ANSWERS

Economics 614, Advanced Macro, Spring 2026 Midterm examination

Before you begin, read *all* of the questions. Possibly, the exam is too long for you to answer all of the questions, so start by answering the questions you are surest about. After you have written down answers to those, move on to questions you are less sure about.

**1)** Consider a model like Romer's "baseline" RBC model, but simpler. All markets are perfectly competitive. There are no taxes or government purchases. There is no population growth or improvement in productivity: the aggregate production function is $Y_t = K_t^{\alpha} L_t^{1-\alpha}$. A representative household has one unit of time which it divides between leisure and work. It maximizes:

$$U_t = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left( \ln C_{t+\tau} - \tfrac{1}{3} L_{t+\tau}^3 \right) \qquad \textit{where } 0 < \beta < 1$$

Points per question

where $C$ is household consumption and $L$ is household labor. Household wealth (capital) in period $t+1$ will be:

$$K_{t+1} = W_t L_t + (1 + r_t) K_t - C_t$$

$r$ is the real interest rate or real return to holding a unit of capital for one period, which is the marginal product of capital minus the depreciation rate $\delta$. $W$ is the real wage.

3 a) Write down the Bellman equation for the agent's problem.

$$V_t = \max_{C_t, L_t} \left\{ \ln C_t - \tfrac{1}{3} L_t^3 + \beta E_t V_{t+1} \right\}$$

Note: $K_{t+1}$ does not belong in here, because it's determined by C & L

3 b) Derive an "intratemporal" equation that gives $L_t$ (on the left-hand side) as a function of the real wage $W_t$ and consumption $C_t$ (on the right-hand side).

$$\frac{\partial V_t}{\partial C_t} = \frac{1}{C_t} + \beta \frac{\partial E_t V_{t+1}}{\partial K_{t+1}} (-1) = 0$$

$$\frac{\partial V_t}{\partial L_t} = -L_t^2 + \beta \frac{\partial E_t V_{t+1}}{\partial K_{t+1}} W_t = 0$$

$$\text{gives } \quad L_t^2 \frac{1}{W_t} = \beta \frac{\partial E \ldots}{\partial K_{t+1}}$$

$$\frac{1}{C_t} = \beta \ldots$$

$$\frac{1}{C_t} = L_t^2 \frac{1}{W_t}$$

$$L_t = \left( \frac{W_t}{C_t} \right)^{1/2}$$

c) Derive an "intertemporal" equation that gives $C_t$ (on the left-hand side) as a function of the agent's beliefs, as of period $t$, about what $C_{t+1}$ will turn out to be, and any other relevant variables. Assume that at time $t$ the agent knows not only $W_t$ and $r_t$ : it also knows exactly what $W_{t+1}$ and $r_{t+1}$ will be. But the agent does *not* know exactly what $C_{t+1}$ or $L_{t+1}$ will turn out to be. Do *not* use "certainty equivalence."

3

Envelope theorem or Benveniste - Scheinkman condition says

$$\frac{\partial V_{t+1}}{\partial K_{t+1}} = \frac{\partial V_{t+1}}{\partial C_{t+1}} \frac{\partial C_{t+1}}{\partial K_{t+1}} \Bigg|_{\partial K_{t+2}=0,\ \partial L_{t+1}=0}$$

From budget constraint, $K_{t+2} = W_{t+1} L_{t+1} + (1+r_{t+1})K_{t+1} - C_{t+1}$

Holding $K_{t+2}$ & $L_{t+1}$ fixed, $\partial C_{t+1} / \partial K_{t+1} = 1 + r_{t+1}$

so $$\frac{\partial V_{t+1}}{\partial K_{t+1}} = \frac{1}{C_{t+1}}(1+r_{t+1})$$

so from first equation in b), using $\frac{\partial E_t V_{t+1}}{\partial K_{t+1}} = E_t \frac{\partial V_{t+1}}{\partial K_{t+1}}$,

$$\frac{1}{C_t} = \beta E_t\left[\frac{1}{C_{t+1}}(1+r_{t+1})\right] = \beta(1+r_{t+1}) E_t\left[\frac{1}{C_{t+1}}\right]$$

known at time t

$$C_t = \beta^{-1}(1+r_{t+1})^{-1}\left(E_t\left[\frac{1}{C_{t+1}}\right]\right)^{-1}$$

d) Let $r^*$ denote the value of $r$ in the nonstochastic steady-state of the economy. Using your answer to c), what is $r^*$?

