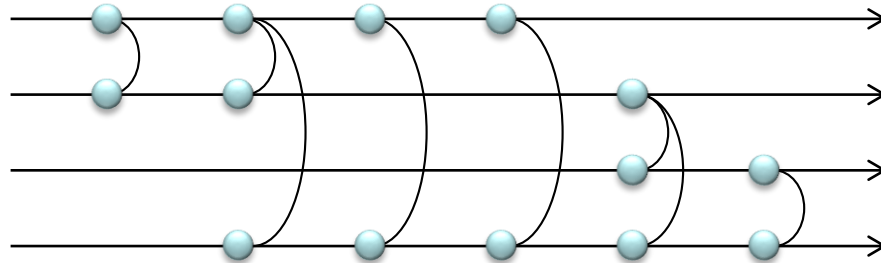


# Temporal Networks



**Hiroki Sayama**  
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# Temporal networks

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- Networks whose topologies and activities change over time
- Heavily data-driven research
  - E.g. human contacts (email, social media, physical proximity)
- Tools still under active development
  - We will need to code a lot ourselves!

# Real-world temporal networks

- **Social (face-to-face) contacts**
- **Email transactions among employees**
- **Data transactions in distributed computer systems**
- **Dynamically changing food webs**

# Example: SocioPatterns.org

## SocioPatterns

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### WELCOME

SocioPatterns is an interdisciplinary research collaboration formed in 2008 that adopts a data-driven methodology to study social dynamics and human activity.

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We make most of the collected data freely available to the scientific community.

### FEATURED: INFECTIOUS SOCIOPATTERNS POSTER



### NEWS

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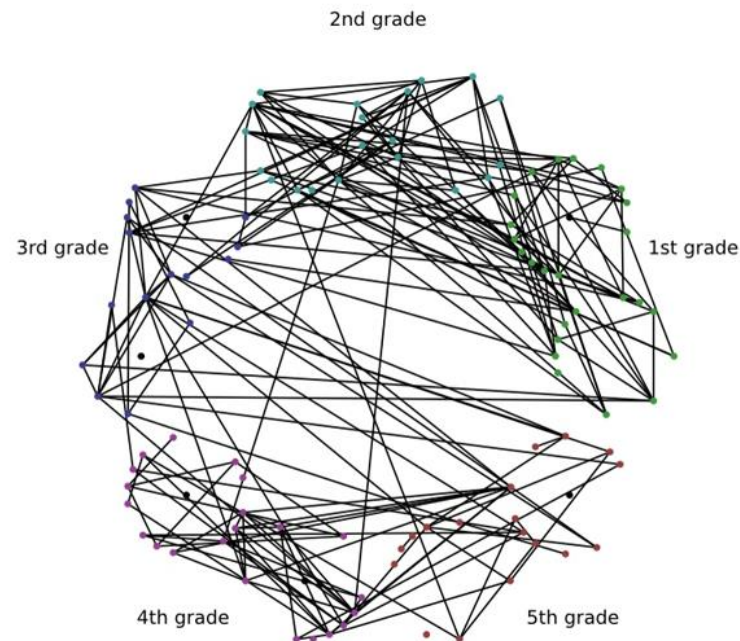
Percentage of removed nodes  
— 0% — 30%

# Example

---

- Dynamic contact patterns in a primary school

<https://vimeo.com/31490438>



Thu, 12:40- 13:20

# Representations of Temporal Networks

# General representation

---

- To represent a temporal network, define the following function:

$$a(i, j, t) = 0 \text{ or } 1 \text{ (or weight)}$$

- $a(i, j, t) = 1$  if node  $i$  is connected to node  $j$  at time  $t$ , otherwise 0

(If time  $t$  is discrete, this could be represented by an adjacency tensor)

# Two popular classes of temporal networks with continuous time

---

- Temporal networks with contact sequence
  - Edges appear momentarily at certain time points and then disappears
- Temporal networks with connection intervals
  - Edges exist for certain time periods



# Contact sequence

---

$i$	$j$	$t$
1	2	1
1	2	2
1	4	2
1	4	3
1	4	4
2	3	5
2	4	5
3	4	6
...		



Edge list with  
contact times

{

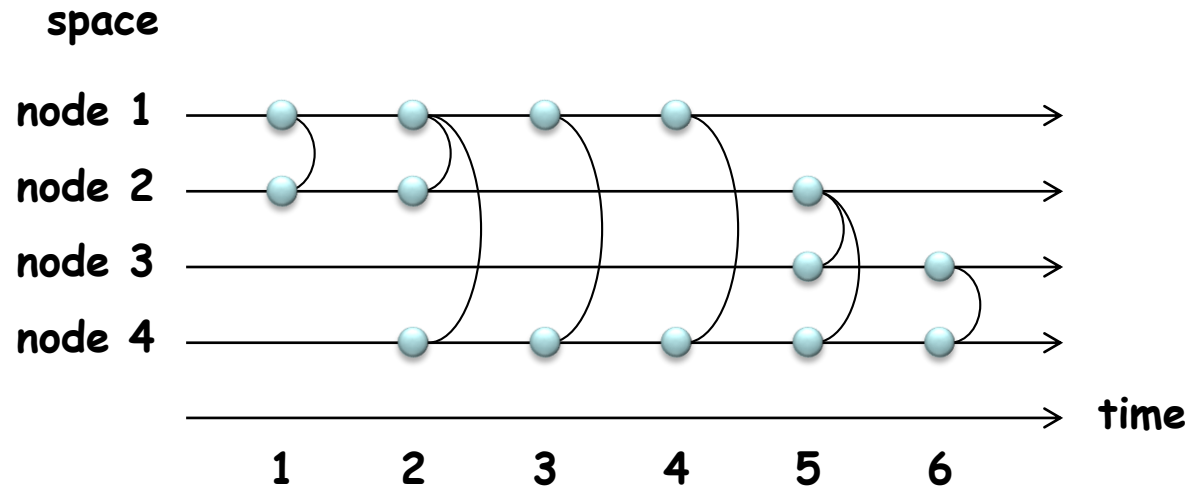
(1, 2):	(1, 2, ...),
(1, 4):	(2, 3, 4, ...),
(2, 3):	(5, ...),
(2, 4):	(5, ...),
(3, 4):	(6, ...)

