ON THE EQUIVALENCE OF TWO NORMALIZATIONS IN ESTIMATING SHADOW COST FUNCTIONS*

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Abstract

This paper shows that the shadow cost system that normalizes one price divergence parameter to unity is equivalent to the Balk normalization (i.e., the cost of the observed inputs at the shadow and observed market prices are the same). We also show that the Balk system can alternatively be expressed as a simultaneous equation system that should be estimated using a system approach instead of the seemingly unrelated regression technique.

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1. Introduction

A generalized version of the neo-classical cost function is often used to accommodate cases of imperfect input markets and/or presence of constraints in input allocation. The arguments of the generalized (shadow) cost function are shadow (virtual) prices and output(s). Shadow prices are often expressed as a proportion of observed prices where the proportionality factors (price divergence parameters) measure the presence and magnitude of distortions. These distortions can also arise when the markets are competitive, especially when producers make mistakes in allocating inputs. These allocative errors are often labeled as allocative inefficiency. Since the shadow cost function is linearly homogeneous in shadow prices, it should be also linearly homogeneous in price divergence parameters. This implies that additional restrictions are required to identify the input-specific price divergence parameters. Two alternative procedures are used to estimate the shadow cost functions with allocative and/or technical inefficiency.

The first (traditional) procedure assumes that the shadow and market prices of an arbitrarily chosen input coincide thereby meaning that its price divergence parameter equals one (Atkinson and Cornwell, 1994, Kumbhakar, 1997, Kumbhakar and Lovell (2000), among others). That is, one of the price divergence parameter is normalized to unity for identification. The second procedure (Balk, 1997) assumes that the cost of actual inputs at the shadow and observed prices are equal (Balk normalization). At the first glance the Balk system seems to be less restrictive and easier to estimate. This is, however, not the case. Recently Maietta (2002) criticized the Balk system on the ground that the estimates derived from the Balk cost system are sensitive to the choice of input price used as a numeraire to impose linear homogeneity restrictions. Since this is not the case with the first procedure, Maietta argued that the first procedure (normalization) has a clear advantage over the second one.

In this paper we show that the system of equations to be estimated under the two procedures are algebraically the same. We also show that the Balk system is a nonlinear simultaneous equation system. Consequently, the use of seemingly unrelated regression (SUR) technique to estimate the model would be inappropriate.

2. Equivalence of the Two Cost Systems

Here we follow Kumbhakar (1997) and write the cost system with both technical and allocative inefficiency as:

$$\ln(w'x) = \ln C(w^*, y) + \ln\left(\sum_{j=1}^{n} s_j^*(w^*, y)\theta_j^{-1}\right) + u \qquad (1)$$

$$s_j = \frac{s_j^*(w^*, y)\theta_j^{-1}}{\sum_{k=1}^{n} s_k^*(w^*, y)\theta_k^{-1}}, \quad j = 1, \ldots, n,$$

where $w'x$ refers to the actual (observed) cost, w is the vector of market/observed input prices, x is the observed input vector, $C(\cdot)$ is the shadow cost function, w^* is the vector of shadow input prices, y is the vector of output quantities, $u \geq 0$ is the input-oriented measure of technical inefficiency, s_j is the observed cost share of input j, $s_j^*(\cdot) = w_j^* x_j / C(w^*, y)$ is the shadow cost share function, and θ is the vector of price divergence parameters that relate the shadow and market prices, viz., $w_j^* = \theta_j w_j$, $\theta_j > 0$.

Balk's (1997) normalization is based on the assumption that $\sum w_j x_j = \sum w_j^* x_j$, which implies that $\sum_{j=1}^{n} s_j^*(w^*, y)\theta_j^{-1} = 1$ as well as $\sum_{j=1}^{n} s_j \theta_j = 1$. Using $\sum w_j x_j = \sum w_j^* x_j$, the cost function in (1) can be expressed as

$$\ln(w'x) = \ln C(w^*, y) + u \qquad (2)$$

$$s_j = s_j^*(w^*, y)\theta_j^{-1}$$

Then we rewrite the restriction $\sum_{k=1}^{n} s_k^*(w^*, y)\theta_k^{-1} = 1$ and express θ_1 as

$$\theta_1 = \sum_{k=1}^{n} s_k^*(w^*, y)\left(\frac{\theta_k}{\theta_1}\right)^{-1} \qquad (3)$$

Finally, use the first input price as numeraire to impose the linear homogeneity restriction on the shadow cost function $C(w^*, y)$ and express it as $(1/w_1\theta_1)C(w^*, y) = C(\tilde{w}^*, y)$ where $\tilde{w}_j^* = w_j^*/w_1^* = (w_j / w_1)(\theta_j / \theta_1)$. With these the cost function in (2) can be written as $\ln(w'x/w_1) = \ln\theta_1 + \ln C(\widetilde{w}^*, y) + u$, and then using the relationship in (3) it can be further modified to express as

$$\ln(w'x/w_1) = \ln C(\widetilde{w}^*, y) + \ln\left(\sum_{j=1}^{n} s_j^*(\widetilde{w}^*, y)\left(\frac{\theta_j}{\theta_1}\right)^{-1}\right) + u \tag{4}$$

$$= \ln C(\widetilde{w}^*, y) + \ln\left(s_1^*(\widetilde{w}^*, y) + \sum_{j=2}^{n} s_j^*(\widetilde{w}^*, y)\left(\frac{\theta_j}{\theta_1}\right)^{-1}\right) + u$$

