Economics 614, Advanced Macro, Spring 2025 Final examination

1) Recall the new Keynesian expectations-augmented Phillips curve $\pi_t = E_t\pi_{t+1} + \kappa y_t$. y is the output gap. Assuming rational expectations, this equation is not consistent with the actual behavior of output and inflation. Explain, using equations.

10 pts

$\pi_{t+1} = E_t \pi_{t+1} + \varepsilon_{t+1}$ so

$\pi_t = \pi_{t+1} - \varepsilon_{t+1} + \kappa y_t$

$\pi_{t+1} - \pi_t = -\kappa y_t + \varepsilon_{t+1}$.

Under rational expectations, error ε_{t+1} must be uncorrelated with y_t. So if model is correct, upcoming change in inflation should be negatively correlated with output gap. In terms of regression...

Not true in data!

2) In the context of financial markets, what is a "fire sale"?

10 pts

See notes.

3) In the Diamond-Dybvig model, what are the two types of "agents"? How are they different?

10 pts

"Patient," who get utility from consumption in periods 1 and 2, and "impatient," who get utility only from consumption in period 1. See notes.

4) Consider a model with a competitive labor market with a market-clearing nominal wage W per unit of labor. A representative-agent household maximizes

$$E_t \sum_{t=0}^{\infty} \beta^t \left[\frac{1}{1-\theta} C_t^{1-\theta} + \frac{1}{1-\nu} (M/P)_t^{1-\nu} - \frac{1}{1+\lambda} L_t^{1+\lambda} \right]$$

The agent's nominal wealth evolves as $A_{t+1} = M_t + (A_t + W_t L_t - P_t C_t - M_t)(1+i_t)$

At time t, the agent takes as given A_t , the wage W_t and the price level P_t , and chooses consumption, labor and his real money balance. Assume "certainty equivalence" holds, so that in the agent's optimization problem you take expected values of future variables to be equivalent to actual known values of future variables.

a) Write down the Bellman equation for the agent's problem.

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$V_t = \max \left[\frac{1}{1-\theta} C_t^{1-\theta} \cdots + \beta E_t V_{t+1} \right]$

s.t. $A_{t+1} = \ldots$

b) Derive an equation that gives the agent's demand for real money balance $(M/P)_t$ as a function of consumption C_t and any other relevant variables.

(4)

$$0 = \frac{\partial V_t}{\partial (M/P)_t} = (M/P)_t^{-\nu} + \beta E_t \frac{\partial V_{t+1}}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial (M/P)_t}$$

$$\frac{\partial A_{t+1}}{\partial (M/P)_t} = [1-(1+i_t)] \frac{\partial M_t}{\partial (M/P)_t} = [1-(1+i_t)] P_t$$

$M_t = (M/P)P$, so....

$$\text{so } \frac{\partial A_{t+1}}{\partial (M/P)_t} = (M/P)_t^{-\nu} + \beta(-i_t) P_t \frac{\partial E_t V_{t+1}}{\partial A_{t+1}}$$

$$0 = \frac{\partial V_t}{\partial C_t} = C_t^{-\theta} + \beta E_t \frac{\partial V_{t+1}}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial C_t}$$

$-P_t(1+i_t)$

$$\text{so } \frac{\partial V_t}{\partial C_t} = C_t^{-\theta} (-P_t(1+i_t)) \frac{\partial E_t V_{t+1}}{\partial A_{t+1}}$$

$$\Rightarrow (M/P)_t = C_t^{\theta/\nu} \left(\frac{i_t}{1+i_t}\right)^{-\frac{1}{\nu}}$$

c) Derive an equation that gives the quantity of labor the household chooses to supply at time t, L_t^S, as a function of the real wage $(W/P)_t$ and consumption C_t.

(4)

$$0 = \frac{\partial V_t}{\partial L_t} = -L_t^{\lambda} + \beta_t E_t \frac{\partial V_t}{\partial A_t} \frac{\partial A_{t+1}}{\partial L_t}$$

$w_t(1+i_t)$

$$\Rightarrow L_t = C_t^{-\frac{\theta}{\lambda}} (w/P)_t^{\frac{1}{\lambda}}$$

d) What is the elasticity of labor supply?