}

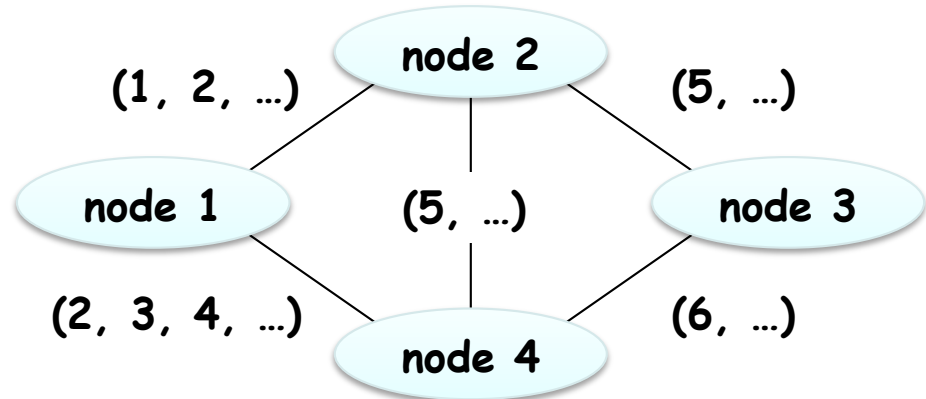
(Time does not have  
to be discrete)

# Contact sequence: Visualized

(1, 2, 1)  
 (1, 2, 2)  
 (1, 4, 2)  
 (1, 4, 3)  
 (1, 4, 4)  
 (2, 3, 5)  
 (2, 4, 5)  
 (3, 4, 6)  
 ...



```
{
  (1, 2): (1, 2, ...),
  (1, 4): (2, 3, 4, ...),
  (2, 3): (5, ...),
  (2, 4): (5, ...),
  (3, 4): (6, ...)
}
```



# Connection intervals

---

$i$	$j$	$t_1$	$t_2$
1	2	0.5	1.0
1	2	1.5	2.5
1	4	2.0	4.0
2	3	3.5	5.0
2	4	5.0	6.0
3	4	5.5	6.5
		...	



Edge list with connection intervals

{

(1, 2): [(0.5, 1.0),  
(1.5, 2.5), ...],

(1, 4): [(2.0, 4.0), ...],

(2, 3): [(3.5, 5.0) ...],

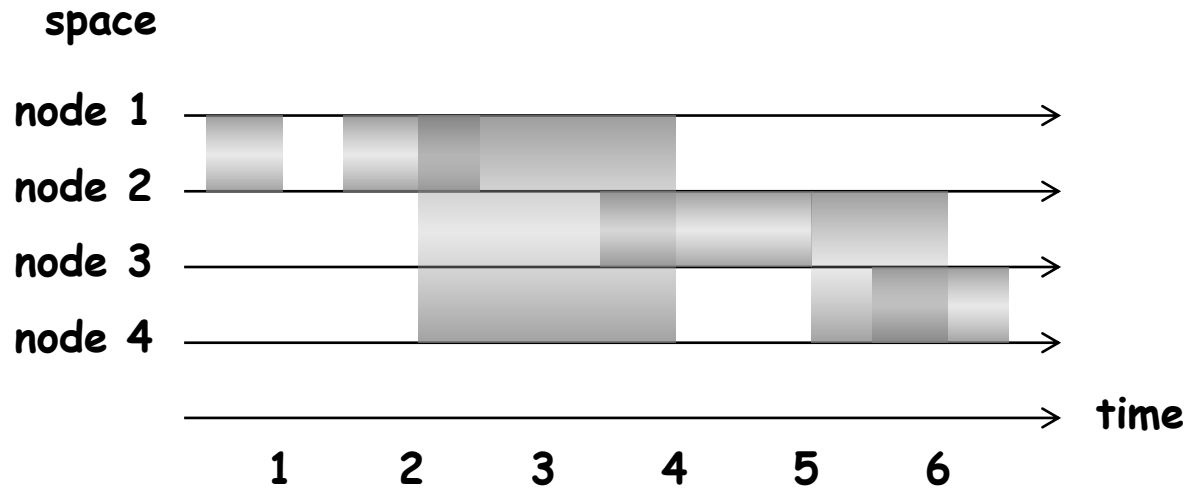
(2, 4): [(5.0, 6.0) ...],

(3, 4): [(5.5, 6.5) ...]

}

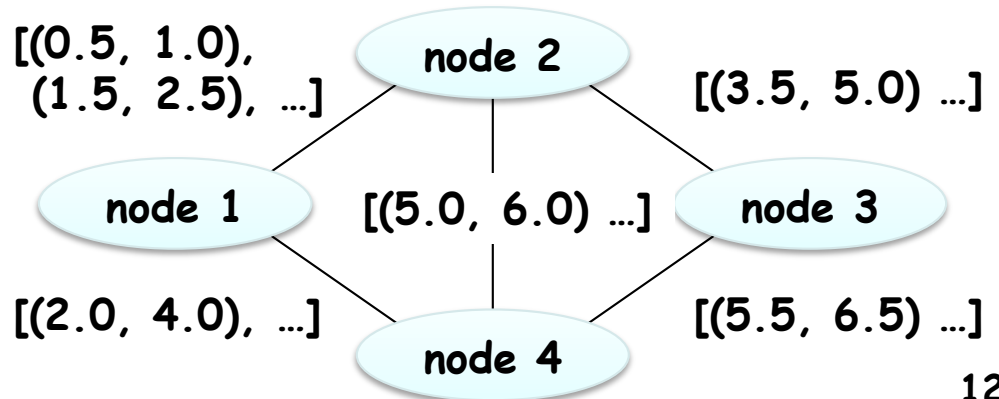
# Connection intervals: Visualized

(1, 2, 0.5, 1.0)  
 (1, 2, 1.5, 2.5)  
 (1, 4, 2.0, 4.0)  
 (2, 3, 3.5, 5.0)  
 (2, 4, 5.0, 6.0)  
 (3, 4, 5.5, 6.5)  
 ...



