Radiation

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Radiation does not require physical contact. All objects radiate energy continuously in the form of electromagnetic waves due to thermal vibrations of the molecules.

1. Black body

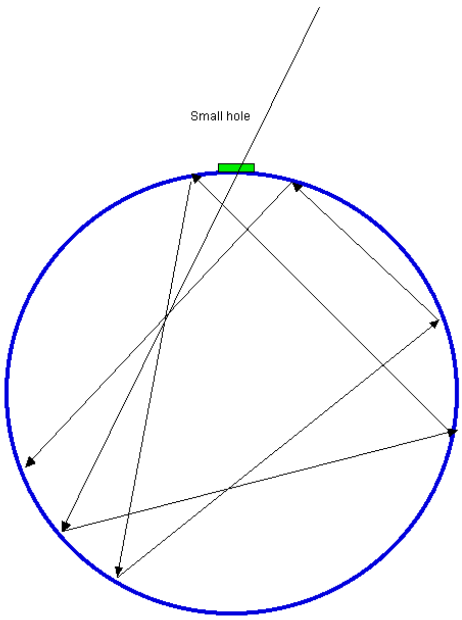


Fig. A photon entering a cavity through a small hole is effectively absorbed, so that the cavity represents a blackbody.

((**Atkins Quanta**))

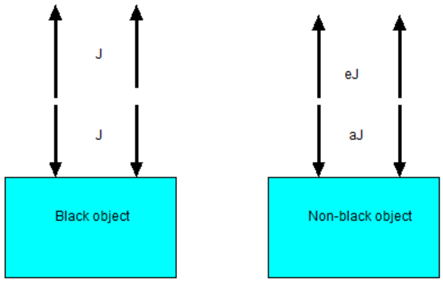
A black body is one that absorbs all the radiation incident upon it. A practical example is a container completely sealed except for a tiny pinhole: this hole behaves as a black body because all light incident on it from outside passes through and, once in, cannot escape through the vanishingly small hole. Inside it experiences an indefinitely large number of reflections before it absorbed, and these reflections have the result that the radiation comes into thermal equilibrium with the wall. Within the cavity we can imagine the electromagnetic field as having a distribution of frequencies characteristic of the temperature of the walls.

The presence of the hole enables a small proportion of this equilibrium radiation to seep out and be detected, and the distribution of wavelength in the black-body radiation is the same as the distribution within the equilibrium enclosure because the pinhole is a negligible perturbation

The radiant energy flux density from a black surface at a temperature T is equal to the radiant energy density emitted from a small hole in a cavity at the same temperature T.

2 Emission and absorption: Kirchhoff law

If a non-black object at T absorbs a fraction a of the radiation incident upon it, the radiation flux emitted by the object will be a times the radiation flux emitted by a black-body at the same temperature. The object must emit at the same rate as it absorbs if equilibrium is to be maintained: $a = e$.



((**Example**))

There exists a very close connection between the emissivity e and the absorptivity a of a body. A good emitter of radiation is also a good absorber of radiation, and vice versa. This is a qualitative statement of Kirchhoff's law.

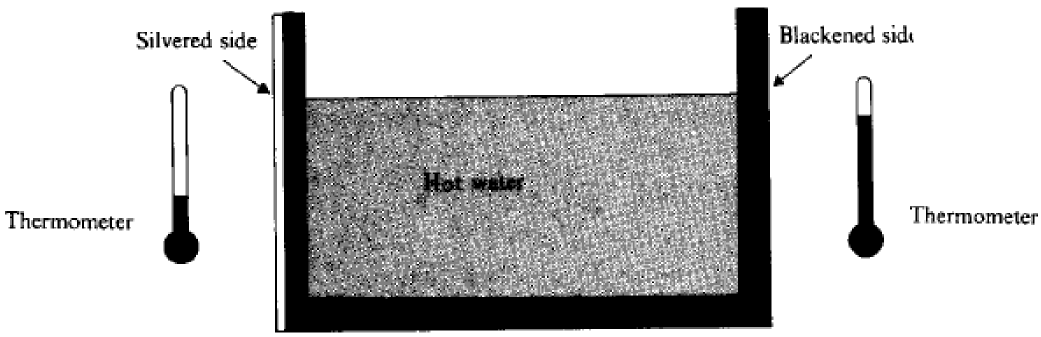


Fig. A classical experiment illustrating Kirchhoff's law. The container is filled with hot water. Its left side is silvered on the outside so that it is a poor absorber. Its right side is blackened so that it is a good absorber. Since the left side is then a poorer emitter of radiation than the right side, the thermometer on the left is found to indicate a lower temperature than the one on the right.