(4)

Take logs

$$l_t = -(\theta/\lambda) c_t + \frac{1}{\lambda}(w-p)_t$$

elasticity

e) Using the usual approximations and definitions, derive an equation that gives C_t as a function of $E_t C_{t+1}$ and the "real interest rate" $r_t = i_t - E_t \pi_{t+1}$.

From above $0 = C_t^{-\theta} + \beta E_t \frac{\partial V}{\partial A_{t+1}} \left(-(1+i_t) P_t\right)$

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Envelope theorem (or Benveniste–Scheinkman) says

$$\frac{\partial V}{\partial A_{t+1}} = \frac{\partial u}{\partial C_{t+1}} \cdot \frac{\partial C_{t+1}}{\partial A_{t+1}} \bigg|_{\partial A_{t+2} = 0} = C_{t+1}^{-\theta} \frac{1}{P_{t+1}}$$

so

$$0 = C_t^{-\theta} + \beta E_t C_{t+1}^{-\theta} \frac{1}{E_t P_{t+1}} \left(-(1+i_t) P_t\right)$$ using certainty equivalence

gives

$$C_t = E_t C_{t+1} \left[\beta (1+i_t) \frac{P_t}{E_t P_{t+1}}\right]^{-\frac{1}{\theta}}$$

using $E_t P_{t+1} = E_t(1+\pi_{t+1})$,

$$C_t = E_t C_{t+1} \left[\beta (1+i_t) \frac{1}{E_t \pi_{t+1}}\right]^{-\frac{1}{\theta}}$$

Using $\frac{1+i_{t+1}}{1+\pi_{t+1}} \approx 1 + r_t$,

$$C_t = E_t C_{t+1} \left[\beta (1+r_t)\right]^{-\frac{1}{\theta}}$$

f) e) Suppose you draw a labor supply curve for this model in log terms, that is a graph with the log of the real wage $(w-p)_t$ on the *vertical* axis and the log quantity of labor per household l_t on the *horizontal* axis, assuming that that $E_t C_{t+1} = \overline{C}$, where $\overline{C}$ denotes the long-run steady state level of consumption. What happens to this labor supply curve in as there is an increase in the real interest rate r_t ? Explain how you know.

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From $L_t = C_t^{-\frac{\theta}{\chi}} (W/P)_t^{\frac{1}{\chi}}$

and $C_t = E_t C_{t+1} \left[\beta(1+r_t)\right]^{-\frac{1}{\theta}}$

we have $L_t = \left(\overline{C}\,(\beta(1+r_t))^{-\frac{1}{\theta}}\right)^{-\frac{\theta}{\chi}} (w/P)_t^{\frac{1}{\chi}}$

$= \overline{C}^{-\frac{\theta}{\chi}} \beta^{\frac{1}{\chi}} (1+r_t)^{\frac{1}{\chi}} (w/P_t)^{\frac{1}{\chi}}$

See that a decrease in C_t increase L^s at any given w/P. That means out/down shift in L^s curve

Take logs

$$l_t = -\frac{\theta}{\chi}\bar{c} - \frac{1}{\chi}\ln\beta + \frac{1}{\chi}\ln(1+r_t) + \frac{1}{\chi}(w-p)_t$$

Approximation $\ln(1+r_t) \approx r_t$

$$l_t = -\frac{\theta}{\chi}\bar{c} - \frac{1}{\chi}\ln\beta + \frac{1}{\chi}r_t + \frac{1}{\chi}(w-p)_t$$

Equation of labor supply curve:

$$\frac{1}{\chi}(w-p)_t = l_t - \frac{\theta}{\chi}\bar{c} - \frac{1}{\chi}\ln\beta - \frac{1}{\chi}r_t$$

$$(w-p)_t = \chi l_t - \theta\bar{c} - \ln\beta - r_t$$

So $r_t\uparrow$ shifts L^s curve down (out).

g) Consider drawing a new Keynesian IS curve for this model with the real interest rate on the vertical axis and the log of output on the horizontal axis, by assuming that output Y_t is equal to consumption C_t and $E_t C_{t+1} = \overline{C}$. Suppose there are two economies described by this model. In economy A, the representative agent is extremely risk averse. In economy B, the representative agent is less risk-averse. Is this IS curve steeper for economy A, steeper for economy B, or do both curves have the same slope? Explain.