{

(1, 2): [(0.5, 1.0),  
 (1.5, 2.5), ...],  
 (1, 4): [(2.0, 4.0), ...],  
 (2, 3): [(3.5, 5.0) ...],  
 (2, 4): [(5.0, 6.0) ...],  
 (3, 4): [(5.5, 6.5) ...]



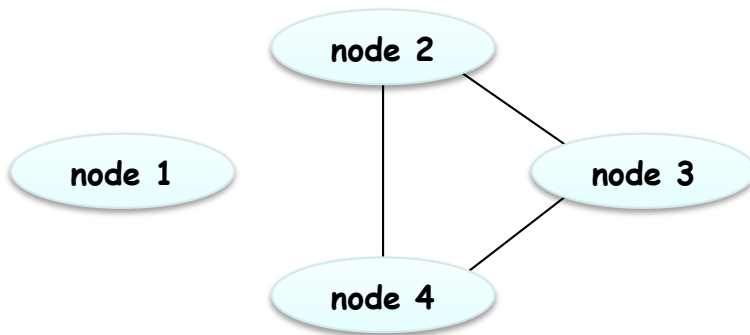
}

# Time-aggregated network

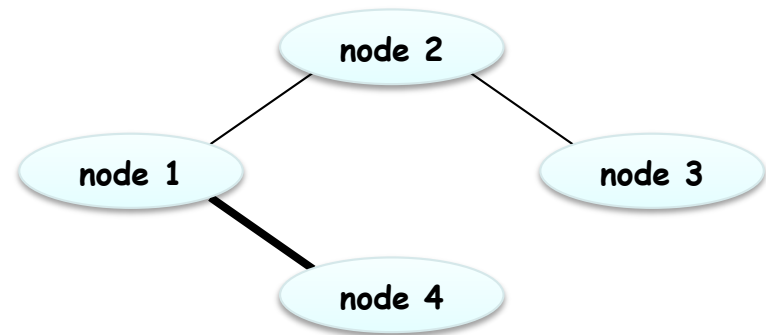
---

- These networks can also be created for a specific time window (e.g., each hour, day, week, etc.)

→ Time-aggregated network



$t = 5, 6$



$t \text{ in } [2.0, 4.0]$

# Seeking real-world data

## SocioPatterns

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Percentage of removed nodes  
— 0% — 50%

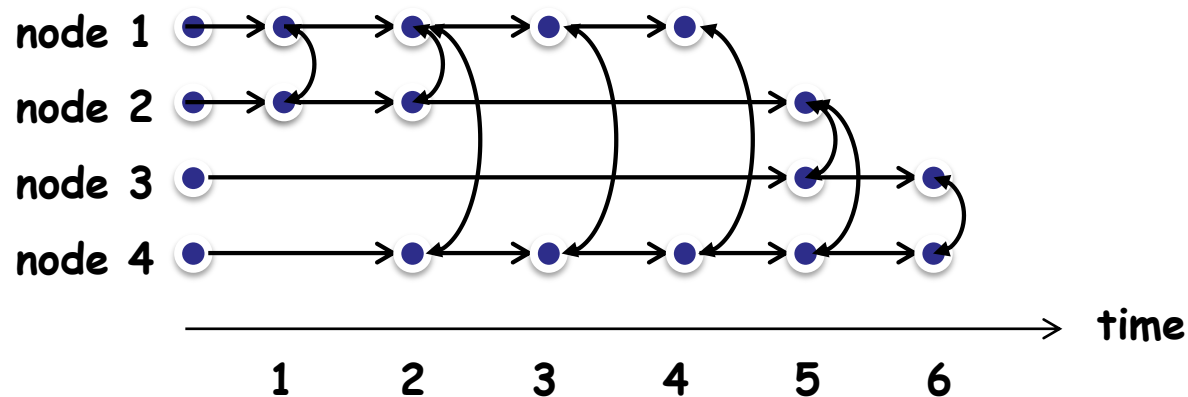
# Exercise

---

- Download the “**Primary school temporal network data**” and look at its contents
- Read its contact sequence data into Python
- Generate and visualize an aggregated network on a specific hour

# Plotting contacts in space-time

1. Create space-time nodes at  $t = 0$  for all nodes
  - This may be skipped if you prefer not to have them
2. Read a new contact
  - If it occurs later than the time points of involved nodes, create new space-time nodes for them, add directed edges from old ones, and update their time points
3. Create edges between involved nodes
4. Go back to 2





# Exercise

---

- Plot the contacts in the primary school temporal network data in detail over space-time (just for the hour chosen in the previous exercise)
  - How to lay out space-time nodes?