3

In nonstochastic LRSS, $C_t = C_{t+1} = C^*$

$$C^* = \beta^{-1}(1+r^*)^{-1} C^*$$

$$1 = \beta^{-1}(1+r^*)^{-1}$$

$$r^* = \beta^{-1} - 1 \quad \text{or} \quad \frac{1}{\beta} - 1$$

(Note: since $\beta < 1$, $\frac{1}{\beta} > 1$, $r^* > 0$.

e) Let $k$ denote the ratio of capital to labor: $k = K / L$. What is $k^*$, the steady-state value of $k$?

3 $r^* = MPK - \delta$ so $MPK = r^* + \delta = \beta^{-1} - 1 + \delta$

$$MPK = \frac{\partial Y}{\partial K} = \alpha K^{\alpha-1} L^{1-\alpha} = \alpha (K/L)^{\alpha-1}$$

so $\alpha k^{*\,\alpha-1} = \beta^{-1} - 1 + \delta$

$$k^{*\,1-\alpha} = \frac{\alpha}{\beta^{-1}-1+\delta}$$

$$k^* = \left(\frac{\alpha}{\beta^{-1}-1+\delta}\right)^{\frac{1}{1-\alpha}}$$ Note $k^* > 0$.

f) Now forget about the steady state; go back to thinking about fluctuations. Derive an equation that gives $L_t$ (on the left-hand side) as a function of the agent's beliefs about $L_{t+1}$ and any other relevant variables. Assume that as of time $t$ the household knows with certainty what $W_{t+1}$ will be and what $r_{t+1}$ will be, but it is uncertain about $L_{t+1}$. Again, do not use "certainty equivalance."

3

b) said $\frac{1}{C_t} = L_t^2 \frac{1}{W_t}$

c) said $\frac{1}{C_t} = \beta(1+r_{t+1}) E_t\left[\frac{1}{C_{t+1}}\right]$

so $L_t^2 \frac{1}{W_t} = \beta(1+r_{t+1}) E_t\left[L_{t+1}^2 \frac{1}{W_{t+1}}\right] = \beta(1+r_{t+1}) \frac{1}{W_{t+1}} E_t\left[L_{t+1}^2\right]$

known at time t.

$$L_t^2 = \beta(1+r_{t+1}) \frac{W_t}{W_{t+1}} E_t\left[L_{t+1}^2\right]$$

$$L_t = \left(\beta(1+r_{t+1}) \frac{W_t}{W_{t+1}}\right)^{1/2} \left(E_t\left[L_{t+1}^2\right]\right)^{1/2}$$

3

g) Using your answer to f), now apply "certainty equivalance" to get an equation that gives $L_t$ (on the left-hand side) as a function of $E_t L_{t+1}$ and any other relevant variables.

C. E. means
1) solve as if no uncertain
2) replace stochastic variable $X_{t+1}$ with $E_t X_{t+1}$

$$L_t = \left(\beta(1+r_{t+1})\frac{W_t}{W_{t+1}}\right)^{1/2}\left(\left(E_t L_{t+1}\right)^2\right)^{1/2}$$

$$= \left(\beta(1+r_{t+1})\frac{W_t}{W_{t+1}}\right)^{1/2} E_t L_{t+1}$$

3

h) The value of $L_t$ from the equation using "certainty equivalance" in g) may not be equal to the "true" value you gave in f). Is the true value of $L_t$ larger than, smaller than or equal to the "certainty equivalance" value of $L_t$? Explain how you know. *This is about "Jensen's inequality."*

MORE ON OTHER SIDE

"True" $L_t = \left( \qquad \right)^{1/2}\left(E_t[L_{t+1}^2]\right)^{1/2}$

Cert. equiv. $L_t = \left( \qquad \right)^{1/2}\left(\left(E_t L_{t+1}\right)^2\right)^{1/2}$

so question is $\underbrace{E_t[L_{t+1}^2]}_{\text{true}} \overset{?}{\gtreqless} \underbrace{(E_t L_{t+1})^2}_{\text{C.E.}}$

$L^2$ is a convex function of $L$, so

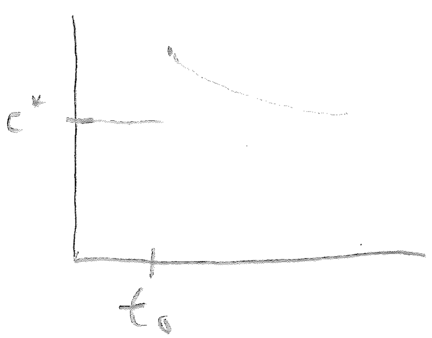
Jensen's inequality says $E_t[L_{t+1}^2] > (E_t L_{t+1})^2$

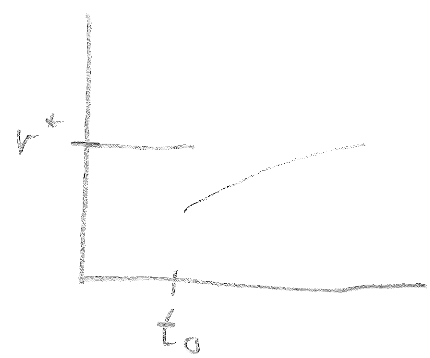
so "true" is bigger.

