Economics 614, Advanced Macro, Spring 2024 Final examination ANSWERS

I do not expect you to have time to answer all of the questions on this exam. So look over the whole exam, identify the questions about which you are most sure of the answer, do those questions first, and then come back to the ones you are less sure about.

1) Consider an IS/MP model:

$Y = E(Y, r, G, T)$ where $r = i - \pi^e$ and $E_Y > 0$, $E_r < 0$, $E_G > 0$, $E_T < 0$

$r = r(Y, \pi)$ where $r_Y > 0$, $r_Y > 0$

$\pi = \pi(Y)$ where $\pi_Y > 0$

a) Derive expressions for $\partial r / \partial G$ and $\partial Y / \partial G$. *4 pts. Endogenous variables are Y, r, π. Exogenous variables are G, T.*

$$\left.\begin{aligned} &\text{From } Y = E(Y, r, G, T), \quad \frac{\partial Y}{\partial G} = E_Y \frac{\partial Y}{\partial G} + E_r \frac{\partial r}{\partial G} + E_G \\ &\text{From } r = r(Y, \pi), \quad \frac{\partial r}{\partial G} = r_Y \frac{\partial Y}{\partial G} + r_\pi \frac{\partial \pi}{\partial G} \\ &\text{From } \pi = \pi(Y), \quad \frac{\partial \pi}{\partial G} = \pi_Y \frac{\partial Y}{\partial G}. \end{aligned}\right\} \text{3 equations, 3 unknowns.}$$

$$\frac{\partial Y}{\partial G} = \frac{E_G}{1 - E_Y - E_r(r_Y + r_\pi \pi_Y)}$$

$$\frac{\partial r}{\partial G} = \frac{\partial Y}{\partial G} r_Y + \frac{\partial Y}{\partial G} \pi_Y r_\pi$$

b) In an IS/MP model, the central bank is adjusting the supply of real money balances to keep r at the value given by the interest-rate rule. Assume $M^S / P = L(i, Y)$ where $L_i < 0$, $L_Y > 0$ and $\pi^e = 0$.

Derive an expression for $\partial(M^S / P) / \partial G$. Hint: use your answer to a)! *4 pts.*

$$\frac{\partial (M^S/P)}{\partial G} = L_i \frac{\partial i}{\partial G} + L_Y \frac{\partial Y}{\partial G} = L_i \frac{\partial r}{\partial G} + L_Y \frac{\partial Y}{\partial G}$$

2) Consider a model like Romer's "baseline" RBC model. To simplify it,
- there are no taxes or government spending
- there is no population growth
- there is no trend growth in total factor productivity.

A representative household has one unit of time which it divides between leisure and work. It maximizes

$$U_t = E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left(\ln C_{t+\tau} + b \ln\, (1 - l_{t+\tau})\right) \qquad \textit{where } 0 < \beta < 1\,,\ 0 < b < 1$$

where C is household consumption per household and l is the fraction of household time devoted to labor.

The household saves by holding capital (it can turn one unit of real income into one unit of capital). The capital it will hold in period $t+1$ is $K_{t+1} = Y_t - C_t + (1-\delta)K_t$ where δ is the depreciation rate and Y is (real) household income.

One part of household income is (real) labor income. The household earns a real wage w for every unit of labor supplied, so real labor income in period t is $w_t l_t$.

The other part of household income is income from renting out capital. The real rental rate or "real interest rate" is r, which is equal to the marginal product of capital minus the depreciation rate. Thus the household's (real) income from renting out capital in period t is $r_t K_t$.

Note that I didn't give the produciton function. This is not to be solved from the point of view of a social planner; it is to be solved from the household's point of view.

a) Write down the value function and the intertemporal budget constraint. *4 pts.*

$$V_t = \max \left[\ln C_t + b \ln (1 - l_t) + \beta E_t V(K_{t+1}) \right]$$

$$\text{s.t. } K_{t+1} = w_t l_t + r_t K_t - C_t + (1-\delta) K_t$$

b) Using the value function, derive current consumption C_t as a function of the representative agent's beliefs about future consumption C_{t+1} and the future real interest rate r_{t+1}. Notice I did not say "assume certainty equivalance." *4 pts. Here you are deriving the Euler equation. You use what I call the "Benveniste-Scheinkman equation," which is an application of the envelope theorem. Intuitively, at the optimum, it should be impossible for the household to increase his lifetime utility by doing anything. One possible thing he could do is to:*

