**A Dynamic Profit Function with Adjustment Costs for Outputs***

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**Abstract**

It is well known that there are adjustment costs associated with many input factors, delaying the firm's response to changes in relative prices. Although adjustment costs is implicitly acknowledged when cost rather than profit function is used, little attention, has been given to adjustment costs for outputs. However, there will in many cases also be adjustment costs associated with changes in the product mix for multioutput firms. In this paper we formulate the firm's optimization problem in a profit maximizing set up that allows adjustment costs for all netputs, from which it follows that adjustment cost for some factors affect the adjustment of both inputs and outputs. We also show that one can test whether a factor is quasi-fixed or fully fixed.

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## 1. Introduction

It is known that firms in many industries face adjustment costs that can delay their response to changes in relative prices. The inflexibility in adjustment leads to over- or under-capacity for some inputs, and can also be manifested through an increase or decrease in the stocks of outputs. In the dynamic theory of firm, adjustment costs are often regarded as output forgone due to internal costs of investment (Lucas, 1967a; Treadway, 1971).[1] According to this definition adjustment costs are directly associated with only input factors. This seems to be the case in the literature as well, because output is mostly treated as fully variable. When using a cost function, one implicitly assumes that adjustment costs for output are prohibitively high in the short run.[2] This provides a rationale for adjustment cost associated with output supply. There are also several strands in the literature that suggest that adjustment costs are solely associated with adjusting output.[3] The first explanation that comes to mind is menu costs. There can also be other costs associated with changing output level that cannot be attributed to any inputs but which represents cost in terms of foregone output. For instance, Rosen, Murphy and Scheinkman (1994) show how one can get price cycles in livestock production due to adjustment costs since production has to be reduced initially to increase the breeding stock if one wants to increase production. For multi-product firms with joint production technologies, switching among different outputs is also a potential source of adjustment costs because production is reduced or possibly stopped to adjust, for example, equipment to the new product mix.

In this paper we pose the competitive firm's problem as a fully dynamic optimization problem and allow for adjustment costs associated with outputs as well as inputs. In this specification one can investigate the dynamics of output supply and how this interacts with factor demand

[1] There is also several approaches used in empirical studies of adjustment costs that takes the dynamic theory of the firm into account to varying degrees. These include Anderson and Blundell (1982), Pindyck and Rotemberg (1983), Friesen (1992) and Paul (2001).

[2] Some studies recognize that if output is a choice variable for the firm, there will be a simultaneity problem when output is modeled as fixed (Segerson and Squires, 1990). This simultaneity problem can be avoided by using instrumental variables instead of using a profit function approach.

[3] Empirical evidence of adjustment cost for outputs has been provided by, e.g., Dixon (1983) and Carlton (1986) using *ad hoc* empirical specifications. Slade (1998) formulates and estimates a theoretically consistent model of adjustment cost. All these studies have in common that there are only adjustment costs associated with output and none with input, and one do not take disequilibrium for one netput into account in the demand/supply for other factors.

and vice versa.[4] The present model extends some of the earlier specifications. In particular, Hamermesh's (1993) notion of dynamic substitutes and complements is extended to accommodate interactions between inputs and outputs to get information about how disequilibrium in factor demand affects output production and vice versa if there are adjustment costs associated with changing output levels. This allows one to investigate, e.g., whether adjustment costs for labor influence changes in output levels and vice versa. Previous approaches will, in economic terms, be special cases of our specification, although they are mathematically more general.[5] Making output and possibly some input factors fixed/variable while retaining adjustment cost for some factors, different flexible accelerator models can be derived.