Similarly, using (3) the second equation in (2) becomes:

$$s_j = s_j^*(\widetilde{w}^*, y)\left(\frac{\theta_j}{\theta_1}\right)^{-1}\frac{1}{\theta_1} = \frac{s_j^*(\widetilde{w}^*, y)}{\left(\frac{\theta_j}{\theta_1}\right)\left[\sum_{k=1}^{n} s_k^*(\widetilde{w}^*, y)\left(\frac{\theta_k}{\theta_1}\right)^{-1}\right]} = \tag{5}$$

$$= \frac{s_j^*(\widetilde{w}^*, y)}{\left(\frac{\theta_j}{\theta_1}\right)\left[s_1^*(\widetilde{w}^*, y) + \sum_{k=1}^{n} s_j^*(\widetilde{w}^*, y)\left(\frac{\theta_k}{\theta_1}\right)^{-1}\right]}$$

If one uses the traditional approach and sets one price divergence parameters to unity then the cost system in (1), after imposing the linear homogeneity (in w^*) restrictions, can be rewritten as

$$\ln(w'x/w_1) = \ln C(\widetilde{w}^*, y) + \ln\left(s_1^*(\widetilde{w}^*, y) + \sum_{j=2}^{n} s_j^*(\widetilde{w}^*, y)\theta_j^{-1}\right) + u \tag{6}$$

$$s_j = \frac{s_j^*(\widetilde{w}^*, y)\theta_j^{-1}}{s_1^*(\widetilde{w}^*, y) + \sum_{k=2}^{n} s_k^*(\widetilde{w}^*, y)\theta_k^{-1}} \tag{7}$$

Since $\theta_1 = 1$ one can replace θ_j in (6) and (7) by θ_j/θ_1 that makes (6) and (7) identical to (4) and (5). This proves our first point that the Balk system and the traditional shadow cost system are the same.

3. Estimation of the traditional and the Balk Cost System

It is shown that the Balk cost system in (4) and (5) and the traditional cost system in (6) and (7) are algebraically equivalent. Both systems consist of equations that are nonlinear in parameters and can be either estimated using the iterative nonlinear seemingly unrelated regression (ITNLSUR) technique or the maximum likelihood method. Since the two systems are the same, estimation results will be the same. However, instead of using the system in (4) and (5) one may wish to estimate the Balk system in alternative form (based on the original formulation). This system (derived from (2) with the linear homogeneity restrictions imposed) can be expressed as follows. From $\sum_{j=1}^{n} s_j^*(\tilde{w}^*, y)\theta_j^{-1} = 1$ and $\sum_{j=1}^{n} s_j \theta_j = 1$ it is clear that

$$\sum_{j=1}^{n} s_j^*(\tilde{w}^*, y)\left(\frac{\theta_j}{\theta_1}\right)^{-1} = \left[\sum_{j=1}^{n} s_j\left(\frac{\theta_j}{\theta_1}\right)\right]^{-1} \tag{8}$$

Then, by using (8) the Balk system in (4) and (5) can be written as

$$\begin{aligned} \ln(w'x / w_1) &= \ln C(\tilde{w}^*, y) - \ln\left(\sum_{j=1}^{n} s_j\left(\frac{\theta_j}{\theta_1}\right)\right) + u \qquad (9) \\ s_j &= s_j^*(\tilde{w}^*, y)\left(\frac{\theta_j}{\theta_1}\right)^{-1} \sum_{j=1}^{n} s_j\left(\frac{\theta_j}{\theta_1}\right) \end{aligned}$$

Estimation of the above system (after appending classical error terms) seems to be less complicated than the traditional cost system either in (1) or in (6) and (7) because the cost function in (9) contains observed cost shares whereas the cost function in (6) contains the shadow cost shares that are functions of data and parameters. This makes the traditional cost system more nonlinear in parameters compared to the Balk system in (9). However, s_j are endogenous variables that appear on both sides of the equations in (9). Consequently, the Balk system in (9) is a nonlinear simultaneous equation system whereas the system in (6) and (7) is a SUR system. This means that the system in (9) should be estimated using either iterative nonlinear 3SLS or FIML method. The latter procedures are computationally more demanding than the iterative nonlinear SUR technique.

4. Conclusions

Balk (1997) proposed an alternative normalization to make estimation of the cost system easy. Following Balk's suggestion Maietta (2002) found that the results from the Balk system is sensitive to the input price used as a numeraire (to impose the linear homogeneity restrictions). The traditional approach which normalizes one price divergence parameter to be unity is invariant to the choice of numeraire. We show that both the Balk and the traditional cost systems are algebraically the same. We also show that the Balk system (the way it is originally formulated) is a nonlinear simultaneous equation system and consequently it cannot be estimated using the standard nonlinear SUR technique. Based on this observation we conclude that there is no apparent advantage in the Balk normalization so far as estimation of the cost system is concerned.

References

Atkinson, S.E. and C. Cornwell, 1994, Parametric Estimation of Technical and Allocative Efficiency with Panel Data, *International Economic Review* 35, 231-243.

Balk, B.M., 1997, The Decomposition of Cost Efficiency and the Canonical Form of Cost Function and Cost Share Equations, *Economics Letters* 55, 45-51.

Kumbhakar, S.C., 1997, Modeling Allocative Inefficiency in a Translog Cost Function and Cost Share Equations: An Exact Relationship, *Journal of Econometrics* 76, 351-356.

Kumbhakar, S.C. and C.A.K. Lovell, 2000, *Stochastic Frontier Analysis*, Cambridge University Press, Cambridge, England.

Maietta, O.W., 2002, The Effect of the Normalization of the Shadow Price Vector on the Cost Function Estimation, *Economics Letters* 77, 381-385.