Muon spin relaxation/rotation/resonance (μSR).

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1. Introduction

We present the principle of muon spin rotation spectroscopy (μSR). There are two kinds of muon, positive muon μ^+ and negative μ^-. The physical properties of these particles are shown below. Note that μ^+ particles are mainly used mainly in the μSR. The lifetime of μ^- particles is very short when they rotate around heavy nucleus. Many of them are absorbed by the nucleus. So it may be better to use μ^+SR, instead of μSR.

In μSR one uses a specifically implanted spin (the muon's) and does not rely on internal nuclear spins. Unlike NMR (nuclear magnetic resonance), the μSR technique does not require any radio-frequency technique to align the probing spin. the implanted muons are not diffracted but remain in a sample until they decay. Only a careful analysis of the decay product (i.e., a positron) provides information about the interaction between the implanted muon and its environment in the sample. The μSR experiments require muon fluxes of the order of 10^4-10^5 muons per second and square centimeter. Such fluxes can only be obtained in high-energy accelerators.

μ^+ (positive muon, antimuon)

charge	+1
mass	105.6583715 MeV/c^2
spin	1/2
gyromagnetic ratio	-135.5342 MHz/T
g-factor	2.0023318406
Magnetic moment	4.836 x 10^{-3} μ_B
life time	2.19695 μs
A light proton (mass = 0.11 m_p)	

μ^- (negative muon)

charge	-1
mass	105.6583715 MeV/c^2
spin	1/2
gyromagnetic ratio	-135.5387 MHz/T
g-factor	2.002331872
magnetic moment	4.836 x 10^{-3} μ_B
life time	2.1948 μs.
spin	1/2

Less in matter because of nuclear capture
A heavy electron (mass = $207m_e$)

Note that the gyromagnetic ratio of negative muon is the same as that of positive muon, while the charge of positive

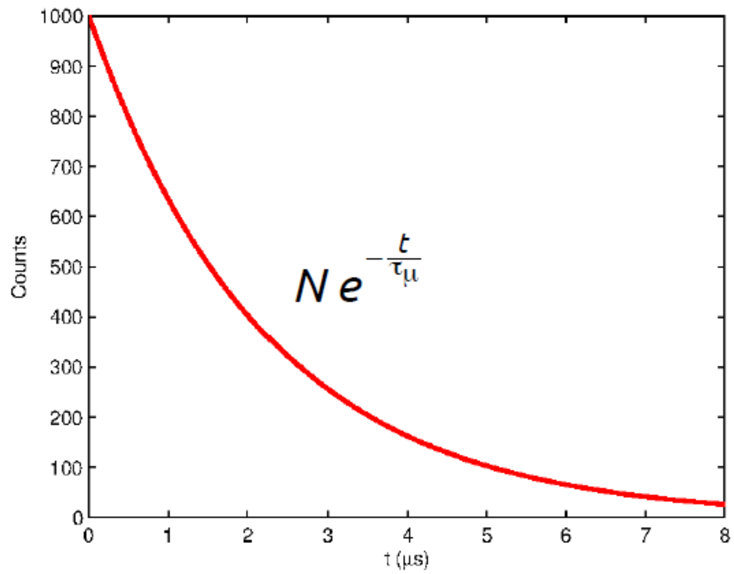


Fig. muon decay with the life time 2.2 μs.

2. The gyromagnetic ratio

The gyromagnetic ratio of the positive and negative muons is given by the ratio of the magnetic moment to the spin angular momentum,

$$\frac{\mu}{S} = \frac{-\frac{g\mu_0}{\hbar}S}{S} = \frac{g\mu_0}{\hbar} = -\frac{g}{2}\frac{e}{m_\mu c},$$

which is negative, where the charge of positive muon is positive [e (>0)], g is the g-factor and is close to 2.0, the magnetic moment μ_0 is given by

$$\mu_0 = \frac{e\hbar}{2m_\mu c}$$

and the mass of positive muon is given by

$m_\mu = 105.6583715$ MeV/c^2.

The direction of the magnetic moment $\boldsymbol{\mu}$ and the spin angular momentum S is antiparallel.Note that

$$\frac{e}{2\pi m_\mu c} = 13.5581(kHz/Oe) = 13.5581\times 10^7 (Hz/T)$$

3. The dynamics of positive and negative muons in the presence of magnetic field

(i) Positive muon μ^+

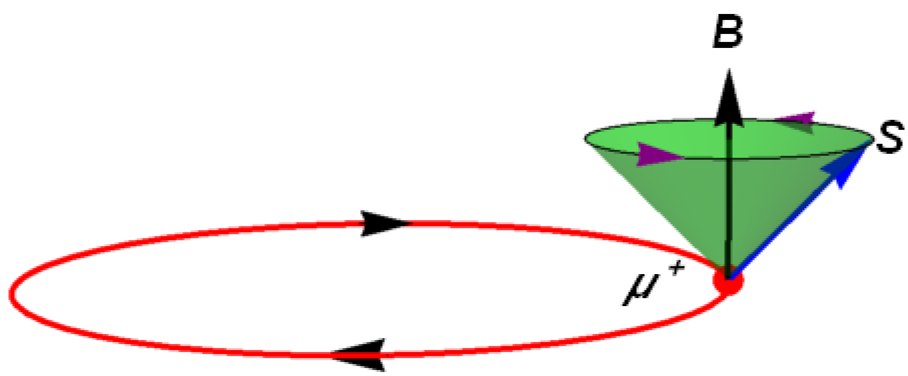


Fig. Spin precession of the positive muon μ^+ around a magnetic field $\boldsymbol{B}$. The spin rotation ($\gamma < 0$, C.C.W). The orbital motion (+*e*, C.W.)