3 Stefan-Boltzman law

The Stefan–Boltzmann law, also known as Stefan's law, states that the total energy radiated per unit surface area of a black body in unit time (known variously as the black-body irradiance, energy flux density, radiant flux, or the emissive power), J, is directly proportional to the fourth power of the black body's thermodynamic temperature T (also called absolute temperature):

$$J = \varepsilon\sigma T^4 \qquad (\mathrm{W/m^2})$$

where J has dimensions of energy per time per unit area and ε is the emissivity of the blackbody. If it is a perfect blackbody, ε = 1. The constant of proportionality σ, called the Stefan–Boltzmann constant or Stefan's constant. The value of the constant is

$$\sigma = \frac{2\pi^5 k_B{}^4}{15c^2h^3} = \frac{\pi^2 k_B{}^4}{60c^2\hbar^3} = 5.670400\text{x}10^{-8}\ \text{W m}^{-2}\ \text{K}^{-4},$$

where k_B is the Boltzmann constant, h is Planck's constant ($h = 2\pi\hbar$) and c is the speed of light in a vacuum. The rate of energy transfer is given by Stefan's law

$$P = AJ = \sigma A e T^4 \qquad \text{(W)}$$

where A is the surface area of the object.

((**Mathematica**))

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Physconst = {kB → 1.3806504 10^-23, c → 2.99792458 10^8, h → 6.62606896 10^-34,
  ℏ → 1.05457162853 10^-34}
```

{kB → 1.38065 × 10^-23, c → 2.99792 × 10^8, h → 6.62607 × 10^-34, ℏ → 1.05457 × 10^-34}

$$\sigma = \frac{2\,\pi^5\,\text{kB}^4}{15\,c^2\,h^3}\ \text{/. h} \to 2\,\pi\,\hbar\ \text{// Simplify}$$

$$\frac{\text{kB}^4\,\pi^2}{60\,c^2\,\hbar^3}$$

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σ /. Physconst
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5.6704 × 10^-8

4 Energy absorption and emission by radiation

With its surroundings, the rate at which the object at temperature T with surroundings at T_0 radiates is

$$P_{net} = \sigma A e (T^4 - T_0{}^4)$$

When an object is in equilibrium with its surroundings, it radiates and absorbs at the same rate. Its temperature will not change.

5 The temperature at the surface of sun

One can measure the average flux of solar energy arriving at the Earth. This value is called the solar constant and is equal to 1370 W/m^2. Imagine a huge sphere with a radius of a = 1 AU (=1.49597870 x 10^{11} m) with the sun at its center. Each square meter of that sphere receives

$$p_0 = 1370 \qquad \text{W/m}^2$$

of power from the sun.

((Note)) The order of the energy (per second) transferred from the sun to the Earth is roughly estimated as

$$\frac{\pi R_E^{\ 2} \times p_0}{4.1858} = 4.18 \times 10^{16} cal/s$$

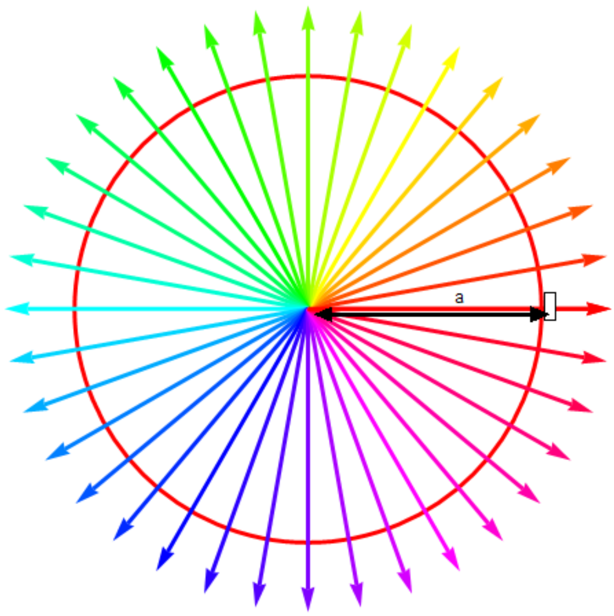


Fig. Luminosity of the sun. a = 1 AU. Sun is located at the center of sphere with radius a (=1 AU).

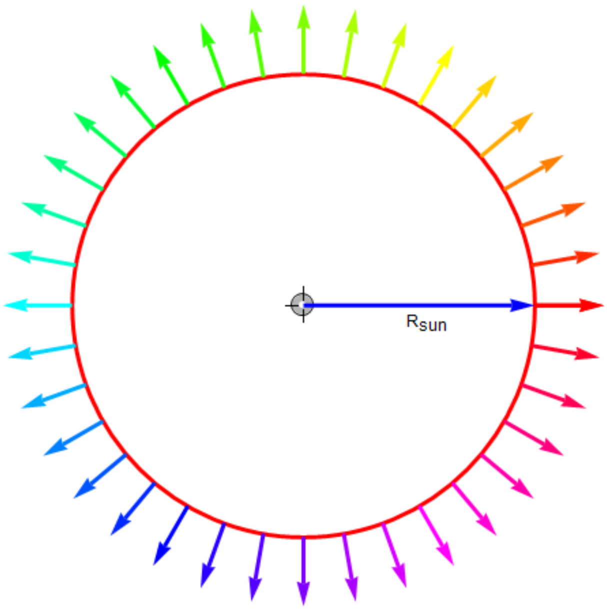
So we can calculate the total energy output of the sun by multiplying the solar constant by the sphere's area with a radius a (= 1 AU). The result, called the luminosity of the sun, is

$$L_{sun} = 4\pi a^2 p_0 = 3.853 \times 10^{26} \quad \text{(W)}$$

where a = 1 AU.

Using the Stefan-Boltzmann law, we can estimate the surface temperature of the sun.

$$L_{sun} = \sigma A_{sun} T_{sun}^{\ 4} = \sigma (4\pi R_{sun}^{\ 2}) T_{sun}^{\ 4} \qquad \text{(W)}$$



or

$$T_{sun} = [(\frac{a}{R_{sun}})^2 \frac{p_0}{\sigma}]^{1/4} = 5780K$$

6. Temperature of the Earth

