Electrical conductivity in metals

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1. Classical theory of DC electrical conductivity

In the presence of an electric field E, the motion of electron (mass *m* and charge -*e*) can be governed by a Newton's second law,

$$m(\frac{dv}{dt}+\frac{v}{\tau}) = F = -eE$$

where t is a relaxation time. In the steady state, we have

$$v = -\frac{eE\tau}{m}$$

The current density *J* is given by

$$J = n(-e)v = \frac{ne^2\tau}{m}E = \sigma E$$

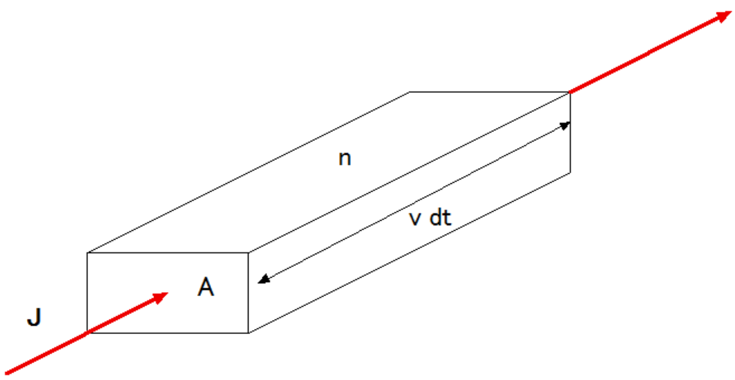


Fig. Current density (current passing through a unit area): $\Delta Q = (-e)nvAdt$;

$$J = \frac{I}{A} = \frac{\Delta Q}{A\Delta t} = (-e)nv$$

The conductivity σ is defined by

$$\sigma = \frac{ne^2\tau}{m}$$

where n is the number density. The unit of σ is

$$[\sigma] = \frac{\frac{1}{cm^3} \cdot erg \cdot cm \cdot s}{erg \cdot \frac{s^2}{cm^2}} = \frac{1}{s}$$

2. Change in Fermi sphere due to the presence of electric field

We consider the equation of motion,

$$\dot{\mathbf{p}} = \hbar \dot{\mathbf{k}} = -eE$$

From this, we get

$$\mathbf{k}(t) - \mathbf{k}(0) = -\frac{e\mathbf{E}}{\hbar} t$$

At $t = 0$, the field $\boldsymbol{E}$ is applied to an electron gas that fills the Fermi sphere centered at the origin of $\boldsymbol{k}$-space. At time t, the Fermi sphere will be displaced to a new center at

$$\delta\mathbf{k} = -\frac{e\mathbf{E}t}{\hbar}$$

Because of collisions with impurities, lattice imperfections, and phonons, the displaced sphere may be maintained in a steady state in an electric field;