In dynamic factor demand systems it is possible to test whether there are adjustment costs or whether the dynamic model can be reduced to a static one. In some specifications (e.g., Epstein and Denny, 1983) one can also test whether a specific factor can be treated as variable rather than quasi-fixed by testing the absence of adjustment costs associated with the specific factor.[6] Apart from assuming and checking that the system is stable, there are no tests to determine whether a factor should be treated as fixed in this context.[7] Factors are fixed when quantity (demanded or supplied) does not respond to a change in relative prices, i.e., there does not exist a long-run relationship (in the data) between prices and quantities (demanded or supplied). The system is then not stable, and the long-run relationship in the estimated equation(s) will be nonstationary (Anderson and Blundell, 1982). This implies that the matrix containing the long-run relationships in the system has less then full rank. This can be tested using the procedure suggested by Johansen (1988; 1991). That is, Johansen's cointegration test can, in this context, be used to investigate whether a specific factor should be treated as fixed or not. An interesting feature of this approach is that it will also provide a test between a profit and cost function specification, since a cost function is a restricted profit function with all outputs treated as fixed (Lau, 1976; McFadden, 1978).

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[4] Lucas (1967b) provided the first general specification of firm's dynamic adjustment problem, and Epstein and Denny (1983) provided a specification based on a cost function. Nadiri and Rosen (1969) noted that disequilibrium in one factor can influence demand for other factors and output supply.

[5] McFadden (1978) claims that a restricted profit function (of which the cost function is a special case) is the most general functional form. While this is true mathematically, a long run profit function is the most general functional form economically because all netputs are allowed to adjust to their long run equilibrium levels.

[6] This is, however, not possible in other specifications (e.g., Anderson and Blundell, 1982).

Thus, this paper makes two important contributions. First, we allow for adjustment costs in the output supply equations, derived from a fully dynamic optimization framework, whereas in other applications output is either treated as variable (profit function) or fixed (cost functions). Second, we test whether a factor should be treated as fixed or not using cointegration tests. More specifically, we test the null of fully variable against quasi-fixedness *and* the null of complete fixedness against quasi-fixedness of input factors.

## 2. The firm's problem

### *2.1 Adjustment costs associated with all netputs*

Let $Y$ be a netput vector where outputs are positive and inputs are negative, and let the production technology be described by a production possibility set or a transformation function with standard regulatory properties.[8] If there are adjustment costs associated with all netputs, the firm's problem can be represented in terms of a value function (McLaren and Cooper, 1980; Epstein, 1981). At any base period $t$ = 0, a price taking firm maximizes the discounted present value of profits by solving the following infinite horizon problem

$$(1) \qquad J(Y_0, s, r) = max_I \int_0^{\infty} e^{-rt}(\Pi(Y, I) + sY)dt$$

subject to

$$(2) \qquad \dot{Y} = I - dY, \quad Y(0) = Y_0 > 0$$

where $\Pi$ is a restricted profit function, $I$ is gross investment or adjustment costs, $s$ is a vector of sales prices and user costs for the quasi-fixed factors and $\delta$ is a diagonal matrix of depreciation rates.[9] The term depreciation rate is somewhat misleading for factors other than capital. More precisely it is the cost of maintaining the stock of different netputs, and it is zero if there is no difference between gross and net adjustment costs.[10] The discount rate $r$ is assumed to be constant for all periods at any time $t$.

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[7] Kulatilaka (1985) and Schankerman and Nadiri (1986) provide tests between static full and partial equilibrium models that can be interpreted as a test of whether factors are completely variable or not. However, they do not separate between quasi-fixed and truly fixed factors.

[8] See, for example, Lau (1976), McFadden (1978) or Diewert (1982).

[9] Note that although this is a generalization in terms of economics, mathematically it is a special case of the problem posed by Lucas (1967) or Epstein and Denny (1983). Important contributions in this line of research can be found in Gould (1969), Treadway (1971), Mortensen (1973). Duality was introduced by McLaren and Cooper (1980) and Epstein (1981).