We can use the Stefan-Boltzmann law to estimate the temperature of the Earth from first principles. The Sun is a ball of glowing gas of radius $R_{Sun} \approx 6.9599 \times 10^5$ km and surface temperature $T_{Sun} \approx 5780$ K. Its luminosity is

$$L_{Sun} = 4\pi R_{Sun}{}^2 \sigma T_{Sun}{}^4 \quad \text{(W)}$$

according to the Stefan-Boltzmann law. The Earth is a globe of radius $R_E \approx 6372$ km located an average distance $a = 1.49597870 \times 10^8$ km (= 1 AU) from the Sun. The Earth intercepts an amount of energy

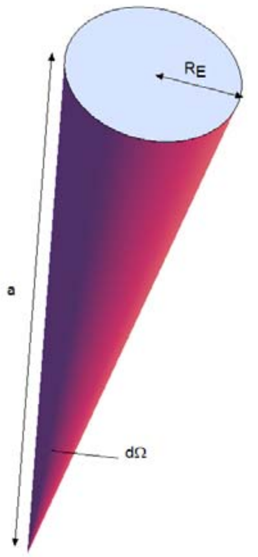
$$\Delta P_{\text{int}\,ercept} = \pi R_E{}^2 P_0 \qquad \text{(W)}$$

or

$$\Delta P_{\text{int}\,ercept} = L_{Sun}\frac{\Delta\Omega}{4\pi} = L_{Sun}\frac{\pi R_E^{\ 2}}{4\pi a^2} = \sigma T_{Sun}^{\ \ 4}\frac{\pi R_{Sun}^{\ \ 2} R_E^{\ 2}}{a^2} \qquad \text{(W)}$$

per second from the Sun's radiative output: *i.e.*, the power output of the Sun reduced by the ratio of the solid angle subtended by the Earth at the Sun to the total solid angle 4π.

$$\pi R_E^{\ 2} = a^2\Delta\Omega\,,$$



The Earth absorbs this energy, and then re-radiates it at longer wavelengths (the Kirchhoff's law). The luminosity of the Earth is

$$L_E = \pi R_E^{\ 2} p_0 = 4\pi R_E^{\ 2}\sigma T_E^{\ 4}, \qquad \text{(W)}$$

according to the Stefan-Boltzmann law, where T_E is the average temperature of the Earth's surface. Here, we are ignoring any surface temperature variations between polar and equatorial regions, or between day and night. In steady-state, the luminosity of the Earth must balance the radiative power input from the Sun, we arrive at

$$\Delta P_{\text{int}\,ercept} = L_E\,,\text{or} \qquad \frac{T_{Sun}}{T_E} = \sqrt{2}\frac{\sqrt{a}}{\sqrt{R_{Sun}}}$$

or

$$T_E = \frac{1}{\sqrt{2}} \frac{\sqrt{R_{Sun}}}{\sqrt{a}} T_{Sun}$$

Remarkably, the ratio of the Earth's surface temperature to that of the Sun depends only on the Earth-Sun distance and the radius of the Sun. The above expression yields T_E = 278.78 K. This is slightly on the cold side, by a few degrees, because of the greenhouse action of the Earth's atmosphere, which was neglected in our calculation. Nevertheless, it is quite encouraging that such a crude calculation comes so close to the correct answer.

7. Evaluation of the average surface temperature of our solar system

The average surface temperature of the planet may be expressed by

$$T_{av} = \frac{1}{\sqrt{2}} \frac{\sqrt{R_{Sun}}}{\sqrt{d}} T_{Sun}$$

where *d* is the mean distance from the Sun. The values of *d*, the calculated surface temperature T_{av}, and the reported surface temperature T_{obs} for each planet are listed in Table.

Planet	*d* (AU)	T_{av} [K]	T_{obs} [K]
Earth	1	278.8	287.2
Mars	1.52	226.1	227
Jupitor	5.20	122.3	165 (1 bar level) 112 (0.1 bar level)
Saturn	9.53	90.3	134 (1 bar level) 84 (0.1 bar level)
Uranus	19.19	63.6	76 (1 bar level) 53 (0.1 bar level)
Neptune	30.06	50.8	72 (1 bar level) 55 (0.1 bar level)
Pluto	39.53	44.3 K	44

((Mathematica))

cal=4.19 J, ,
σSB=Stefan-Boltzmann constant (W/m^2 K^4),
Mea = 5.9736 x 10^{24} kg; Mass of the earth,
Rea=6372.797 km, radius of the earth,
Msun=mass of sun (kg) =Solar mass
Rsun=radius of Sun (m)=Solar radius
light year=a distance light travels in a vacuum in one year=9.4605 x 10^{15} m,
Parsec (pc) = a unit of distance = 3.26 light yeras = 30.857 x 10^{15} m,
AU = astronomical unit = average distance between the Earth and the Sun = 1.49597870 x10^{11} m
p0 = solar constant (W/m2)