# Exercise: Dynamic animation

---

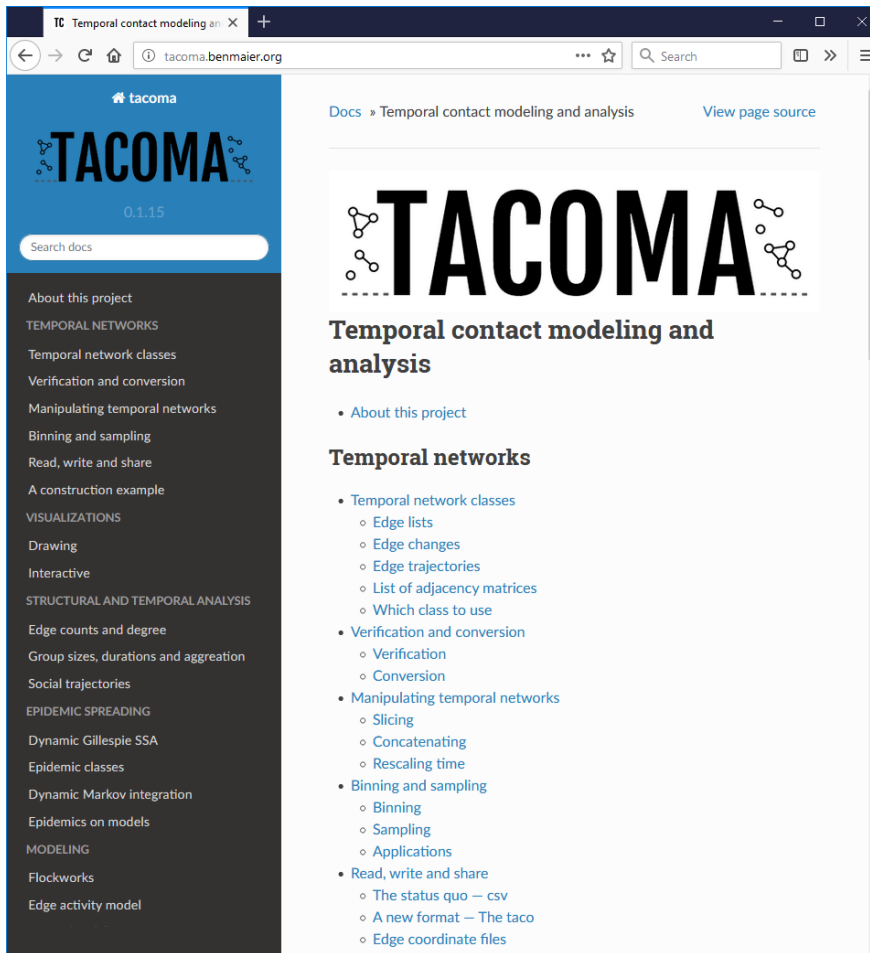
- You could also dynamically visualize the contacts using PyCX's `pycxsimulator.py`
- Visualize the behavior of the primary school temporal network data using PyCX

# Exercise

---

- Calculate the number of contacts in the primary school temporal network data at each time point and plot it over time
- Plot the distribution of time intervals between contacts on edge (1437, 1563) (= most active edge)
- What do you see there?

# New tool: TACOMA



The screenshot shows the TACOMA website homepage. The browser address bar displays "tacoma.benmaier.org". The page features a blue header with the TACOMA logo and version "0.1.15". A search bar is located below the header. The main content area is titled "Temporal contact modeling and analysis" and includes a large "TACOMA" logo. A navigation menu on the left lists various sections: "About this project", "TEMPORAL NETWORKS", "VISUALIZATIONS", "STRUCTURAL AND TEMPORAL ANALYSIS", "EPIDEMIC SPREADING", "MODELING", and "EDGE ACTIVITY MODEL". The main content area lists "Temporal networks" with sub-sections: "Temporal network classes", "Verification and conversion", "Manipulating temporal networks", "Binning and sampling", and "Read, write and share".

Docs » Temporal contact modeling and analysis [View page source](#)