$t = \tau$ collision time

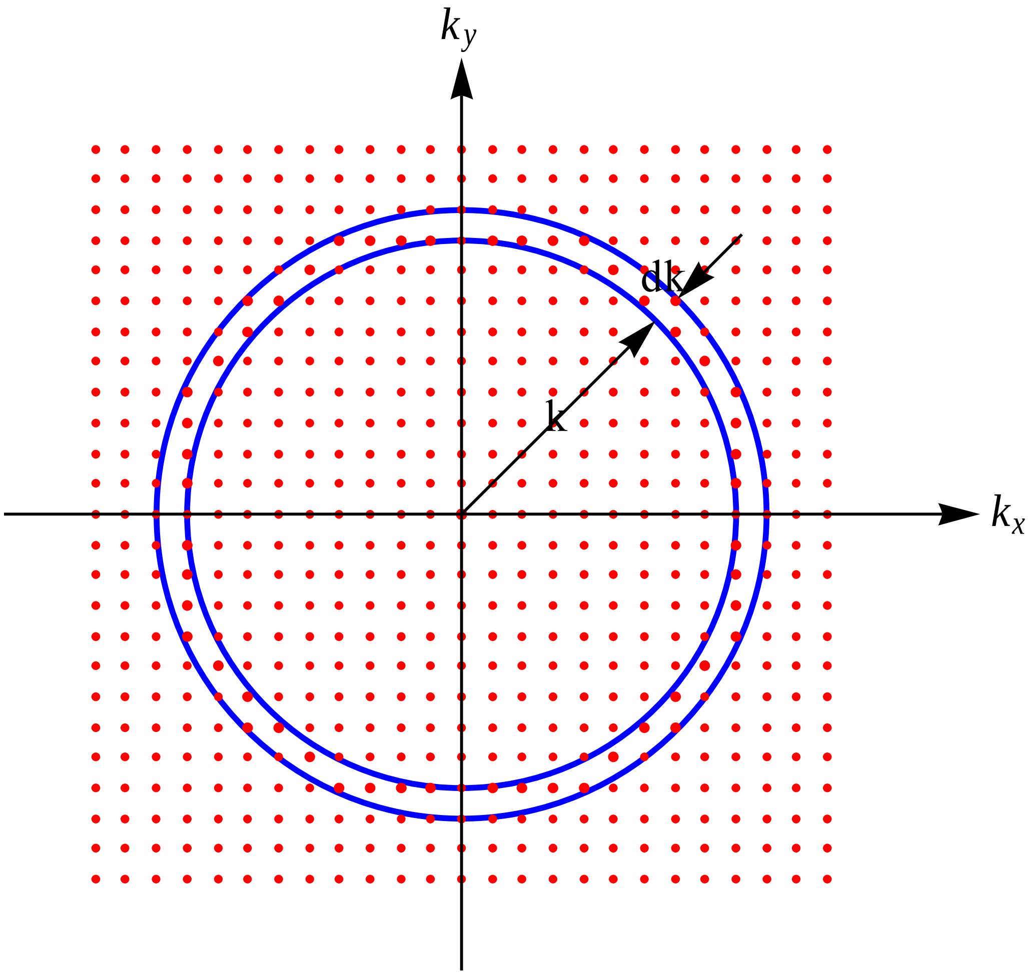
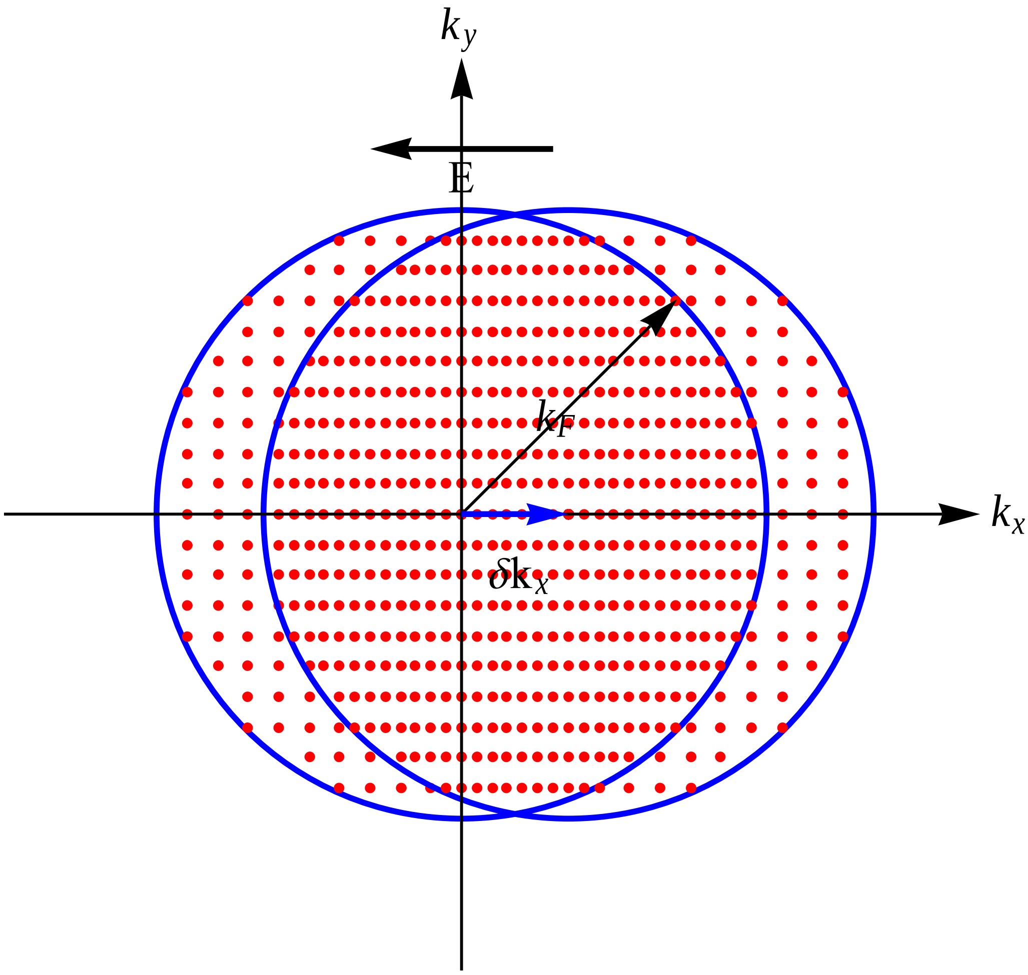