The cost function used by Epstein and Denny (1983) has the same mathematical structure as the restricted profit function used here since the cost function is a special form of the restricted profit function (Lau, 1976). Consequently the assumptions and proofs for the duality between the restricted profit function and the value function will be similar to those provided by Epstein and Denny. In particular, if standard regulatory conditions apply (Arrow and Kurz, 1970; Epstein, 1981), the value function $J$ satisfies the Hamilton-Jacobi equation, which takes the following form

(2') $$rJ(Y,s) = max_I \{\Pi(Y,I) + sY + J_Y(Y,I)(I - dY)\}$$

where a subscript on $J$ denotes its derivative with respect to that netput. Duality between the value and the profit function can then be shown as in Epstein (1981).

Using the intertemporal form of Hotelling's lemma in Epstein (1981) and Cooper and McLaren (1980) one can obtain the policy functions associated with equation (1). This gives the dynamic supply and demand equations in the form of a flexible accelerator model, viz.,

(3) $$\dot{Y} = J_{sY}^{-1}\left[rJ_s^T + Y\right] - dY$$

where the subscript $T$ denotes a transpose.

*2.2 Some factors completely variable:*

Specifications where some factors are assumed to be completely variable are often used in the literature. Such a specification is of interest because it is the model under the null hypothesis of no adjustment costs for some of the factors. Our specification can easily accommodate variable factors. Assuming that $X$ is a vector of variable netput factors with price vector $w$, the firm's problem at time $t$ = 0 becomes

(4) $$V(Y_0,s,r) = max_{I,w} \int_0^\infty e^{-rt}(\Pi(Y,I,w) + sY)dt$$

subject to

(5) $$\dot{Y} = I - dY, \quad Y(0) = Y_0 > 0$$

This specification is more general than most of the specifications used in the literature, because it allows for a multi-output production technology and adjustment cost for some outputs (if not all outputs are treated as completely variable). Note that if some factors are

[10] See , for example, Hamermesh and Pfann (1996) for a discussion of the difference between net and adjustment

treated as completely fixed, the problem will have the form analyzed by Epstein and Denny (1983). Moreover, if these fixed factors are all the outputs, then the restricted profit function will have the form of a cost function as in Epstein and Denny (1983).

For problem stated above, one can use the intertemporal form of Hotelling's lemma of Epstein (1981) and Cooper and McLaren (1980) to obtain the policy functions. When variable factors are included in the problem, demand and supply functions for these factors can also be obtained from the value function. These functions are

(6) $\dot{Y} = J_{sY}^{-1}\left[rJ_s^T + Y\right] + dY$

(7) $L = -rJ_w^T + J_{wY}(\dot{Y} - dY)$

The relationship between (6) and (7) becomes clearer if we rewrite (6) as

(6') $Y = -rJ_s^T + J_{sY}(\dot{Y} - dY)$

If the relevant columns of the $J_{sY}$ matrix contain only zeros, then the corresponding factor is variable. This means that by restricting the appropriate column of the $J_{sY}$ matrix to be zero, one can test the null hypothesis of no adjustment cost for a factor. Moreover, since the form of (6) and (7) is similar to (3) in the presence of adjustment costs associated with all netputs, one can test whether there are adjustment cost associated with any of netputs in (3), by testing whether any of the columns in the $J_{sY}$ matrix in (3) is zero.

Note that if there are adjustment costs associated with any factors in the system given by equations (6) and (7), these will in general also influence the demand and supply for the completely variable factors through the last term in equation (7). Hence, adjustment costs for one factor will in general make demand and supply of all factors deviate from their long-run values when the stock of this variable deviates from steady state. Nadiri and Rosen (1969) first noted that disequilibrium in the demand for one input due to adjustment costs might also influence the demand for other inputs. This is also true for outputs, as the above discussion shows, when the firm's problem is formulated as a profit maximization rather than a cost minimization problem. This result in turn implies that phenomenon like labor hoarding that is attributed to adjustment cost for labor, can affect not only demand for other input factors but also output levels.

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costs.