```
Physconst = {cal → 4.19, Mea → 5.9736 10^24,
  σSB → 5.670400 10^-8, Rea → 6.372 10^6 , Msun → 1.988435 10^30,
  Rsun → 6.9599 10^8, ly → 9.4605 10^15, pc → 30.857 10^15,
  AU → 1.49597870 10^11, p0 → 1370}
```

```
{cal → 4.19, Mea → 5.9736×10^24, σSB → 5.6704×10^-8,
 Rea → 6.372×10^6, Msun → 1.98844×10^30, Rsun → 6.9599×10^8,
 Mmoon → 7.3483×10^22, ly → 9.4605×10^15,
 pc → 3.0857×10^16, AU → 1.49598×10^11, p0 → 1370}
```

Total heat (cal) per sec to the Earth from the Sun

$$\frac{\pi\, \mathrm{Rea}^2\, \mathrm{p0}}{\mathrm{cal}} \text{ /. Physconst}$$

4.17069×10^{16}

Luminosity of Sun

```
Lsun = 4 π (AU)^2 p0 /. Physconst
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3.85284×10^{26}

Temperature of Sun

```
eq1 = Solve[((4 π Rsun^2 σSB Tsun^4) /. Physconst) == Lsun, Tsun];
Tsun /. eq1[[4]]
```