Fig. Fermi sphere with radius k_F.



k_y
E
k_F
k_x
δk_x

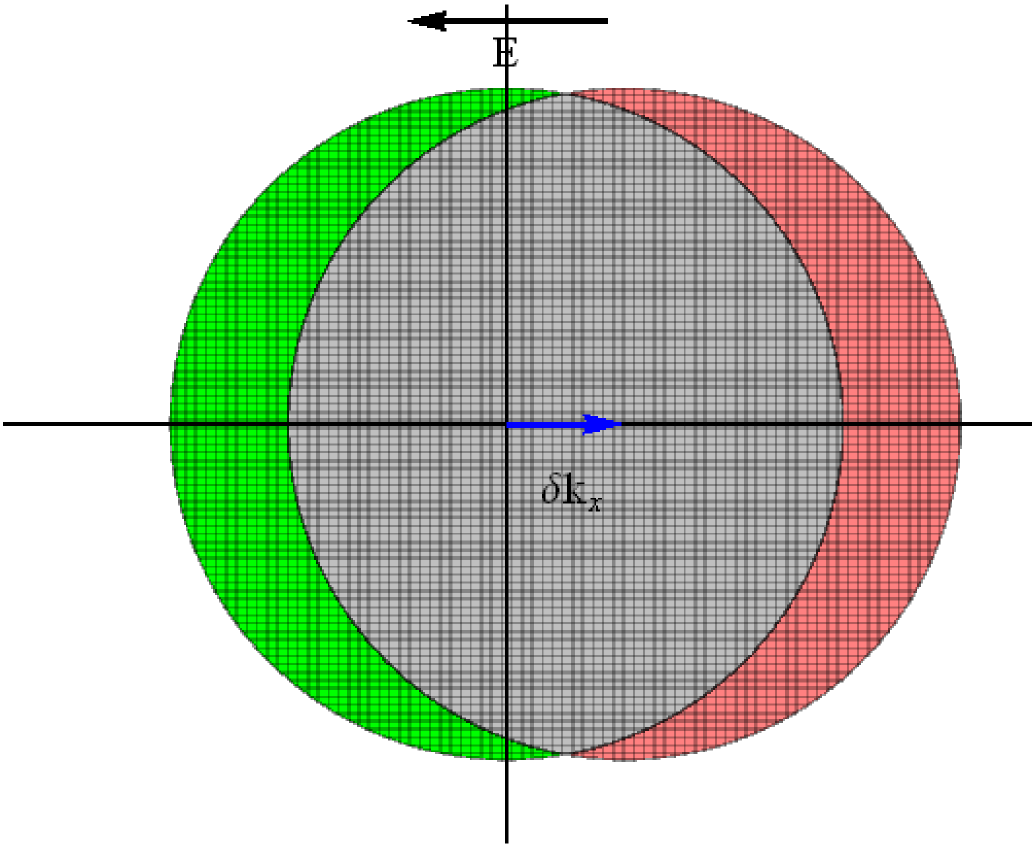


Fig. The shift of the Fermi sphere in the presence of an electric field along the negative *x* direction.