The dynamic interaction in the flexible accelerator model is captured by the off-diagonal elements in the adjustment matrix $J_{sY}$. To capture the interaction between input factors, Hamermesh (1993, pp. 233) introduced the following notion: Two factors are dynamic complements if disequilibrium in one factor slows the adjustment for the other. In terms of the model presented here, dynamic complements are represented as positive off-diagonal elements of the adjustment matrix. Two factors are dynamic substitutes if disequilibrium in one factor speeds up the adjustment for the other. Dynamic substitutes are represented as negative off-diagonal elements in the adjustment matrix.[11]

When the firm's problem is formulated in such a way that factor demand interacts with output supply, disequilibrium due to adjustment cost for the input factors will also influence the output levels unless all the off-diagonal elements in the appropriate row of the adjustment matrix are zero. Similarly, if there are adjustment costs associated with the output levels, these may influence input demand as well. The notion of dynamic substitutes and complements can also be extended to include this interaction. In particular, two outputs will be dynamic complements (substitutes) if the off diagonal element in the adjustment matrix is positive (negative). Inputs will be dynamic complements (substitutes) for outputs if the associated off-diagonal elements in the adjustment matrix are negative (positive) and outputs will be dynamic complements (substitutes) for outputs if the associated off-diagonal elements in the adjustment matrix are positive (negative).

*2.3 Single output case with no adjustment cost*

It is worth noting here that the more common formulation used by, e.g., Lucas (1967) and Epstein (1981) can be found by putting more structure on $\Pi(\cdot)$. Let $\Pi(Y,I,w)= pF(L,K,I)-wL$, where $p$ is the output price, $L$ is a vector of variable input factors and all the quasi-fixed factors be inputs represented with the vector $K$. The firm's problem is then

(8) $$J(Y_0,s,r,w) = max_{I,L}\int_0^{\infty} e^{-rt}(pF(L,K,I)-wL-sK)dt$$

subject to

(9) $$\dot{K} = I - dK, \quad K(0) = K_0 > 0$$

[11] Strictly speaking, Hamermesh uses the terminology dynamic *p*-complements and substitutes.

where $F(\cdot\ )$ is the production function. The main difference between (1) and (8) is that the specification in (8) is restricted to a single output and it assumes no adjustment costs for output and a set of variable factors.

## 3. Functional form

For empirical work, one must assign an explicit functional form to the value function. We will here use the form suggested by Epstein and Denny (1983), and also assume $r$ to be constant.[12] With a subset of the factors treated as quasi-fixed and the remaining factors as completely variable, the value function can be written as[13]

$$J(Y,p,w) = a_0 + \begin{bmatrix} a_1^T & a_2^T \end{bmatrix} \begin{bmatrix} s \\ w \end{bmatrix} + \frac{1}{2} \begin{bmatrix} s^T & w^T \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} s \\ w \end{bmatrix}$$

(10)

$$+ \begin{bmatrix} s^T A_{sY}^{-1} & w^T A_{wY}^{-1} & a_Y^T \end{bmatrix} Y + \frac{1}{2} Y^T A_{YY} Y$$

where the $a_i$ and $A_{ij}$ are vectors and matrices of appropriate dimensions. The above value function generates the following policy functions

(11) $\dot{Y} = A_{sY} \left( r(a_1^T + A_{11}s + A_{12}w + A_{sY}^{-1}Y + Y) \right)$

(12) $L = r(a_2 + A_{12}s + A_{22}w + A_{wY}^{-1}Y) + A_{wY}^{-1}\dot{Y}$

Equation (11) can be written as

(11’) $\dot{Y} = A_{sY} \left( ra_1^T + rA_{11}s + rA_{12}w \right) + (r + A_{sY})Y$

This is a flexible accelerator with adjustment matrix $M = (r + A_{sY})$.

