_{t+1} - m_t = \varepsilon_{t+1}$$

$$Y_{t+3} = m_{t+3} - P_{t+3} = m_{t+3} - X_{t+2} = m_{t+3} - m_{t+2} = \varepsilon_{t+3}$$

Note there is no persistence (serial correlation) in Y.