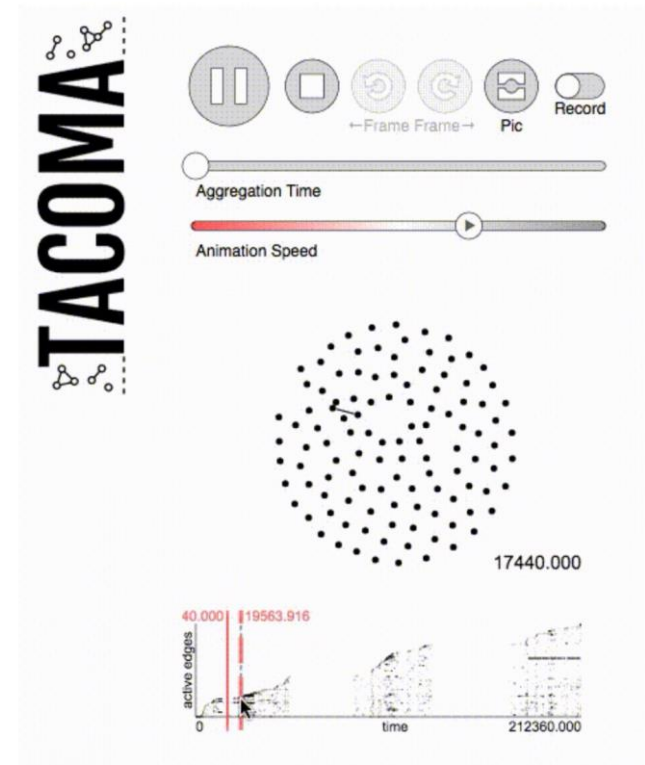
## TACOMA

### Temporal contact modeling and analysis

- About this project

### Temporal networks

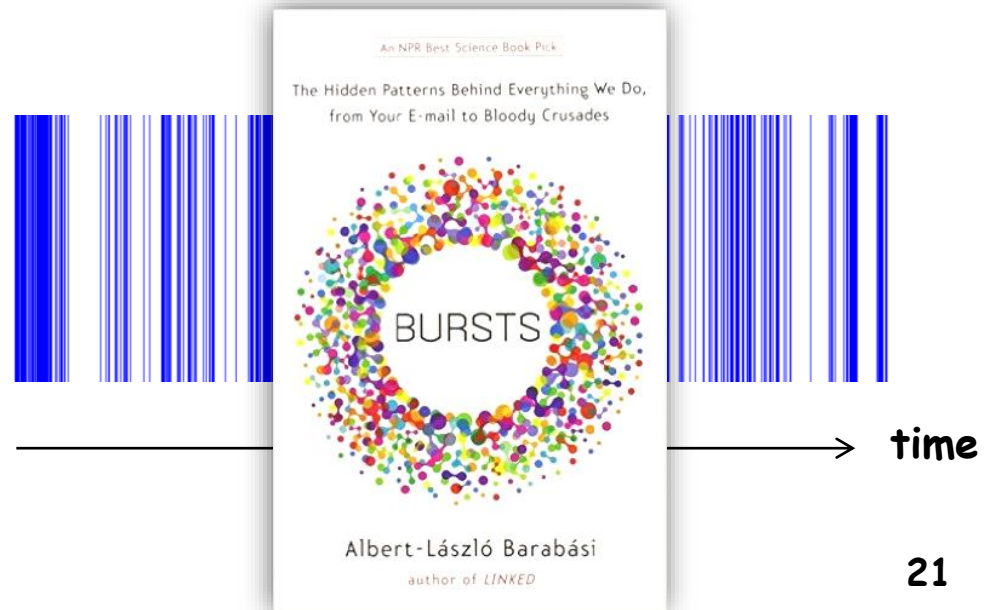
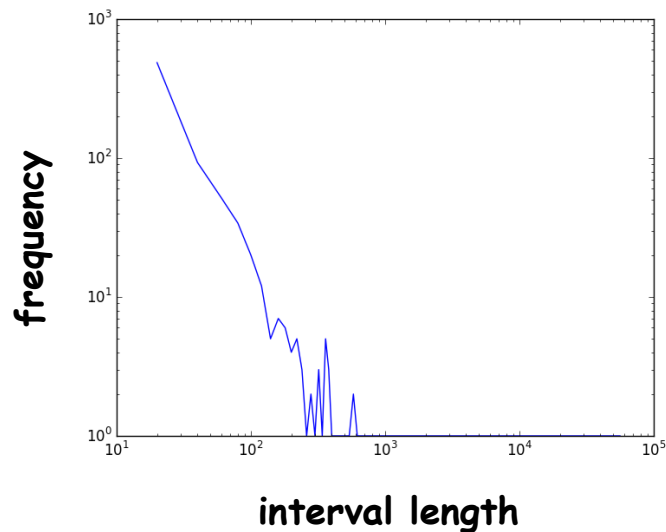
- Temporal network classes
  - Edge lists
  - Edge changes
  - Edge trajectories
  - List of adjacency matrices
  - Which class to use
- Verification and conversion
  - Verification
  - Conversion
- Manipulating temporal networks
  - Slicing
  - Concatenating
  - Rescaling time
- Binning and sampling
  - Binning
  - Sampling
  - Applications
- Read, write and share
  - The status quo – csv
  - A new format – The taco
  - Edge coordinate files



<http://tacoma.benmaier.org/>

# Burst!

- Many real-world social contacts show “bursty” behavior
  - Intervals between two consecutive contacts show a power-law distribution



# Implications of bursts

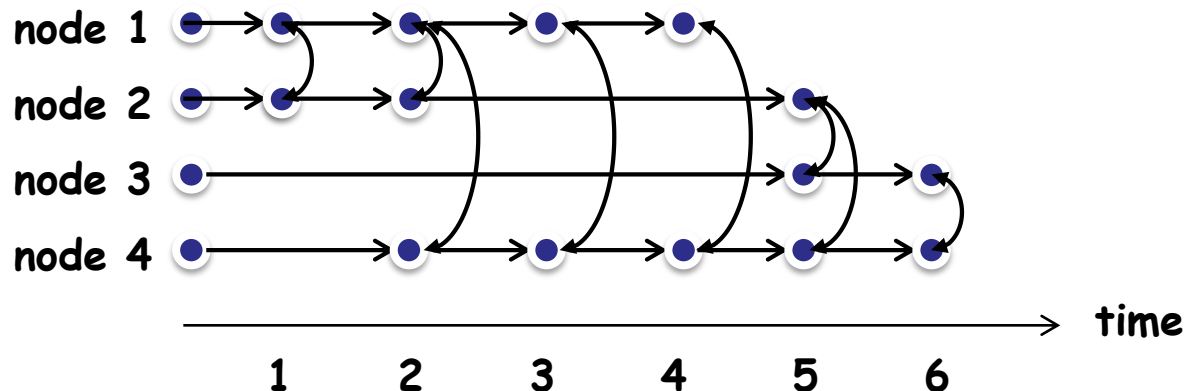
---

- Events are interrelated over time too
- Temporal behavior of the system may not be fully captured using time averages or Poisson distributions
- Dynamics on the network (e.g., disease spreading) may be fundamentally different from what would be expected on non-bursty networks

# Time-Respecting Paths

# Time-respecting path

- A sequence of contacts with non-decreasing times
  - E.g. There are time-respecting paths from node 1 to 3 but not the other way



