Econ 614 problem set on loss-minimizing central bank

I) Consider a new Keynesian model in which:

$y_t = E_t y_{t+1} - sr_t + \epsilon$ $\qquad\qquad$ $\pi_t = E_t \pi_{t+1} + \kappa y_t$

where y is the output gap. r is the difference between the real interest rate and the natural rate of interest. ϵ is a mean-zero i.i.d. "spending shock" with variance σ^2 .

In every period the central bank chooses r_t *before* it has observed the realized value of ϵ, taking as given the public's $E_t y_{t+1}$ and $E_t \pi_{t+1}$ to minimize a loss function:

$L = E\left[\frac{1}{2}\left(\pi_t^2\right) + \frac{1}{2}a\left(y_t^2\right)\right]$. Recall that $E[X^2] = (E[X])^2 + \sigma_X^2$.

This means the central bank's desired inflation rate is zero. Its desired value for the output gap is also zero. That is, the central bank wants output to be equal to the natural rate of output.

1) Find the value of r_t that the central bank will choose for any given values of $E_t y_{t+1}$ and $E_t \pi_{t+1}$.

2) Given this value of r_t, what will be the realized values of y_t and π_t ?

II) Now consider a new Keynesian model in which:

$y_t = E_t y_{t+1} - sr_t$ $\qquad\qquad$ $\pi_t = E_t \pi_{t+1} + \kappa y_t$

Notice that there is no spending shock here. In every period the central bank chooses r_t, taking as given the public's $E_t y_{t+1}$ and $E_t \pi_{t+1}$. The *public's* loss function is:

$$L = E\left[\frac{1}{2}\left(\pi_t^2\right) + \frac{1}{2}a\left(y_t - y^*)^2\right)\right]$$

where $y^* > 0$. This means that the public's desired level of inflation is zero, but its desired value for the output gap is positive. That is, the public wants output to be greater than the natural rate of output. The parameter a describes the relative importance of output versus inflation to the public. We want to know what the long-run equilibrium will look like, for various possible sets of preferences on the part of the central bank. Here, because there are no disturbance (stochastic) terms, such an equilibrium means $y_t = E_t y_{t+1} = 0$ and $\pi_t = E_t \pi_{t+1} = \bar{\pi}$ where $\bar{\pi}$ is the long-run steady state inflation rate.

1) Suppose the central bank sets r_t to minimize the public's own loss function.

a) Conjecture that $E_t y_{t+1} = 0$ and $E_t \pi_{t+1} = 0$.

i) Figure out the interest rate that the central bank will choose taking these expectations as given. What is realized output? What is realized inflation?

ii) Is this outcome a rational expectations equilibrium? That is, is it possible to have a long-run steady state inflation rate $\bar{\pi} = 0$?

b) Now conjecture that $E_t y_{t+1} = 0$ but $E_t \pi_{t+1} = \hat{\pi}^e > 0$. That is, expected inflation is some positive value.

i) Figure out the interest rate that the central bank will choose, and the resulting values of y_t and π_t. These values will be functions of $\hat{\pi}^e$.

ii) In long-run equilibrium realized inflation π_t equals $\hat{\pi}^e$. Find the value of $\hat{\pi}^e$ that can hold in a long-run equilibrium. This is the steady state inflation rate $\overline{\pi}$.

2) Now suppose that the central bank does *not* set *r* to minimize the public's loss function. Instead, the central bank's policy committee is filled with "conservatives" whose loss function is not the same as the public's. These conservatives care relatively little about output. The value of "a" in their loss function is $\hat{a} < a$. Using your answer to 1) b), consider the rational expectations equilbrium resulting from this situation. Is the public better off when the central bank has the same value of *a* as the public, or when the central bank's policy committee is filled with conservatives?

3) Finally, suppose the central bank acts to minimize a loss function with the same value of *a* as the public's, but a desired value of output equal to the natural rate of output. That is, suppose the central bank acts *as if* $y^*=0$. Using your answer to 1) b), consider the rational expectations equilbrium resulting from this situation. Is the public better off when the central bank acts to maximize the public's loss function with $y^* > 0$, or a loss function with $y^* = 0$?