Keynesian DSGE

NK IS/LM

Persistent disturbances & interest-rate rule

Review

$$u_t = \rho u_{t-1} + \varepsilon_t$$

$$E_t u_{t+i} = \rho^i u_t$$

(same coefficient)

$$x_t = z u_t$$

$$E_t x_{t+i} = z \rho^i u_t = \rho^i x_t$$

$$x_t = \alpha E_t x_{t+1} + \eta u_t \qquad 0 < \alpha < 1$$

LRSS: $x_\infty = x_{\infty+1} = \dots$ means $x_\infty = 0$, $u_\infty = 0$

$$x_{\infty-1} = \eta u_{\infty-1}$$

$$x_{\infty-2} = \alpha \eta u_{\infty-1} + \eta u_{\infty-2}$$

$$x_{\infty-3} = \alpha^2 \eta u_{\infty-1} + \alpha \eta u_{\infty-2} + \eta u_{\infty-3}$$

hence

$$x_t = \eta u_t + \alpha \eta E_t u_{t+1} + \alpha^2 \eta E_t u_{t+2} + \dots = E_t \eta \sum_{i=0}^{\infty} \alpha^i u_{t+i}$$

$$= \eta \sum_{i=0}^{\infty} \alpha^i \rho^i u_t = \eta \frac{1}{1-\alpha\rho} u_t$$

("math trick")

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--- & interest-rate rule

$$y_t = {}_t y^e_{t+1} - s r_t + u^{IS}_t \quad \text{where} \quad u^{IS}_t = \rho_{IS} u^{IS}_{t-1} + \varepsilon^{IS}_t$$

$$\pi_t = {}_t \pi^e_{t+1} + k y_t + u^{AS}_t \quad \text{where} \quad u^{AS}_t = \rho_\pi u^{AS}_{t-1} + \ldots$$

$$r_t = \phi_\pi \pi_t + \phi_y y_t + u^{mp}_t \quad \text{where} \ldots \quad \rho_{mp} \ldots$$

y, π & r will be functions of u^{IS}, u^{π}, u^{mp}

For simplicity, $r_t = \phi \pi_t$

this satisfies condition for LRSS w/ stable $\hat{\pi}$:

if $\pi^e = \hat{\pi}$ (here, zero) then $r = 0$ (real interest rate = natural rate)

etc.

Say u's $= 0$, $y^e_{t+1} = 0$ & $\pi_t = {}_t\pi^e_{t+1} + k y_t$

then if $\pi^e = \hat{\pi}$, $r = 0 \rightarrow \pi_t = \pi^e$

if $\pi^e > \hat{\pi}$, $r > 0 \rightarrow y < 0 \rightarrow \pi_t < \pi^e$

etc.

We'll take each disturbance one by one, set others to zero.

interest-rate rule

1) Effect of u^{IS}

Conjecture $y^e_{t+1} = \rho_y y_t$

hence from $\pi_t = \pi^e_{t+1} + \kappa y_t = \kappa \frac{1}{1-\rho_y} y_t$

($\alpha = 1$) (γ) (apply math trick)

hence $r_t = \phi \kappa \frac{1}{1-\rho_y} y_t$

$$y_t = y^e_{t+1} - s\phi\kappa \frac{1}{1-\rho_y} y_t + u^{IS}_t$$

$$\Rightarrow y_t = a\, y^e_{t+1} + a\, u^{IS}_t \quad \text{where } a = \frac{1}{1 + s\phi\kappa \frac{1}{1-\rho_y}}$$

(α) (γ)

apply math trick

$$y_t = a \frac{1}{1 - a\rho_y} u^{IS}_t \qquad \text{so } \rho_y = \rho_{IS}$$

$$= \frac{1}{\frac{1}{a} - \rho_{IS}} u^{IS}_t$$

$$\pi_t = \kappa \frac{1}{1-\rho_{IS}} y_t$$

$$r_t = \phi \pi_t$$

so $u^{IS} \uparrow \rightarrow y\uparrow, \pi\uparrow, r\uparrow$

2) Effect of u^{IS} (cont.)