The above model must be given a discrete approximation, to make it useful in empirical analysis. Equation (11) can be written as

(13) $\Delta Y_t = A_{sY}\left( ra_1^T + rA_{11}s_t + rA_{12}w_t \right) + MY_{t-1}$

which can also be expressed as a partial adjustment model, viz.,

(13’) $Y_t = A_{sY}\left( ra_1^T + rA_{11}s_t + rA_{12}w_t \right) + (I + M)Y_{t-1}$

Similarly, equation (12) can be written as

(14) $L_t = r(a_2 + A_{12}s + A_{22}w_t + A_{wY}^{-1}Y_t) + A_{wY}^{-1}\Delta Y_t$

[12] This is a common assumption and a good discussion on this can be found in Epstein (1981).

The form can then be further simplified by multiplying out the composite parameters to obtain a reduced form that is linear. Let

$b_1 = A_{sY} r a_1^T, \quad c_1 = rA_{11}, \quad d_1 = rA_{12}, \quad b_2 = ra_2, \quad c_2 = rA_{12}, \quad d_2 = rA_{wY}, \quad e = A_{wY}^{-1}$. The system can then be written as

(15) $$Y_t = b_1 + c_1 s_t + d_1 w_t + (I + M)Y_{t-1}$$

(16) $$L_t = b_2 + c_2 s_t + d_2 w_t + e\Delta Y_t$$

Equation (15) will be the starting point of our empirical analysis. A test of whether $(I+M) = 0$ will be a test of no adjustment costs in the system. Whether any of the equations in (15) can be given the form of equation (16) will be a test of whether a specific factor can be treated as variable or not.

A problem with the policy functions discussed above is that they are not homogenous of degree zero. Epstein and Denny (1983) addressed this issue by modeling one factor as variable and normalize all prices by this factor. This approach is not optimal here since we will test whether adjustment costs are present for all netputs. We, therefore, normalize the second order price terms by the price of one of the netputs, giving the value function the form of Diewert and Ostensoe's (1988) normalized quadratic profit function. This is very similar to the approach of Epstein and Denny (1983), but differs in that the first order terms of the value function are not normalized so that all price terms in the value function are homogenous of degree zero. With a subset of the factors treated as quasi-fixed and the remaining factors as completely variable, the resulting value function can be written as

(17) $$J(Y,s,w) = a_0 + \begin{bmatrix} a_1^T & a_2^T \end{bmatrix} \begin{bmatrix} s \\ w \end{bmatrix} + \frac{1}{2} \begin{bmatrix} s_*^T / s_1 & w^T / s_1 \end{bmatrix} \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} s / s_1 \\ w_1 \end{bmatrix} + \begin{bmatrix} s^T A_{sY}^{-1} & w^T A_{wY}^{-1} & a_Y^T \end{bmatrix} Y + \frac{1}{2} Y^T A_{YY} Y$$

where $s_*$ is the $s$ vector with the first element deleted. The subscript 1 indicates the first element of the corresponding vector. The policy function of the netput that one normalizes upon will have a different functional form from the ones discussed above, as shown in Epstein and Denny (1983). We have specified the normalizing factor as one of the quasi-fixed factors, to allow the functional form to reduce to a form without variable factors. However, when

[13] Empirical applications of versions of this functional form for cost functions can, in addition to Epstein and Denny (1983), be found in Bernstein and Nadiri (1988) and Luh and Stefanou (1996), and in Luh and Stefanou (1991) for a profit function.

variable factors are present one of them is a better candidate for the normalizing factor, and one can obtain most of the economically relevant information without estimating this equation. It is to be noted here that other functional forms can also be used in this context to ensure that the policy functions are homogenous of degree zero. For instance, Luh and Stefanou (1991) used a version of the Generalized Leontief function.