- It can be obtained simply as a path in the space-time representation



# Exercise

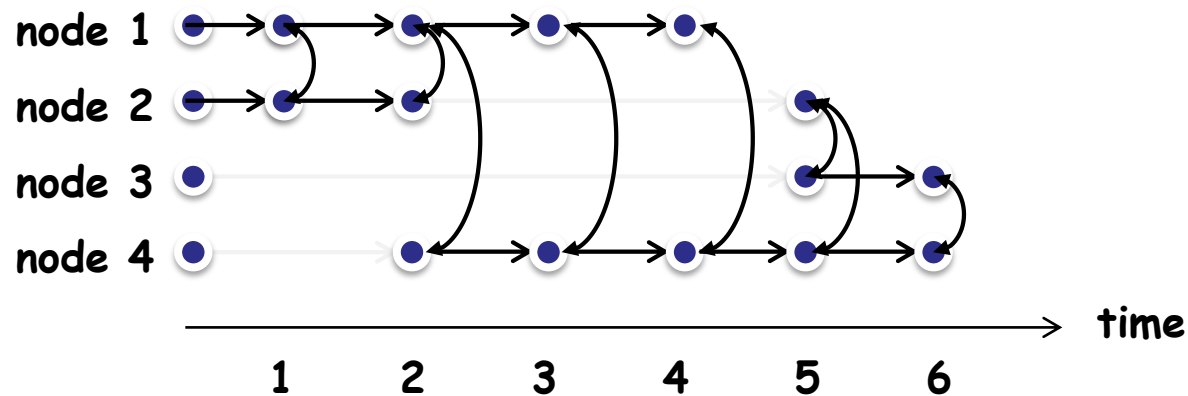
---

- Check if there are any time-respecting paths between randomly chosen two nodes in the space-time network constructed previously

# Time-respecting path with limited waiting time

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- Time-respecting path that does not contain same-node edges longer than a certain threshold (waiting time)
  - Represents loss of infectivity, etc.

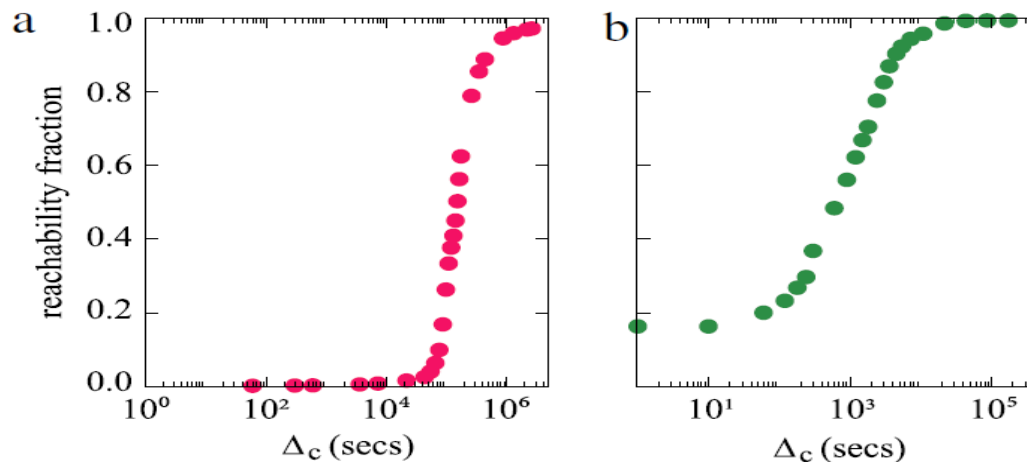


(Not allowed to stay on the same node for >1 time step)

# Reachability

---

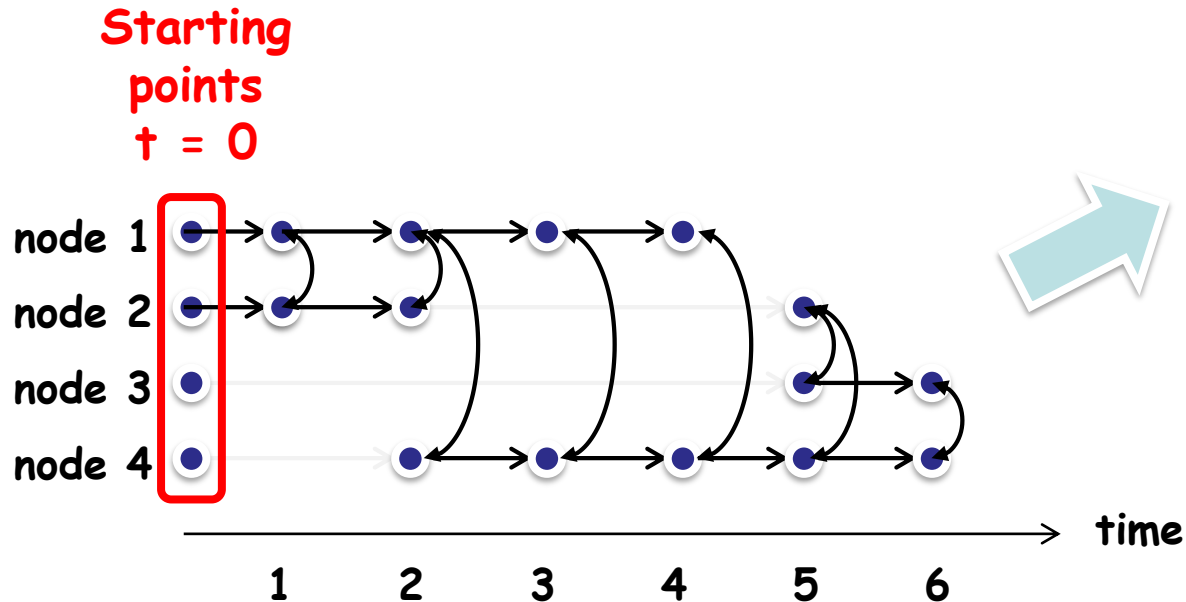
- Where can you reach from a given node at a specific time point, following only time-respecting paths?
  - Within a certain time window
  - With limited waiting time



Reachability ratios (a: mobile phone call network, b: airline connection network)  
Source: Holme & Saramäki (2012)

# Reachability network

- A static network in which reachable nodes are connected by edges



# Exercise

---

- Draw the reachability network of the primary school temporal network data during the hour chosen in the previous exercises
  - With max. waiting time = 1 minute
  - What happens if you increase the limit?