1) increase C_t by ∂C_t, holding all other choice variables in period t unchanged, which reduces K_{t+1} by an amount $\partial K_{t+1} / \partial C_t$. The increase in C_t affects his lifetime utility according to the marginal utility of consumption.
2) reduce C_t by enough to leave all other t+1 variables and also K_{t+2} unchanged. The decrease in C_{t+1} will reduce his utility according to the discounted marginal utility of consumption.

$$0 = \frac{\partial V_t}{\partial C_t} = \frac{1}{C_t} + \beta E_t \left[\frac{\partial V}{\partial K_{t+1}} \cdot \frac{\partial K_{t+1}}{\partial C_t} \right]$$

$$\frac{\partial K_{t+1}}{\partial C_t} = -1 \quad \text{and} \quad \left.\frac{\partial V}{\partial K_{t+1}}\right|_{\partial K_{t+2} = 0} = \frac{1}{C_{t+1}} \cdot \left.\frac{\partial C_{t+1}}{\partial K_{t+1}}\right|_{\partial K_{t+2} = 0} = \frac{1}{C_{t+1}} (1 + r_{t+1})$$

so $$0 = \frac{1}{C_t} + \beta E_t\left[(-1)\frac{1}{C_{t+1}}(1+r_{t+1})\right]$$

$$C_t = \frac{1}{\beta}\,\frac{1}{E_t\left[\frac{1+r_{t+1}}{C_{t+1}}\right]}$$

Note: without certainty equivalence, above is not necessarily equal to $\frac{1}{\beta} E_t \frac{C_{t+1}}{1+r_{t+1}}$

c) Now consider the "nonstochastic long-run steady state." Assume that there is no trend growth in total factor productivity, what is the steady state real interest rate r^*? 4 pts. In nonstochastic LRSS, $C_t = C_{t+1} = \bar{C}$. So take answer to b), substitute that in, solve for r.

$$\bar{C} = \frac{1}{\beta}\,\bar{C}\,\frac{1}{1+\bar{r}}$$

$$1 = \frac{1}{\beta}\,\frac{1}{1+\bar{r}}$$

$$1+\bar{r} = \frac{1}{\beta}$$

$$\bar{r} = \frac{1}{\beta} - 1$$ Note: since $\beta < 1$, $\bar{r} > 0$.

3) Recall that King and Rebelo (1999), "Resuscitating Real Business Cycles," incorporate "indivisible labor" and "consumption insurance" into their model.

a) What is the set of assumptions that we refer to as "indivisible labor" and "consumption insurance"?

b) Why do King and Rebelo make these assumptions for their model? Why add this complication?

6 pts total. See notes and paper. This is not about variable capacity utilitzation, by the way.

4) Consider a Keynesian model in which the expectations-augmented Phillips curve is of the Friedman-Phelps type:

$$y_t = -r_t \qquad \pi_t = E_{t-1}\pi_t + y_t + \epsilon_t$$

where y is the output gap (the difference between log output and the log of the natural rate of output). r is the difference between the real interest rate and the natural rate of interest. $E_{t-1}\pi_t$ is the public's expected value for this period's inflartion rate, as of last period. ϵ is a "supply shock" or "cost-push shock" to the Phillips curve. ϵ is "white noise" (no serial correlation, expected value equal to zero) with variance σ_ϵ^2.

The central bank sets r to minimize a loss function: $L_t = \frac{1}{2}E[y_t^2] + \frac{1}{2}E[\pi_t^2]$, where $E[x]$ denotes the central bank's expected value for a variable x. This is not necessarily the same as the public's expected vakue for that variable.

At the time the central bank chooses r_t, it knows $E_{t-1}\pi_t$ and it *also* knows ϵ_t.

a) Write down an equation that gives loss L_t as a function of r_t. *2 pts. Some of you did not notice that I said the central bank knows ϵ_t when it sets r_t. Whoops!*

$$L_t = \tfrac{1}{2}(-r_t)^2 + \tfrac{1}{2}(\pi^e - r_t + \varepsilon_t)^2$$

b) Use your answer to a) to derive the value of r_t that the central bank will set, as a function of $E_{t-1}\pi_t$ and ϵ_t. *2 pts.*

$$0 = \frac{\partial L}{\partial r_t} = (-r_t)(-1) + (\pi^e - r_t + \varepsilon_t)(-1)$$

$$r_t = \frac{\pi^e + \varepsilon_t}{2}$$

c) Given your answer to b), what will π_t be, as a function of $E_{t-1}\pi_t$ and ϵ_t? *2 pts.*

