Intrinsic parity

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When the number of particles can change we have to assign an intrinsic parity to each particle. This is not possible in classical theory, where particles cannot be created or destroyed. This is why parity is not discussed in classical theory. In some cases one cannot find an appropriate set of intrinsic parities such that $\hat{\pi}$ is conserved. In the case of weak interaction the parity conservation is violated.

1. Intrinsic parity

We would expect the operator $\hat{\pi}$ to act separately on each particle when the particles are far apart, and if $\hat{\pi}$ commutes with the Hamiltonian, it would then continue to act separately on each particle when they come together, so the extra phase in the transformation in a multi-particle state would be the product of the phases η_1, η_2, ..for the individual particles

$$P\psi(\mathbf{r}_1,\sigma_1;\mathbf{r}_2,\sigma_2,..) = \eta_1\eta_2\cdots\psi(-\mathbf{r}_1,\sigma_1;-\mathbf{r}_2,\sigma_2,..),$$

where the σ_s are spin 3-components, and the phase factor η_n depends only on the species of particle *n*. These factors are known as the *intrinsic parities* of the different particle types. The operator P^2 commutes with all coordinates, momenta, and spins. It could be an internal symmetry. All the intrinsic parities η_n are just either +1 or −1. Note that the intrinsic parity of the electron, proton, and neutron is +1.

When parity is conserved, it then restricts the kind of decay processes that can take place. Let us consider, for example, particle *A* decaying in its rest frame into particles *B* and *C* (B and C are identical particles)

$$A \rightarrow B + C .$$

The conservation of parity in the decay then implies that

$$\eta_A = \eta_B\eta_C(-1)^l .$$

where η_A and η_B are the intrinsic parity of the two particles and *l* is their relative orbital angular momentum; *l* = 0, 2, 4,… (even parity).

2. ((Example)): the disintegration of the 1s state of deuteron by pion ((G.L.Squires))

The π meson was predicted by Yukawa. This particle plays a significant role in nuclear forces. The particle comes in three charge states, π^+, π^0, π^-. It was found to have spin zero. What is the intrinsic parity of π^-, assuming that the known particles, the proton, and the neutron, have positive intrinsic parity?

We consider the classical example, the absorption of very low-energy (or, as is usually termed, "stopping") π^- mesons on deuterium nuclei (deuteron), giving a pair of neutrons,

$$\pi^- + d \to n + n$$

where d is the deuteron (1s state; $l = 0$) and π^- is the pion (negatively charged spin zero particle), and n is the neutron (spin 1/2).

(i) The right-hand side reaction: *n* + *n*

The parity of $n + n$ (identical particles) is given by

$$\eta_n \eta_n (-1)^l = (-1)^l$$

where η_n (=1) is the intrinsic parity of neutron and l is the relative orbital angular momentum for the two identical particles. The neutron has a spin 1/2. So the total spin is $S = 0$ (antisymmetric) or $S = 1$ (symmetric). Because the neutron is a fermion, we have the two cases: ($S = 0$, $l = 0$.) leading to $J = 0$, or ($S = 1$, $l = 1$) leading to $J = 2, 1, 0$. Note that the total angular momentum should be conserved between the right-hand side and the left-hand side.

(ii) The left-hand side: $\pi^- + d$

The parity of $\pi^- + d$ is given by

$$\eta_d \eta_{\pi^-}$$

where the pion has zero spin and the deuteron has a spin 1. So the total spin of the left-hand side is $S = 1$. The orbital angular momentum is zero in the lowest Bohr state; $l = 0$. Then the $d - \pi^-$ atom has total angular momentum; $J = 1$. Because of the angular momentum conservation between the left-hand and right-hand side, it follows that the state of $n + n$ should be denoted by $S = 1$, $l = 1$, and $J = 1$.

(iii) Intrinsic parity of π^-

Then we have

$$\eta_d \eta_{\pi^-} = (-1)^l = -1$$

or

$$\eta_{\pi^-} = -\eta_d = -1.$$

since the intrinsic parity of the deuteron is +1.

3. Example (the possibility of $\rho^0 \to \pi^0 + \pi^0$)

The ρ^0 meson has spin 1, and the π^0 meson has spin 0. We show that the decay

$$\rho^0 \to \pi^0 + \pi^0$$

is impossible.

((**Solution**)) **((G.L.Squires))**

The ρ^0 meson has spin 1, and the π^0 meson has spin 0. Therefore, if the decay were to occur, the two π^0 mesons would have to be in an l = 1 orbital state in order to conserve angular momentum. However, an l = 1 state has odd parity, which is an antisymmetric space function. (For a system of two particles, the parity and symmetry of the space function are the same, because reflecting a spin through the origin leaves it unchanged, so the parity of a spin function is always positive). Since the mesons have zero spin, the spin state function is symmetric. Therefore the overall state function is antisymmetric. But the π^0 meson is a boson and must have symmetric function. The decay is therefore impossible.

((**Note**))

The left-hand side of $\rho^0 \to \pi^0 + \pi^0$;

S = 1 for ρ^0 meson. The total angular momentum is J = 1

The right-hand side of $\rho^0 \to \pi^0 + \pi^0$;

$$D_0 \times D_0 = D_0 \; (S = 0) \qquad \text{(symmetric)}$$

From the total angular momentum conservation, the relative orbital angular momentum is l = 1 (antisymmetric).

The overall state function is antisymmetric. The π^0 meson is a boson and so the overall state function must be symmetric. Note that

$$\eta_{\rho^0} = \eta_{\pi^0}\eta_{\pi^0}(-1)^l = (-1)^l = -1$$

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APPENDIX

Spin-parity of some commonly known particles:

State	spin	Parity	Particle
pseudoscalar (0^-)	0	-	π, K
scalar (0^+)	0	+	a_0, Higgs
vector (1^-)	1	-	γ, ρ, ω, ϕ, ψ, Y
pseudovector (axial vector) (1^+)	1	+	a_1,