Then we have

$$v = \frac{\hbar \delta \mathbf{k}}{m} = -\frac{\hbar}{m}\frac{e\mathbf{E}\tau}{\hbar} = -\frac{e\mathbf{E}\tau}{m}.$$

The current density J is given by

$$\mathbf{J} = n(-e)\mathbf{v} = \frac{ne^2\tau}{m}\mathbf{E} = \sigma\mathbf{E}$$

which is the Ohm's law. The electrical conductivity s is defined by

$$\sigma = \frac{ne^2\tau}{m}.$$

The electrical resistivity ρ is defined by

$$\rho = \frac{1}{\sigma} = \frac{m}{ne^2\tau}$$

((Note)) The unit of ρ is [s].

3. Relaxation time of electron for Cu at 300 K

It is instructive to estimate from the observed conductivity the order of magnitude of the relaxation time τ. We observe Cu at 300 K,

$\rho = 1.7\,\mu\Omega cm$ or $\sigma = 6.0\times10^5 (\Omega cm)^{-1}$ (practical units)

We note that

$$\Omega cm = \frac{V}{A} cm = \frac{\frac{1}{300} statV}{3\times10^9 statA} cm = \frac{1}{9\times10^{11}} esu$$

Then we have

$$\rho = \frac{1}{9\times10^{11}} \times 1.7\times10^{-6} = 1.89\times10^{-18} \text{ [s]}$$

or

$$\sigma = 5.29\times10^{17} \text{ s}^{-1}$$

((Note)) 1 statV = 300 V, 1A = 2.997924536.8431 x 10^9 = 3 x 10^9 statA

Using the number density of electron in Cu,

n = 8.47 x 10^{22}/cm^3.

$\tau = \frac{\sigma m}{ne^2}$ = 2.47 x 10^{-14} s T = 300 K.

4. Number density

ρ: density (g/cm^3)
A: mass of atom (g/mol)

Suppose that there are Z electrons per atom. Then the number density is

$$n = \frac{Z\rho}{A} N_A$$

where N_A is the Avogadro number.

	Z	n (10^{22}/cm^3)
Li	1	4.7
Na	1	2.65
K	1	1.40
Rb	1	1.15
Cs	1	0.91
Cu	1	8.47
Ag	1	5.86
Au	1	5.90
Be	1	24.7
Mg	2	8.61
Ca	2	4.61
Sr	2	3.55
Ba	2	3.15

5. Mean free path

$$l = v_F \tau$$

where v_F is the Fermi velocity. The conductivity of Cu at 4 K is nearly 10^5 times that at 300 K.

$$\sigma(4K) = 5.29 \times 10^{17} \times 10^{5} = 5.29 \times 10^{22}$$

$$\tau = \frac{\sigma m}{ne^2} = 2.47 \text{ x } 10^{-9} \text{ s} \qquad T = 4 \text{ K.}$$

When v_F = 1.57 x 10^8 m/s for Cu, then the mean free path l is

$$l(4K) = 1.57 \times 10^{8} \times 2.47 \times 10^{-9} = 0.3 cm$$

and

$$l(300K) = 1.57 \times 10^{8} \times 2.47 \times 10^{-14} = 3 \times 10^{-6} cm$$

6. Temperature denepdence of electrical resistivity of metals

The relaxation time t is described as

$$\frac{1}{\tau} = \frac{1}{\tau_L} + \frac{1}{\tau_i}$$

where τ_L and τ_i are the collision times for scattering by phonons (lattice vibration) and by imperfections, respectively. The net resistivity is given by

$$\rho = \rho_L + \rho_i$$

((**Matthiessen's rule**))

ρ_L is the resistivity caused by thermal phonons. ρ_i is the resistivity caused by scattering of the electron waves by static defects that disturb the periodicity of the lattice.

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7. Residual resistivity

The residual resistivity $\rho_i(T = 0$ K), is the extrapolated resistivity at T = 0 K.

$$\rho_L(T) \rightarrow 0 \qquad \text{as } T \rightarrow 0.$$

$$\rho_i \rightarrow \rho_i(0) \qquad \text{as } T \rightarrow 0.$$

Often ρ_i is independent of T.

Residual resistivity ratio = $\dfrac{\rho(T = 300K)}{\rho_i}$, which is a convenient approximate indicator of sample purity.