10 **2)** Consider King and Rebelo's version of a real business cycle model in "Resuscitating Real Business Cycles." In this model there is a positive relationship between the real interest rate $r$ and "capital utilization" $Z$. This relationship is a vital feature of King and Rebelo's model. Explain briefly. *See notes.*

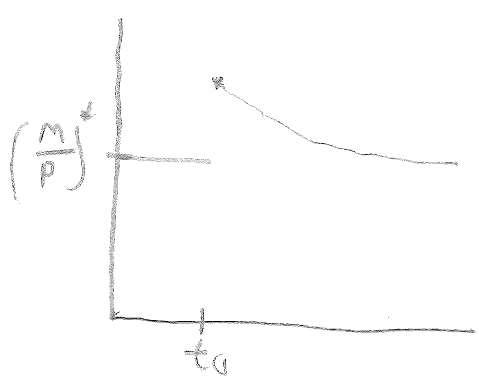
10 **3)** Consider a simple version of Romer's textbook basic real business cycle model, in which the population is fixed and there is no trend growth in total factor productivity (no improvement in "technology"). Deviation in the log of government purchases of output from its long-run steady state value is $\tilde{G}_t = \rho_G \tilde{G}_{t-1} + \epsilon_{G,t}$ where $\epsilon_G$ is mean-zero i.i.d. Deviation in the log of "technology" from its long-run value is $\tilde{A}_t = \rho_A \tilde{A}_{t-1} + \epsilon_{A,t}$ where $\epsilon_A$ is mean-zero i.i.d. The economy uses money. Demand for real money balance depends on the nominal interest rate and aggregate *consumption* $C$ (rather than output) such that $(M/P)^D = L(i, C)$ *where* $L_i < 0,\ L_C > 0$. The money supply $M^S$ is fixed. The price level always immediately adjusts to the value needed to keep the economy in its Pareto-optimal equilibrium. Suppose the economy is in its LRSS. Then, at time $t_0$, there is a *negative* shock to $G$ ($\epsilon_G < 0$). Draw a graph that shows the path of the price level over time, marking the time of the shock as $t_0$. *For simplicity, assume that expected future inflation is always equal to zero* (whether or not this turns out to be true).

*In the model, the responses of consumption and the real interest rate to that shock are:*

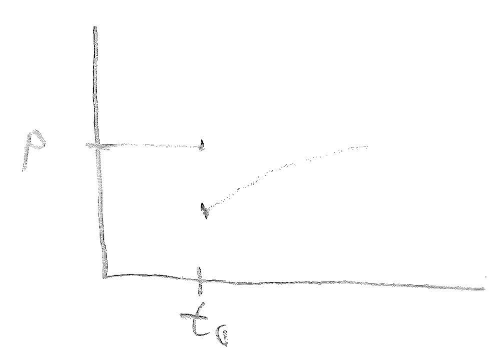



*Given that expected future inflation is always zero, that path for the real interest rate is also the path for the nominal interest rate. From the money demand function, the paths of the nominal interest rate and consumption imply that the path for real money balance demand is:*




*Given that the money supply is fixed, the path the price level must follow is:*



**4)** Consider an economy described by the Lucas supply function model, so that:

$$y_t = \frac{b}{1+b}(m_t - E_{t-1}m_t) \qquad p_t = E_{t-1}m_t + \frac{1}{1+b}(m_t - E_{t-1}m_t)$$

Agents know that nominal aggregate demand $m$ has two components: $m_t = m_t^* + v_t$. $m_t^*$ is set by a central bank, based on information from period (t-1); this information includes the value of $v_{t-1}$ but not the value of $v_t$. $v$ evolves as a random walk: $v_t = v_{t-1} + u_t$, $u$ is mean-zero i.i.d. with variance $\sigma_u^2$. Both the public and the central bank know this, and have rational expectations.

(4)

a) What determines the value of $b$? *See notes.*

(6)

b) Demonstrate the "policy irrelevance" result, that is the proposition that the variance of output is the same whether the central bank follows a "passive" monetary policy, or alternatively tries to stabilize output and the price level as much as possible.