$$\pi_t = \pi^e - \frac{\pi^e + \varepsilon_t}{2} + \varepsilon_t = \tfrac{1}{2}(\pi^e + \varepsilon_t)$$

d) Now assume that the public has rational expectations. At the time that the public forms its expectation $E_{t-1}\pi_t$, it does not know what ϵ_t will turn out to be, but it does know the distribution of ϵ. So what is the value of $E_{t-1}\pi_t$? *4 pts.*

$$E_{t-1}\pi_t = \tfrac{1}{2}E_{t-1}\pi_t + \tfrac{1}{2}E_{t-1}[\varepsilon_t] = \tfrac{1}{2}E_{t-1}\pi_t$$

so $E_{t-1}\pi_t = 0$

5) Here is a simple model of output determination. y is the deviation of output from its LRSS path. c is the deviation of consumption from its LRSS path. v is the deviation of investment from its LRSS path. r is the deviation of the real interest rate from its LRSS path. $y_t = c_t + v_t$

Consumption is a positive function of current output: $c_t = \beta y_t$

Investment depends on expected future output and the real interest rate gap: $v_t = \alpha E_t y_{t+1} - \lambda r_t$

Finally, r is known to follow an AR(1) process: $r_t = \rho r_{t-1} + \epsilon_t \quad where\ \epsilon\ is\ i.i.d.\ and\ 0 < \rho < 1$

a) Using the first three equations, write an equation that gives y_t in terms of $E_t y_{t+1}$ and r_t. *4 pts.*

$$y_t = c_t + v_t = \beta y_t + \alpha \overset{e}{y}_{t+1} - \lambda r_t$$

$$y_t = \frac{\alpha}{1-\beta} \overset{e}{y}_{t+1} - \frac{\lambda}{1-\beta} r_t$$

b) From your answer to a), you can see that you can solve this model if you make an assumption about the relative value of some parameters. What is this assumption? *4 pts. To solve back from the long-run steady state, the coefficient on the expected future variable must be positive and less than one. So:*

$$0 < \frac{\alpha}{1-\beta} < 1 \quad \text{so} \quad \alpha < 1-\beta, \quad \beta < 1$$

c) Assume the condition in b) holds and solve the model: get y_t, c_t, v_t as functions of ϵ_t.

4 pts. Using the math trick,

$$y_t = -\frac{\lambda}{1-\beta-\alpha\rho} r_t = -\frac{\lambda}{1-\beta-\alpha\rho}(\rho r_{t-1} + \varepsilon_t)$$

$$c_t = \beta y_t$$

$$v_t = \alpha\rho y_t - \lambda r_t = \ldots$$

6) Consider an economy whose economy can be described by the "Lucas supply function" model. In the past, the country has been subject to many random, unpredictable money-supply shocks. At time t_0 the country unexpectedly adopts a new monetary regime under which there are fewer unpredictable money-supply shocks. The change in regime is immediately communicated to the public, and understood by them. Consider the expectations-augmented Phillips curve: $\pi_t = E_{t-1}\pi_t + \beta y_t$. At time t_0 , does the parameter β increase, decrease, or remain the same? Explain.

8 pts for a good explanation. This is a decrease in V_M . See the notes and textbook.

7) Consider the following "three-equation" model:

(i) $y_t = E_t y_{t+1} - s r_t + u_t$ where $u_t = \rho u_{t-1} + \epsilon_t$ and ϵ is mean-zero i.i.d.

(ii) $\pi_t = E_t \pi_{t+1} + \kappa y_t$

(iii) $r_t = \phi \pi_t$

where y is the output gap and r is the gap between the real interest rate and the natural rate of interest. Expectations are rational.

Assume there is a long-run steady state where $y = 0,\ \pi = 0$.

a) Derive equations that give π_t and r_t as functions of u_t .

5 pts. See the notes.

b) Let $\bar{r}_t$ denote the natural rate of interest in a period, so that r_t as defined above is equal to the real interest rate in the period *minus* $\bar{r}_t$. Make an equation that gives the nominal interest rate i_t as a function of u_t and $\bar{r}_t$.

5 pts. $i_t = \bar{r} + r_t + E_t \pi_{t+1}$ where $E_t u_{t+1} = \rho u_t$. Substitute in from answers to a).