### 4. Fixed versus quasi-fixed factors

When specifying a static system of factor demand equations, some factors are often treated as fixed due to an assumption of adjustment costs independent of whether the netputs are fixed or quasi-fixed (Brown and Christensen, 1981). In dynamic specifications, all netputs with adjustment cost are treated as quasi-fixed with the exception of cost function approaches, where output(s) are treated as completely fixed. A netput is fixed in a system if it does not respond to changes in relative prices due to large adjustment costs. However, this basically means that there is no relationship between changes in the quantity demanded or supplied of the relevant netput and changes in its price. This will be clearer if equation (15) is given an Error Correction Model (ECM) representation, viz.,

(18) $$\Delta Y_t = c_1 \Delta s_t + d_1 \Delta w_t - M\left(Y_{t-1} - M^{-1}\left(b_1 + c_1 s_t + d_1 w_t\right)\right)$$

The expression inside the parenthesis is the long-run relationship. Hence changes in $Y$ are due to changes in the prices and deviations of the actual netput vector $Y$ from its optimal level. However, if $M = 0$, there will be no changes in the netput vector $Y$ due to deviations from the long-run steady state. If so, a stable solution to the system does not exist. This will be the case if the data series are nonstationary, and no long-run relationship exists (Evans and Savin, 1981, Engle and Granger, 1987). The adjustment matrix $M$ will have full rank only if all the proposed long-run relationships in fact are long-run relationships (Johansen, 1991).[14] Hence, Johansen's cointegration tests, which can be interpreted as a test of whether $M$ has full rank or not can be used to test if all the supply and demand equations are long-run relationships. If $M$ has less than full rank, one can then re-estimate the system with the appropriate rank treating an appropriate number of factors as fixed. If the fixed factor(s) is (are) correctly modeled, one should have a system with the same rank as the full system. However, if one models a factor as fixed that should not be treated as fixed, the equations for the fixed factors will be present

[14] Johansen (1992) show that if some variables are treated as exogenous in the system and this exogeneity assumption is true, LIML estimators will be equivalent to FIML also in this context.

also in the reduced system, and the Johansen test will then indicate an even lower rank for the adjustment matrix in this system.

The distinction between quasi-fixed and fixed factors is also of interest econometrically. In particular, an argument against using production functions is that the input quantities are choice variables for the firm. This creates a simultaneity problem (Marschak and Andrews 1944; Hoch, 1958). The same problem has been noted with respect to output in a cost function specification (Segerson and Squires, 1990). It should be clear from the discussion above that all quasi-fixed factors can be regarded as choice variables for the firm, although possibly with slow adjustment. Hence, the simultaneity problem associated with production functions will also be present in partial static equilibrium models, if these factors are quasi-fixed and not truly fixed. In truly dynamic models this problem, however, is avoided as the adjustment process is explicitly modeled.

## 5. An empirical application

Here we provide an empirical illustration and analyze the dynamic structure of US agriculture. We use the annual data for the period 1948-1994. This data set and the construction of the variables are described in Ball *et al*. (1997). It contains prices and quantity of livestock, crop, labor, capital and intermediate inputs. Before estimation, a linear and a quadratic trend variables were also added to the system to capture technological change and a dummy variable for years involving bad weather since weather strongly influence crop production. We start by investigating the time series properties of the data using Augmented Dickey-Fuller tests. As indic ated in Table 1, all data series seem to be $I(1)$. The next step is to investigate whether all proposed policy functions are long-run relationships. The results from the Johansen test are reported in Table 2. The results show that all four equations are long-run relationships. Hence, there is no evidence that any factors should be treated as completely fixed.

We now describe tests to further investigate the dynamic structure of U.S. agriculture. Tests for different dynamic structures are reported in Table 3. The first is a test for no adjustment costs in the system, i.e., whether the equation system reduces to a static specification. This hypothesis is clearly rejected. The next hypothesis we test is whether the adjustment matrix is diagonal. This is necessary if the dynamics in each equation are to be treated independent of other factors in the system as is commonly done in single equation dynamic specifications. We can see from Table 3 that this hypothesis is also rejected. Finally, we test whether any of the factors can be treated as completely variable by testing whether the relevant columns of the $(I+M)$ matrix contain only zeros. The null hypothesis of the absence of adjustment costs cannot be rejected for crops, but is rejected for all other netputs. Hence, there seems to be some adjustment costs associated with each of the input factors and with livestock production.