with $Y_t = C_t$,

f) $Y_t = E_t Y_{t+1} [\beta(1+r_t)]^{-\frac{1}{\theta}}$

with $E_t C_{t+1} = \overline{C}$,

$Y_t = \overline{Y} [\beta(1+r_t)]^{-1/\theta}$

Logs:

$y_t = \overline{y} - \frac{1}{\theta} \ln\beta - \frac{1}{\theta} \ln(1+r_t)$

with $\ln(1+r_t) \approx r_t$,

$y_t = \overline{y} - \frac{1}{\theta} \ln\beta + \frac{1}{\theta} r_t$

Equation of IS curve:

$r_t = \theta \overline{y} - \theta y_t - \ln\beta$

Slope is $-\theta$.

More risk averse means θ bigger (MU of C diminishes faster) so IS curve is steeper in economy A.

5) Consider a new Keynesian model in which prices are "indexed" to the long-run steady-state inflation rate $\bar{\pi}$. The fixed probability that a firm can fully reoptimize is price in a period is α . Derive an equation that gives π_t in terms of x_t, p_{t-1} and $\bar{\pi}$, where p_t is the log of the price level in period *t*, x_t is the log of the price set by firms that are able to fully reoptimize their prices in period *t*, and an inflation rate is approximated as $\pi_{t+1} = p_{t+1} - p_t$.

P

$$p_t = \alpha x_t + (1-\alpha)(\bar{\pi} + p_{t-1})$$

$$= \alpha x_t + (1-\alpha)\bar{\pi} + p_{t-1} - \alpha p_{t-1}$$

$$p_t - p_{t-1} = \alpha x_t + (1-\alpha)\bar{\pi} - \alpha p_{t-1}$$

$$\pi_t = (1-\alpha)\bar{\pi} + \alpha(x_t - p_{t-1})$$

6) Consider the following "three-equation" model:

(i) $y_t = E_t y_{t+1} - r_t$

(ii) $\pi_t = E_t \pi_{t+1} + y_t + u_t$ where $u_t = \rho u_{t-1} + \epsilon_t$ and ϵ is mean-zero i.i.d.

(iii) $r_t = \phi \pi_t$

where y is the output gap and r is the gap between the real interest rate and the natural rate of interest. Expectations are rational.

Assume there is a long-run steady state where $y = 0$, $\pi = 0$.

a) Derive equations that give π_t and y_t as functions of u_t.

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See notes, "effect of u" AS n

Conjecture $E\pi_{t+1} = \rho_\pi \pi_t$

$y_t = E y_{t+1} - r_t = E y_{t+1} - \phi \pi_t$

Apply math trick

$y_t = -\frac{\phi}{1-\rho_\pi} \pi_t$

Substitute into (ii)

$\pi_t = E\pi_{t+1} - \frac{\phi}{1-\rho_\pi} \pi_t + u_t$

$\left(1+\frac{\phi}{1-\rho_\pi}\right)\pi_t = E\pi_{t+1} + u_t$

$\pi_t = a E\pi_{t+1} + a u_t$ where $a = \frac{1}{1+\phi/1-\rho_\pi}$

Apply math trick.

$\pi_t = \frac{a}{1-a\rho_\pi} u_t$ See that conjecture was correct; and $\rho_\pi = \rho$

$\pi_t = \frac{1}{\frac{1}{a}-\rho} = \frac{1}{1+\frac{\phi}{1-\rho}-\rho} u_t$

$y_t = -\frac{\phi}{1-\rho} \frac{1}{1+\rho+\frac{\phi}{1-\rho}} u_t = -\frac{\phi}{(1-\rho)^2+\phi} = -\frac{1}{\frac{(1-\rho)^2}{\phi}+\frac{1}{1-\rho}}$

b) Derive an equation that gives the nominal interest rate i_t as a function of u_t and the natural rate of interest $\overline{r_t}$.