# Temporal Network Measurements

# Latency

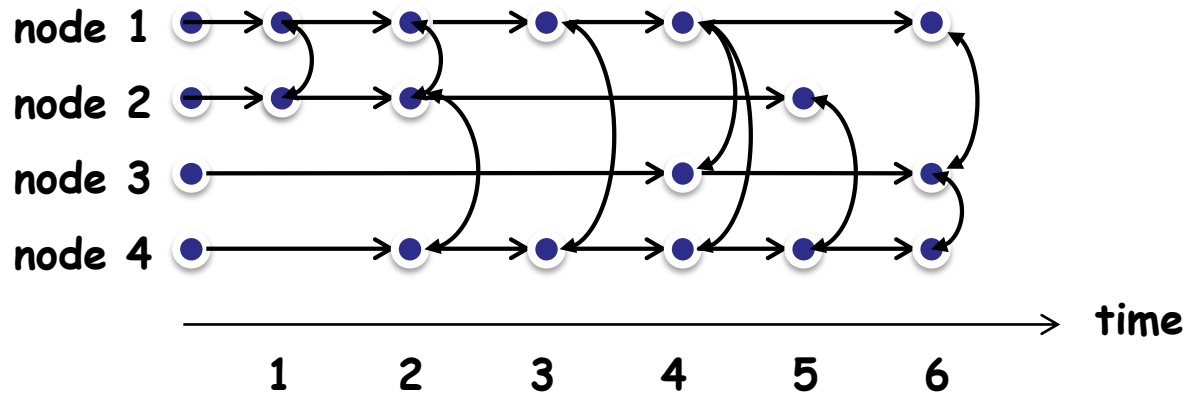
---

- Length of time needed to move from node  $i$  to node  $j$  using the *fastest* time-respecting path (within a given time window)
  - Use the “fastest”, not necessarily the “shortest” path
- Average latency captures how quickly spreading is occurring in the network

# Exercise

---

- What is the latency between each pair of nodes? *Average?*



- What if the time window is  $[4, 6]$ ?



# Exercise

---

- Calculate the average latency of the primary school temporal network data during each hour
- Plot how the average latency changes over time

# Centralities

---

- Measurable on aggregated networks
- Can be generalized to temporal networks by replacing:
  - Degree by node activity
  - Shortest path by fastest time-respecting path (for closeness, betweenness)
- Simulating propagation of importance through temporal edges gives centralities similar to eigenvector ones

# Exercise

---

- Calculate the temporal closeness centralities of nodes in the primary school temporal network data during a certain hour
  - This can be calculated by averaging latencies from the focal node to all other nodes

# Exercise

---

- Simulate propagation of importance on the primary school temporal network data with the following assumptions:
  - Each node initially has importance 1
  - Each time two nodes have a contact, 10% of their respective importance is moved to the other side through the contact
  - This repeats for the entire data set

# Randomization of Temporal Networks

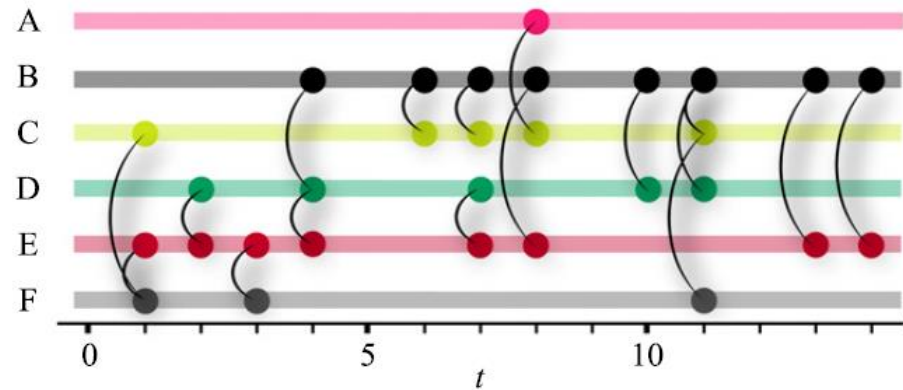
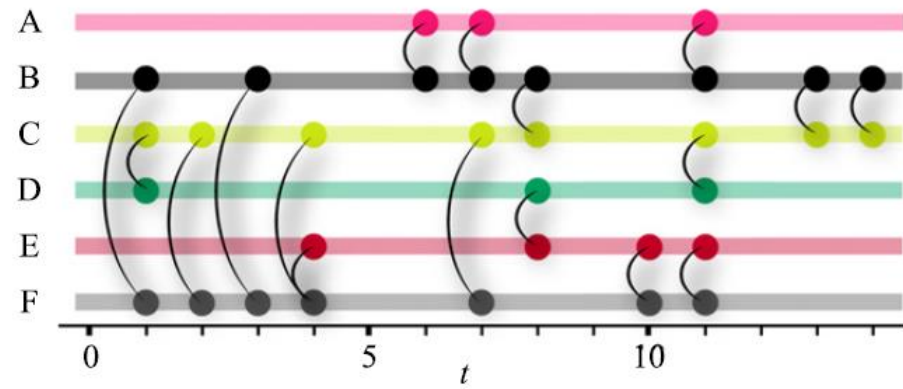
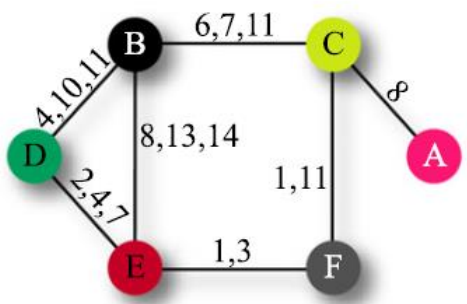
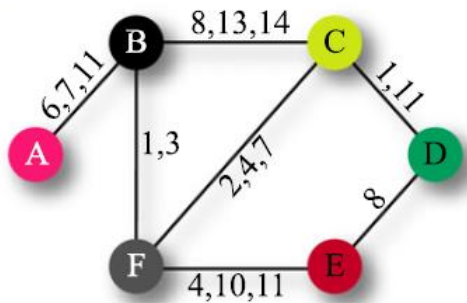
# Randomization of temporal networks

---

- Two main approaches:
  - Spatial randomization
    - E.g. randomized edges
  - Temporal randomization
    - E.g. randomly permuted times

# Randomized edges (RE)

- Randomizes the topology of the aggregated network by double edge swaps, etc.