A dynamic system with one dynamic supply function for livestock and three demand equations is then estimated where the price terms are normalized using the price for crop as numeraire. In Table 4, the estimated parameters for the four equations are reported. With respect to the dynamics, the parameters of the $I + M$ matrix are of interest. These are reported in the upper section of the table as the estimated coefficients on the lagged quantity variables. The parameters that would be labeled as the adjustment parameters in single equation specifications can be found along the diagonal. We see that the "own" adjustment seem to relatively slow for livestock with an adjustment parameter of 0.5270, giving some support to cost function specifications for this output as output deviates from long run static equilibrium. For all input factors the "adjustment" parameters are all statistically significant. The "own" adjustment parameter (0.478) for labor is also relatively slow. However, it is even slower compared to the other two factors since the adjustment parameters for intermediate factors and capital are above 0.8. We also see that seven of the off-diagonal parameters indicate dynamic complementarity, while five indicate dynamic substitutability. This result indicates a relatively even mix between dynamic substitutes and complements. Hence, adjustment costs lead to overutilization of some factors and underutilization of others during the adjustment process. Given that not much focus has been given to this issue, it is hard to say if this result is surprising.

## 6. Concluding remarks

In this paper we formulate the firm's dynamic optimization problem in the form of a profit function that allows adjustment costs associated with both inputs and outputs. By treating outputs and inputs symmetrically, it is easy to show that disequilibrium in input factor demand associated with adjustment cost influences not only demand for other input factors but also output levels and vice versa. This also makes intuitive sense since one would expect that additional use of a factor also will influence production. And one needs more of some inputs to produce a higher level of output also when output deviates from long-run equilibrium. Thus, labor hoarding, for example, can also lead to higher production. Similarly, adjustment costs and slow adjustment for output can influence the demand for input factors. Furthermore, for multi-product firms, adjustment costs for one output can influence production levels of other outputs as well.

Dynamic specifications like the one considered here can be used to test whether there are adjustment costs associated with a given factor (Epstein and Denny, 1983). However, there are no tests to distinguish quasi-fixed from truly fixed factors due to problems associated with unit roots. In this paper we show that Johansen's (1988, 1991) reduced rank tests can be used to separate these hypotheses. The adjustment matrix will have full rank only if there are no truly fixed factors. Furthermore, a sequence of tests will uncover factors that are fixed if the adjustment matrix has reduced rank. Since the cost function is a special form of the profit function, this test can also be used to test which of these specifications are most appropriate.

We provide an empirical application using time series data on U.S. agriculture. A profit function containing two outputs (livestock and crop) and three inputs (capital, labor and intermediate inputs) is estimated. All these netputs are found to be choice variables, since we reject the hypothesis that there are fixed netputs in the system. We also find that there are adjustment costs associated with the production of livestock as well as use of all the inputs. However, we find no adjustment costs associated with crop output. We also reject the hypothesis that the adjustment matrix is diagonal. Thus, single equation estimation of adjustment costs would be inappropriate for this industry.