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$$i_t = \overline{r}_t + r_t + E_t \pi_{t+1} = \overline{r}_t + \phi \pi_t + E_t \pi_{t+1}$$

$$= \overline{r}_t + \phi \frac{1}{1-\rho+\frac{\phi}{1-\rho}} u_t + \frac{1}{1-\rho+\frac{\phi}{1-\rho}} E_t u_{t+1}$$

$$E_t u_{t+1} = \rho u_t \quad \text{so}$$

$$i_t = \overline{r}_t + \frac{\phi}{1-\rho+\frac{\phi}{1-\rho}} u_t + \frac{1}{1-\rho+\frac{\phi}{1-\rho}} \rho u_t$$

$$= \overline{r}_t + \frac{\phi+\rho}{1-\rho+\frac{\phi}{1-\rho}} u_t$$

c) Consider an alternative model in which (i) and (ii) hold, but (iii) does not. Instead of (iii), there is a central bank that chooses r_t to minimize:

$$L = \sum_{\tau=0}^{\infty} \beta^{\tau} E\left[\frac{1}{2} y_{t+\tau}^2 + \frac{1}{2} \pi_{t+\tau}^2\right]$$

after observing u_t, $E_t\pi_{t+1}$ and $E_t y_{t+1}$. Derive equations that give π_t and y_t as functions of u_t. Hint: remember Clarida, Gali and Gertler, "The Science of Monetary Policy: A New Keynesian Perspective," and note two things. First, the central bank has "discretion" in setting r_t. Second, the central bank knows with certainty the values of y_t and π_t that will result from any given r_t, because it knows u_t, $E_t\pi_{t+1}$ and $E_t y_{t+1}$.

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Under discretion, minimizing that loss function is equivalent to minimizing single-period loss period by period. When the central bank knows with certainty the values of y_t and π_t that will result from any given r_t, you can derive the central bank's behavior by describing it as choosing y directly (not r).

$$L = \tfrac{1}{2} y^2 + \tfrac{1}{2}(\pi^e + y + u)^2$$

$$0 = \frac{\partial L}{\partial y} = \tfrac{1}{2} 2y + \tfrac{1}{2} 2 (\pi^e + y + u) \cdot 1 = y + \pi^e + y + u$$

$$y = -\tfrac{1}{2}(u + \pi^e)$$

$$\pi = \pi^e - \tfrac{1}{2}(u + \pi^e) + u = \tfrac{1}{2}\pi^e + \tfrac{1}{2}u$$

Look! Equation for π can be solved by math trick!

Math trick: soln. to $x_t = \alpha x^e_{t+1} + \eta u_t$ is $x_t = \eta \frac{1}{1-\alpha\rho} u_t$

so here

$$\pi_t = \tfrac{1}{2} \frac{1}{1 - \frac{1}{2}\rho} u_t = \frac{1}{2-\rho} u_t$$

Or you could think about solving back from $t + \infty$.

$$\pi_{\infty-1} = \tfrac{1}{2} u_{\infty-1}$$

$$\pi_{\infty-2} = \tfrac{1}{2} \cdot \tfrac{1}{2} u_{\infty-1} + \tfrac{1}{2} u_{\infty-2}$$

$$\pi_{\infty-3} = \tfrac{1}{2} \cdot \tfrac{1}{2} \cdot \tfrac{1}{2} u_{\infty-1} + \tfrac{1}{2} \cdot \tfrac{1}{2} u_{\infty-2} + \tfrac{1}{2} u_{\infty-3}$$

so $\pi_t = \frac{1}{2} \sum_{\tau=0}^{\infty} \frac{1}{2}^{\tau} u_{t+\tau} = \frac{1}{2} \sum_{\tau=0}^{\infty} \frac{1}{2}^{\tau} \rho^{\tau} u_t = \frac{1}{2} \frac{1}{1-\frac{1}{2}\rho} u_t$

$$y_t = -\frac{1}{2}\left(u_t + \pi^e_{t+1}\right) = -\frac{1}{2} u_t - \frac{1}{2-\rho} u^e_{t+1}$$

$$= -\frac{1}{2} u_t - \frac{1}{2-\rho} \rho u_t$$

$$= -\left(\frac{1}{2} + \frac{\rho}{2-\rho}\right) u_t$$

7) Consider a model in which a representative-agent household maximizes $E_t \sum_{\tau=0}^{\infty} \beta^{\tau} \left[\frac{1}{1-\theta} C_{t+\tau}^{1-\theta} - \frac{1}{1+\lambda} L_{t+\tau}^{1+\lambda} \right]$