5780.13

Luminosity of Earth

```
Lea = p0 (π Rea^2) /. Physconst
```

1.74752×10^{17}

Temperature of Earth

```
eq2 = Solve[((4 π Rea^2 σSB Tea^4) /. Physconst) == Lea, Tea];
Tea /. eq2[[4]]
```

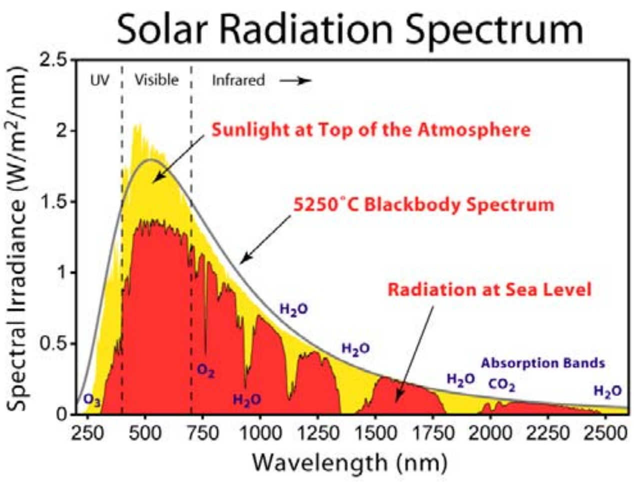
278.78

8. Radiation spectrum of the sun

We have a useful relation for the blackbody radiation, which is called a Wien's displacement law

$$\lambda_{\max} = \frac{2.897768551}{T} \times 10^6 \text{ (nm)}.$$

where T is the absolute temperature [K] and $\lambda_{\max}$ is the wavelength (nm) at which the radiation intensity maximum. This law is directly derived from the Planck's law of black-body radiation. The peak of the sun's spectrum is 484 nm.



Using the Wien's displacement law, the temperature of the sun can be estimated as

$$T_{sun} = \frac{2.89776851 \times 10^6}{484} \approx 5987K$$

9. Cosmic background radiation; direct evidence of Big Bang

Arno Penzias and Robert Wilson (Bell Lab, 1965)

No matter where in the sky they pointed their antenna, they detected faint background noise. They had discovered the cooled-down cosmic background radiation left over from the hot Big Bang. The spectrum of the cosmic microwave background shows a peak at $\lambda_{\max}$ = 1.063 mm. Using the Wien's displacement law, the temperature of the cosmic background radiation is estimated as

$$\begin{aligned} T &= \frac{2.89776851 \times 10^6}{\lambda_{\max}(nm)} \\ &= \frac{2.89776851 \times 10^6}{1.063 \times 10^6} \qquad \text{(Wien's displacement law)} \\ &= 2.726K \end{aligned}$$

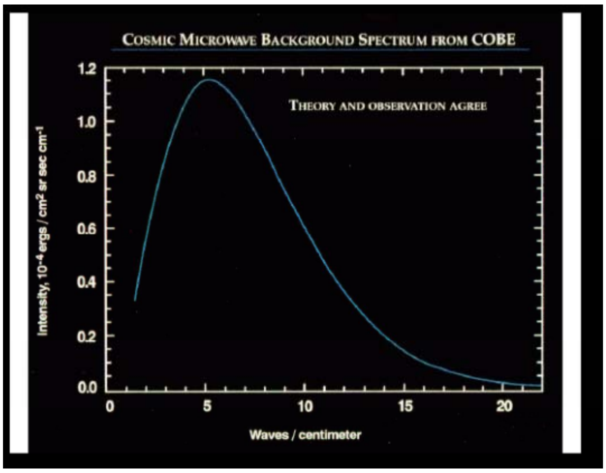


Fig. Experimental measurements of the spectrum of the cosmic blackbody radiation. The x axis is the wavenumber k defined by $k = 2\pi/\lambda$.

Dicke and Peebles (1960)

Early universe had been at least as hot as the Sun center. The hot early universe must therefore have been filled with many high-energy, short-wavelength photons, which formed a radiation field with that can be given by Planck's blackbody law. The universe has expanded so much since those ancient times that all those short-wavelenghth photons have their wavelengths stretched by a tremendous factor. As a result, they have becomes low-energy, long-wavelength photons.

The detail of the Hubble law is discussed in the Appendix and Chapter 17. (Chapter 17 will be taught in Phys.132).