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**Table 1. Augmented Dickey-Fuller tests**

| Variable | Test statistic, levels | Test statistic, first differences |
|---|---|---|
| Plivestock | -1.1380 | -4.5075* |
| Qlivestock | -0.4983 | -5.0336* |
| Pmaterials | -2.0337 | -4.5853* |
| Qmaterials | -2.2269 | -5.0891* |
| Plabor | -0.4736 | -6.8972* |
| Qlabor | -2.1744 | -4.6194* |
| Pcapital | -2.2407 | -4.2071* |
| Qcapital | -0.8158 | -4.4751* |

* indicates significant at a 5% level

**Table 2. Johansen test for full rank of the adjustment matrix**

| $H_0$:rank=$p$ | Max test | Critical value 5% | Trace test | Critical value 5% |
|---|---|---|---|---|
| $p <= 0$ | 80.24* | 33.5 | 211.5* | 68.5 |
| $p <= 1$ | 60.82* | 27.1 | 131.3* | 47.2 |
| $p <= 2$ | 37.15* | 21.0 | 70.45* | 29.7 |
| $p <= 3$ | 19.74* | 14.1 | 33.30* | 15.4 |
| $p <= 4$ | 13.57* | 3.8 | 13.57* | 3.8 |

* indicates significant at a 5% level

**Table 3. Dynamic restrictions**

| Test | Test statistic | Df | *p*-value |
|---|---|---|---|
| Instantaneous adjustment | 307.01* | 25 | 0.0000 |
| Diagonal adjustment | 65.941* | 20 | 0.5035 |
| *Instantaneous adjustment for* | | | |
| Livestock | 20.422* | 5 | 0.0010 |
| Crop | 5.531 | 5 | 0.3546 |
| Materials | 44.855* | 5 | 0.0072 |
| Labor | 13.683* | 5 | 0.0178 |
| Capital | 148.33* | 5 | 0.0000 |

* indicates significant at a 5% level

**Table 4. Estimated parameters**

| | Equation: | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| | Livestock | | Intermediate | | Labor | | Capital | |
| Variable | Coefficient | *p*-value | Coefficient | *p*-value | Coefficient | *p*-value | Coefficient | *p*-value |
| $Qlivestock_{t-1}$ | 0.5270 | (0.0006) | 0.6193 | (0.0265) | -0.0065 | (0.9853) | 0.3186 | (0.0483) |
| $Qintermediate_{t-1}$ | -0.0752 | (0.2464) | 0.8425 | (0.0000) | -0.0496 | (0.7574) | 0.2483 | (0.0012) |
| $Qlabor_{t-1}$ | 0.0957 | (0.1028) | 0.0668 | (0.5423) | 0.4780 | (0.0019) | 0.0975 | (0.1321) |
| $Qcapital_{t-1}$ | 0.0453 | (0.4361) | 0.1431 | (0.1987) | -0.2309 | (0.1167) | 0.8048 | (0.0000) |
| Price livestock | -0.0766 | (0.1826) | -0.1668 | (0.1279) | -0.0328 | (0.8170) | -0.0114 | (0.8564) |
| Price intermediate | -0.0998 | (0.4482) | 0.5669 | (0.0279) | -0.5948 | (0.0755) | 0.1778 | (0.2249) |
| Price labor | 0.1973 | (0.0956) | -0.4374 | (0.0535) | 0.8100 | (0.0079) | -0.1223 | (0.3440) |
| Price capital | 0.0179 | (0.4816) | 0.0532 | (0.2732) | 0.0513 | (0.4202) | 0.0407 | (0.1527) |
| Trend | -0.0075 | (0.2284) | -0.0145 | (0.2186) | 0.0422 | (0.0090) | -0.0117 | (0.0922) |
| $Trend^2$ | 0.0001 | (0.1039) | 0.0000 | (0.7554) | -0.0005 | (0.0215) | 0.0001 | (0.1957) |
| Constant | 0.7109 | (0.0136) | -0.2864 | (0.5850) | -1.6019 | (0.0248) | 0.0206 | (0.9462) |
| $R^2$ | 0.9914 | | 0.9698 | | 0.9975 | | 0.9708 | |
| LM [a] | 1.6710 | (0.2036) | 0.0291 | (0.9714) | 1.7603 | (0.1878) | 1.1187 | (0.3388) |

[a] LM is a LM test against up to 2nd order autocorrelation distributed as F(2,34).