The agent can hold capital K and choose the capital utilization rate u_t. The agent earns a nominal rental rate R for each unit of "effective capital" $(uK)_t$. The agent's nominal wealth is A. At time t, the agent takes as given A_t, the wage W_t, the nominal interest rate i_t and the price level P_t. She chooses consumption, labor, investment I_t (purchases of more capital, or sales in which case I is negative) and u_t, among other things. The price of a unit of capital is always equal to the price of a unit of consumption. The effective-capital rental rate is determined in a competitive market. For a firm, capital is a "variable factor" like labor.

a) Suppose that the agent's nominal wealth evolves as:

$$A_{t+1} = (1+i_t)[A_t + W_t L_t + R_t u_t K_t - P_t(C_t + I_t + u_t^2 K_t)] + P_{t+1}[I_t + (1-\delta)K_t]$$

The term u^2 represents a current cost of running a unit of capital harder. δ is the depreciation rate (a fixed parameter). Starting from the Bellman equation, derive the value of u_t that the agent will set for period t.

$$V_t = \max \left[\frac{1}{1-\theta} C_t^{1-\theta} - \frac{1}{1+\lambda} L_t^{1+\lambda} + \beta E_t V_{t+1} \right] \text{ s.t. } A_{t+1} = \ldots$$

$$0 = \frac{\partial V_t}{\partial u_t} = \beta \frac{\partial E V_{t+1}}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial u_t}$$

$$= \beta \frac{\partial E V_{t+1}}{\partial A_{t+1}} \left((1+i_t)[R_t K_t - P_t 2 u_t K_t] \right)$$

$$0 = R_t K_t - P_t 2 u_t K_t$$

$$u_t = \tfrac{1}{2} (R/P)_t$$

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b) Now suppose that the agent's nominal wealth evolves as:

$$A_{t+1} = (1+i_t)[A_t + W_t L_t + R_t u_t K_t - P_t(C_t + I_t)] + P_{t+1}[I_t + (1-\theta u_t^2)K_t]$$

Here there is no current cost of running capital harder, but the depreciation rate increases with u_t.

Starting from the Bellman equation, derive the value of u_t that the agent will set for period t in terms of the real interest rate r, using the usual approximation that $(1+r)_t \approx (1+i_t)/(1+E_t\pi_{t+1})$ where π is the inflation rate. (5)

$$V_t = \ldots$$

$$0 = \frac{\partial V_t}{\partial u_t} = \beta \frac{\partial E V_{t+1}}{\partial A_{t+1}} \frac{\partial A_{t+1}}{\partial u_t}$$

$$0 = \beta \frac{\partial E V_{t+1}}{\partial A_{t+1}} \left((1+i_t) R_t K_t + P_{t+1} (-2\theta u_t) K_t \right)$$

$$(1+i_t) R_t K_t = P_{t+1} 2 u_t \theta K_t$$

$$u_t = \frac{1}{2\theta} \frac{1}{P_{t+1}} (1+i_t) R_t$$

Want to get it in terms of r_t where $1+r_t = \frac{1+i_t}{1+\pi_{t+1}}$

I see P_{t+1} in there. $P_{t+1} = P_t(1+\pi_{t+1})$

So

$$u_t = \frac{1}{2\theta} \frac{1}{P_t(1+\pi_{t+1})} (1+i_t) R_t = \frac{1}{2\theta} \frac{1+i_t}{1+\pi_{t+1}} \frac{R_t}{P_t}$$

$$u_t = \frac{1}{2\theta} \left(\frac{R}{P}\right)_t (1+r_t)$$

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c) In the model of King and Rebelo, "Resuscitating Real Business Cycles," capital utilization is variable.
i) Which of the two cases above, a) or b), does the model resemble?

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Case b)

ii) Why did the authors include this feature in their model? What purpose did it serve?

It allowed for cyclical downturn, an aboslute drop in output, to occur when productivity growth is below trend but not lower than it was in the previous period (no absolute drop required).

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d) In the model of Christiano, Eichenbaum and Evans, "Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy," capital utilization is variable.
i) Which of the two cases above does the model resemble?

Case a)

ii) Why did the authors include this feature in their model? What purpose did it serve?

It makes for greater real rigidity, which helps the model fit the data in several ways.