How do structural coefficients affect response?

$$y_t = \frac{1}{1-\rho_{IS} + \frac{s \phi k}{1-\rho_{IS}}} u_t^{IS}$$

$$\pi_t = \frac{1}{\frac{(1-\rho_{IS})^2}{k} + s\phi} u_t^{IS}$$

$$r_t = \frac{1}{\frac{(1-\rho_{IS})^2}{\phi k} + s} u_t^{IS}$$

If $\phi \uparrow$ (coeff. on π in irr is bigger),

$u^{IS}\uparrow$ has bigger effect on r, smaller effect on y & π.

3) Effect of u^{AS}

Conjecture $\pi^e_{t+1} = \rho_\pi \pi_t$

$$y_t = y^e_{t+1} - s r = y^e_{t+1} - s \phi \pi_t$$

($\alpha = 1$) ($\eta = -s\phi$)

apply math trick

$$y_t = -s\phi \frac{1}{1-\rho_\pi} \pi_t$$

hence

$$\pi_t = \pi^e_{t+1} - k s \phi \frac{1}{1-\rho_\pi} \pi_t + u^{AS}_t$$

$$\Rightarrow \pi_t = a\, \pi^e_{t+1} + a u^{AS}_t \quad \text{where } a = \frac{1}{1 + sk\phi \frac{1}{1-\rho_\pi}}$$

apply math trick

$$\pi_t = a \frac{1}{1 - a\rho_\pi} u^{AS}_t \quad \text{so } \rho_\pi = \rho_{AS}$$

$$= \frac{1}{\frac{1}{a} - \rho_{AS}} u^{AS}_t$$

$$y_t = -s\phi \frac{1}{1-\rho_{AS}} \pi_t$$

$$r_t = \phi \pi_t$$

3) Effect of u^{AS} (cont.)

$$\pi_t = \frac{1}{1-\rho_\pi + \frac{sk\phi}{1-\rho_{AS}}} u_t^{AS}$$

$$Y_t = -\frac{1}{\frac{(1-\rho_{AS})^2}{s\phi} + \frac{k}{1-\rho_{AS}}} u_t^{AS}$$

$$r_t = \frac{1}{\frac{1-\rho_\pi}{\phi} + \frac{sk}{1-\rho_{AS}}} u_t^{AS}$$

If $\phi \uparrow$, $u^{AS} \uparrow$ has bigger effect on r,

smaller effect on π

but <u>bigger</u> effect on Y.

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3) Effect of u^{mp}

Again conjecture ρ_y so $\pi_t = \kappa \frac{1}{1-\rho_y} y_t$

so $r_t = \phi \pi_t + u_t^{mp} = \phi \kappa \frac{1}{1-\rho_y} y_t + u_t^{mp}$

so $y_t = y_{t+1}^e - s r = y_{t+1}^e - \frac{s\phi\kappa}{1-\rho_y} y_t - s u_t^{mp}$

$\rightarrow y_t = a\, y_{t+1}^e - s a\, u_t^{mp}$ where $a = \frac{1}{1 + \frac{s\phi\kappa}{1-\rho_y}}$

(a ↑ α; sa ↑ γ)

apply math trick

$y_t = -s a \frac{1}{1 - a\rho_y} u_t^{mp}$ so $\rho_y = \rho_{mp}$

$= -\frac{s}{\frac{1}{a} - \rho_{mp}} u_t^{mp}$

$\pi_t = -\kappa \frac{1}{1-\rho_{mp}} y_t$

$r_t = \phi \pi_t + u_t^{mp}$

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3) Effect of u^{mp} (cont.)

$$y_t = -\frac{1}{\frac{1-\rho^{mp}}{s} + \frac{s k \phi}{1-\rho^{mp}}} u_t^{mp}$$

$$\pi_t = -\frac{1}{\frac{1-\rho^{mp}}{s k} + \frac{\phi}{1-\rho^{mp}}} u_t^{mp}$$

$$r_t = \frac{\frac{1-\rho^{mp}}{s k \phi} + \frac{\rho^{mp}}{1-\rho^{mp}}}{\frac{1-\rho^{mp}}{s k \phi} + \frac{1}{1-\rho^{mp}}} u_t^{mp}$$

$u_t^{mp} \uparrow \rightarrow r \uparrow, y \downarrow, \pi \downarrow$

If $\phi \uparrow$, u^{mp} has smaller effect on y, π, r