Economics 614 Macroeconomic Theory II
Problem on money demand and consumption

Suppose an economy's representative household is infinitely-lived and acts to maximize a utility function subject to *no* uncertainty - future values of variables are known with certainty. Felicity is increasing in consumption C_t , decreasing in the quantity of labor supplied L_t , and increasing in holdings of real money balance M_t/P_t , as follows:

$$U = \sum_{t=0}^{\infty} \beta^t \left(\ln(C_t) - \tfrac{1}{2}\theta L_t^2 + \frac{1}{1-\sigma} (M_t/P_t)^{1-\sigma} \right) \qquad \textit{where} \qquad 0<\beta<1$$

$$\text{subject to} \quad Z_{t+1} = \frac{P_t}{P_{t+1}} \left[\frac{M_t}{P_t} + (1+i_t)\ (Z_t - \frac{M_t}{P_t}) + W_t L_t - C_t \right]$$

where Z_{t+1} is real wealth entering period *(t+1)*, M_t is the nominal money balance held across period *t*, i_t is the nominal interest rate paid on nonmoney assets held across period *t*, and W_t is the real wage. At time *t*, the household takes Z_t as given and chooses consumption, labor supply, and real money balances to maximize this lifetime utility function.

a) Write down the value function for the household's problem. Hint: *Z* is a state variable.

b) Derive the quantity of real money balance M_t/P_t that a household will choose to hold, as a function of consumption C_t and the nominal interest rate i_t .

c) Derive C_t as a function of C_{t+1} , i_{t+1} and $\frac{P_t}{P_{t+1}}$

d) If the inflation rate is denoted π , then $P_{t+1} = (1+\pi)P_t$.
As long as *i* and π are realistically small (e.g. no more than 5 percent or 0.05), $1+r \approx \dfrac{1+i}{1+\pi}$

Using this notation rewrite your answer to c) in terms of C_{t+1} and the *real* interest rate *r*.