8) Consider a model with a competitive labor market with a market-clearing nominal wage W per unit of labor. A representative-agent household maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\theta} C_t^{1-\theta} + \frac{1}{1-\nu} X_t (M_t / P_t)^{1-\nu} - \frac{1}{1+\lambda} L_t^{1+\lambda} \right]$$

X is an *exogenous* variable that can vary over time, but fluctuations in X are transitory and unpredictable so that $E_t X_{t+1} = 1$ in all periods. The agent's nominal wealth evolves as $A_{t+1} = M_t + (A_t + W_t L_t - P_t C_t - M_t)(1 + i_t)$

At time t, the agent takes as given A_t , the wage W_t and the price level P_t , and chooses consumption, labor and his real money balance. Assume "certainty equivalence" holds, so that in the agent's optimization problem you take expected values of future variables to be equivalent to actual known values of future variables.

a) Derive an equation that gives the agent's demand for real money balance $(M / P)_t$ as a function of consumption C_t , the nominal interest rate i_t and any other relevant things. *4 pts. You should know how to do this from the notes etc.*

$$\frac{\partial A_{t+1}}{\partial (M/P)_t} = \frac{1}{P_t} \frac{\partial A_{t+1}}{\partial M_t} = \frac{1}{P_t}\left(1 - (1+i_t)\right) = \frac{1}{P_t}(-i_t)$$

$$\left(\frac{M}{P}\right)_t = X_t^{1/\nu}\, C_t^{\theta/\nu} \left(\frac{i}{1+i}\right)_t^{-1/\nu}$$

b) Derive an equation that gives C_t as a function of E_tC_{t+1}, i_t, P_t and E_tP_{t+1}. *4 pts. Some of you gave answers that did not have P_t in them, or did not have E_tP_{t+1} in them. You should have noticed this, tried to figure out what was wrong.*

$$C_t = E_t C_{t+1} \left[\beta (1+i_t) \frac{P_t}{P_{t+1}} \right]^{-1/\theta}$$

c) Let π denote the inflation rate so that $P_{t+1} = (1+\pi_{t+1})P_t$. Derive an equation that gives the log of consumption c_t as a function of E_tc_{t+1} and the "real interest rate" $r_t = i_t - E_t\pi_{t+1}$. Use the standard approximations including $\ln(1+r) \approx r$ (for "small" r). *4 pts. See notes.*

$$c_t = E_t c_{t+1} - \frac{1}{\theta} \ln\beta - \frac{1}{\theta} r_t$$

d) Suppose that the representative agent lives in a closed economy. The production function is fixed, with no variations in productivity. The capital stock is fixed, so there is no investment. There is a government that purchases a quantity of output G_t in a period. Thus output $Y_t = C_t + G_t$. The *share* of output purchased by the government is $\varphi_t = (G/Y)_t$ where φ is variable and exogenous, but always "small" like the real interest rate. Derive an equation that gives the log of output y_t as a function of E_ty_{t+1}, r_t, and any other relevant variables. *4 pts. See notes.*

$$y_t = E_t y_{t+1} - \frac{1}{\theta} \ln\beta - \frac{1}{\theta} r_t + \varphi_t - E_t \varphi_{t+1}$$

e) Now suppose that all shocks to this economy are transitory and unpredictable so that $E_tY_{t+1} = \bar{Y}$, $E_t\varphi_{t+1} = \varphi$ and $E_t\pi_{t+1} = 0$ in all periods. Recall that above I already said $E_tX_{t+1} = 1$ in all periods. Suppose also that there is a central bank that wants to stabilize output. It can operate by fixing the interest rate for a period, or by fixing the money supply. Either way, it must choose the period-t value of its control variable (interest rate or money supply) *before* the realization of X_t and φ_t for the period. Would it be better for the central bank to fix the interest rate, or the money supply, or is it ambiguous? Explain your answer. *5 pts. This should have made you think about the Poole (11970) paper. The spending equation, in log terms, is:*

$$y_t = \bar{y} - \frac{1}{\theta} \ln\beta - \frac{1}{\theta} i_t + \varphi_t - \bar{\varphi}$$

The LM equation is:

$$(m^s - p)_t = \frac{1}{\nu} \ln x_t + \frac{\theta}{\nu} (y_t - \varphi_t) - \frac{1}{\nu} \ln\left(\frac{i}{1+i}\right)_t$$

Note that the variable x affects just the LM curve, while the variable φ affects both the IS and LM curves. A good answer would be "Ambiguous. If X shocks are big and φ shocks are small, it is best to fix i. If φ shocks are big and X shocks are small, it is still ambriguous because φ shocks affect both equations." (Try drawing it.)