"Passive" means holding $m^*$ fixed, call it $\bar{m}$.

$$\text{so } E_{t-1}m_t = \bar{m} + E_{t-1}v_t = \bar{m} + v_{t-1}$$

$$m_t - E_{t-1}m_t = \bar{m}_t + v_t - (\bar{m} + v_{t-1}) = u_t$$

$$\text{so } p_t = \bar{m} + v_{t-1} + \frac{1}{1+b}u_t$$

$$y_t = \frac{b}{1+b}u_t \qquad Var(y) = \left(\frac{b}{1+b}\right)^2 \sigma_u^2$$

"Active" is $m_t^* = -E_{t-1}v_t = -v_{t-1}$

$$\text{so } E_{t-1}m_t = -v_{t-1} + E_{t-1}v_t = -v_{t-1} + v_{t-1} = 0$$

$$m_t - E_{t-1}m_t = -v_{t-1} + v_t - 0$$

$$= -v_{t-1} + (v_{t-1} + u_t) = u_t$$

$$\text{so } p_t = 0 + \frac{1}{1+b}u_t$$

$$y_t = \frac{b}{1+b}u_t, \quad Var(y) = \left(\frac{b}{1+b}\right)^2 \sigma_u^2$$

5) In Romer's textbook static (one-period) model of imperfect competition, each household $i$ operates a monopoly firm and supplies labor to a perfectly competitive labor market. A household-firm does not use its own labor in production, but instead hires labor from the perfectly competitive labor market at a market-clearing nominal wage $W$ per unit of labor. Each household acts to maximize:

$U_i = C_i - \frac{1}{\gamma} L_i^{\gamma}$ where $C_i$ is a function of the household's consumption of individual goods. The production function is

$Y_i = L_i$ , where $L$ is the number of labor units hired from the labor market. Demand for the good produced by a household-firm is $Y_i^D = (P_i / P)^{-\eta} Y$ where $P$ is the price level and $Y$ is average real income or real GDP per household.

(4) a) From profit-mazimization on the part of the firm, *derive* an equation that gives an individual firm's profit-maximizing price $P_i^*$ as a function of the market nominal wage $W$. (By "profit-maximizing," I mean ignoring menu costs or anything else that creates price rigidity.) *I'll do it as maximization of nominal profit, not real profit as in the textbook.*

$$R_i = P_i Y_i - W L_i = P_i Y_i - W Y_i = P_i P_i^{-\eta} P^{\eta} Y - W P_i^{-\eta} P^{\eta} Y$$

$$= P_i^{1-\eta} P^{\eta} Y - W P_i^{-\eta} P^{\eta} Y$$

$$\frac{\partial R_i}{\partial P_i} = (1-\eta) P_i^{-\eta} P^{\eta} Y - W(-\eta) P_i^{-\eta-1} P^{\eta} Y = 0$$

$$(1-\eta) P_i^{-\eta} P^{\eta} Y = W(-\eta) P_i^{-\eta} P_i^{-1} P^{\eta} Y$$

$$\Rightarrow P_i = \frac{\eta}{\eta - 1} W = \frac{1}{1 - \frac{1}{\eta}} W$$

note this is + because $\eta > 1$

(4) b) From utility-maximization on the part of the household, *derive* an equation that gives labor supply per household $L_i^S$ as a function of the real wage $W/P$.

$$u = C - \frac{1}{\gamma} L^{\gamma} \quad \text{where} \quad C = (WL + R)/P$$

$$\rightarrow u = \frac{WL + R}{P} - \frac{1}{\gamma} L^{\gamma}$$

$$\frac{\partial u}{\partial L} = \frac{W}{P} - L^{\gamma - 1} = 0$$

$$\Rightarrow L = \left(\frac{W}{P}\right)^{\frac{1}{\gamma - 1}}$$

4

c) Derive what I called the "real rigidity equation," which gives the log of a firm's profit-maximizing price as a function of the log price level and the log of output.

$$P_i = \frac{\eta}{\eta-1} W \quad \& \quad L = \left(\frac{W}{P}\right)^{\frac{1}{\gamma-1}}$$

Logs: $p_i = \ln(\eta/\eta-1) + w \qquad l = y = \frac{1}{\gamma-1}(w-p)$

so $(\gamma-1)y = w - p$

$w = (\gamma-1)y + p$

$$p_i = \ln(\eta/\eta-1) + (\gamma-1)y + p$$

or $p_i - p = c + \phi y$

4

d) Derive the "natural rates" of output and employment, that is the values that prevail when nothing stops firms from setting prices equal to $P_i^*$.

From $P_i = \frac{\eta}{\eta-1} W$, $\quad \frac{W}{P} = \left(\frac{\eta-1}{\eta}\right)$

$\& \; p_i = p$

so $L = Y = \left(\frac{\eta-1}{\eta}\right)^{\frac{1}{\gamma-1}}$