# Properties of RE method

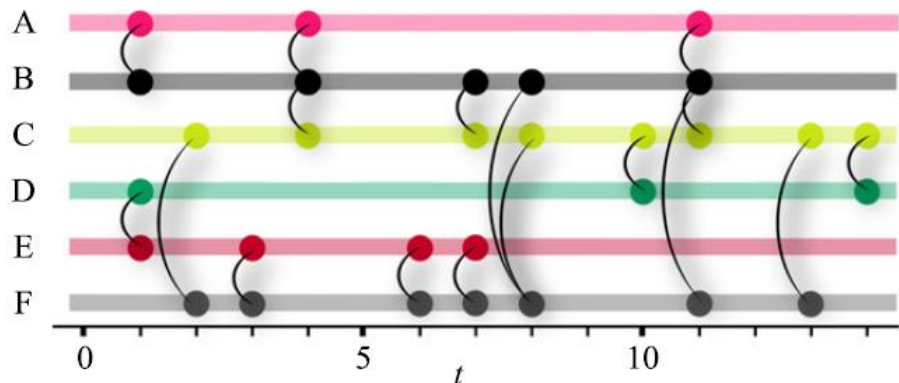
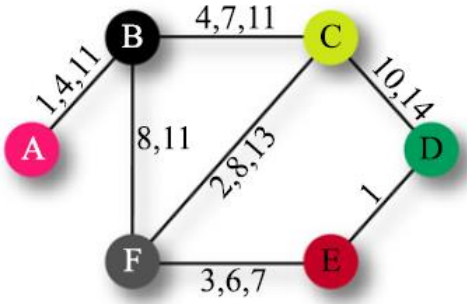
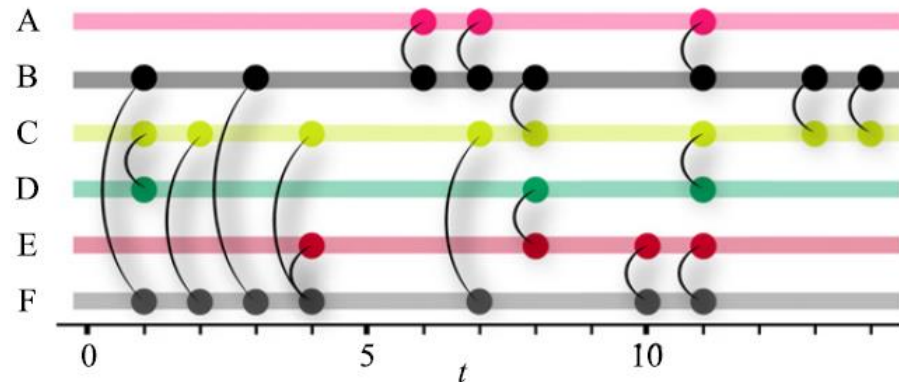
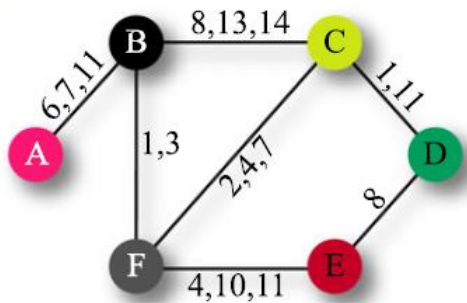
---

- RE destroys spatial (topological) correlations
- Preserved properties
  - Node degrees (but not # of contacts)
  - Contact sequences
  - Activity dynamics (both micro and macro)
- Works for both contact sequences and connection intervals



# Randomly permuted times (RP)

- Randomly reshuffles time stamps of contacts



# Properties of RP method

---

- RP destroys temporal correlations
- Preserved properties
  - Network topology
  - Number of contacts on each node/edge
  - Macroscopic activity dynamics (but not burstiness on individual edges)
- Very easy to implement
  - Needs overlap check for networks with connection intervals

# Other randomization methods

---

- **RE + RP**
  - Randomizes both space and time
  - Preserves only macro activity dynamics
- **Random times (RT)**
  - Time stamps of contacts are sampled randomly from a uniform distribution
  - Destroys temporal correlations and also macroscopic activity dynamics
- **RE + RT etc...**

# Exercise

---

- Implement several randomization methods (RE, RP, RE+RP, RT, RE+RT, etc.)
- Apply them to the primary school temporal network data
- Measure some network properties and see how each randomization affects
- Interpret the results

# Dynamical Processes on Temporal Networks

# Dynamics on temporal networks

- Temporal networks are particularly suitable for studying the dynamics of spreading processes
  - Infectious diseases via social contacts
  - Computer viruses via email transactions
  - Information spreading on social media
- How do the properties of the network affect the spreading process?

# Example: SIS model

---

- Infection rate per contact:  $p_i$
- Recovery rate per unit of time:  $p_r$
- Epidemic process can be simulated on empirically observed temporal network data
  - Dynamics of contact formation could also be simulated using a stochastic model (not discussed much in this class)

# Exercise

---

- Simulate the SIS model on the primary school temporal network data
- Obtain parameter values ( $p_i$ ,  $p_r$ ) with which epidemics occurs
- Then, randomize the network data using RE, RP, RE+RP, RT, RE+RT, etc.
  - How does this affect simulation results?