(ii) Negative muon μ^-

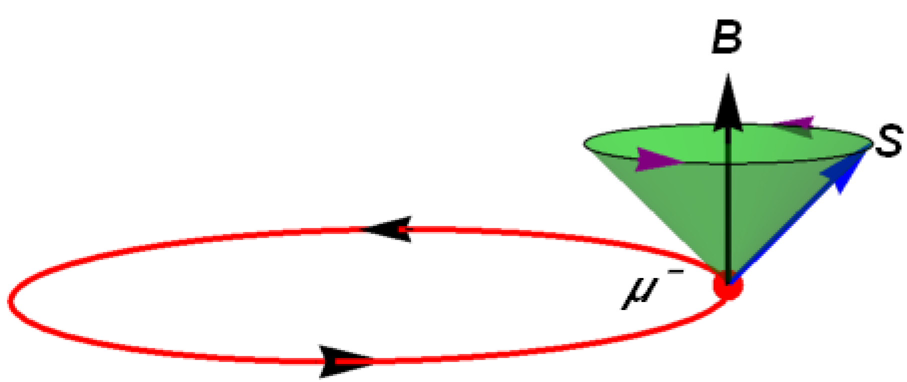


Fig. Spin precession of the negative muon μ^- around a magnetic field ***B***. The spin rotation ($\gamma < 0$, C.C.W). The orbital motion (-*e*, C.C.W.)

4. Production of pions

Protons of 600 to 800 MeV kinetic energy interact with protons or neutrons of the nuclei of a light element target (typically graphite) to produce pions (π^+).

$$p + p \rightarrow \pi^+ + p + n$$

$$p + n \rightarrow \pi^+ + n + n$$

where

n: neutron, *p*: proton

π^+: pion (positive charge, *S* = 0, boson)
the gyromagnetic ratio is negative.

Pions are unstable (lifetime 26.03 ns). They decay into muons (and neutrinos):

$$\pi^+ \rightarrow \mu^+ + \nu_\mu$$

5. Production of muon

The pion decay is a twobody decay. Suppose that the pions are produced at rest in the laboratory frame. To conserve momentum, the muon and the neutrino must have equal and opposite momentum. The pion has zero spin so the muon spin must be opposite to the neutrino spin. One useful property of the neutrino is that its spin is aligned antiparallel with its momentum (it has negative helicity), and this implies that the muon-spin is similarly aligned. Thus by selecting pions which stop in the target (and which are therefore at rest when they decay) one has a means of producing a beam of 100% spin-polarized muons.

The muonium: attractive interaction electron e- and μ^+

$$\pi^+ \rightarrow \mu^+ + \nu_\mu, \qquad \pi^- \rightarrow \mu^- + \overline{\nu}_\mu$$

μ^+: positive muon, charge +*e*, spin 1/2

μ^-: negative muon, charge -*e*, spin 1/2

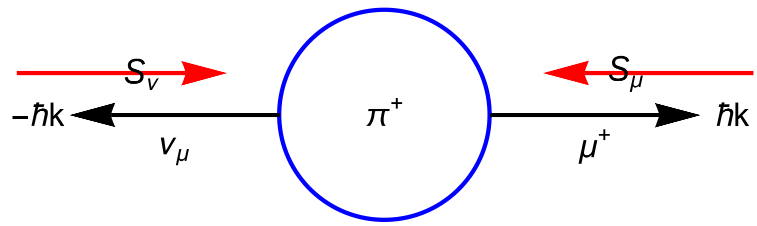
ν_μ: muon neutrino; uncharged, *S* = 1/2

$\overline{\nu}_\mu$: muon antineutrino; uncharged, *S* = 1/2

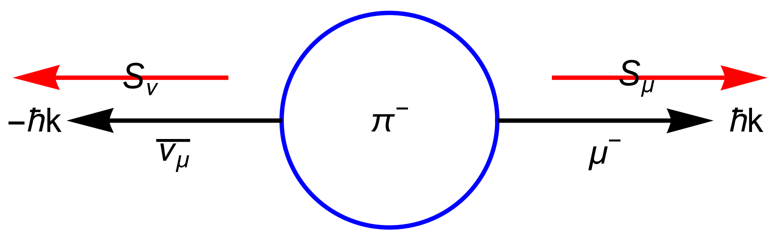
$D_{1/2} \times D_{1/2} = D_1 + D_0$ (spin angular momentum addition)

μ^+: spin pointing in the direction of flight.

μ^-: spin pointing opposite to the direction of flight.



at rest (p = 0)



at rest (p = 0)

6. (*g*-2) experiment

(i) Positive muon μ^+

Spin rotation (C.C.W around the magnetic field)
Orbital rotation (C.W. around the magnetic field)

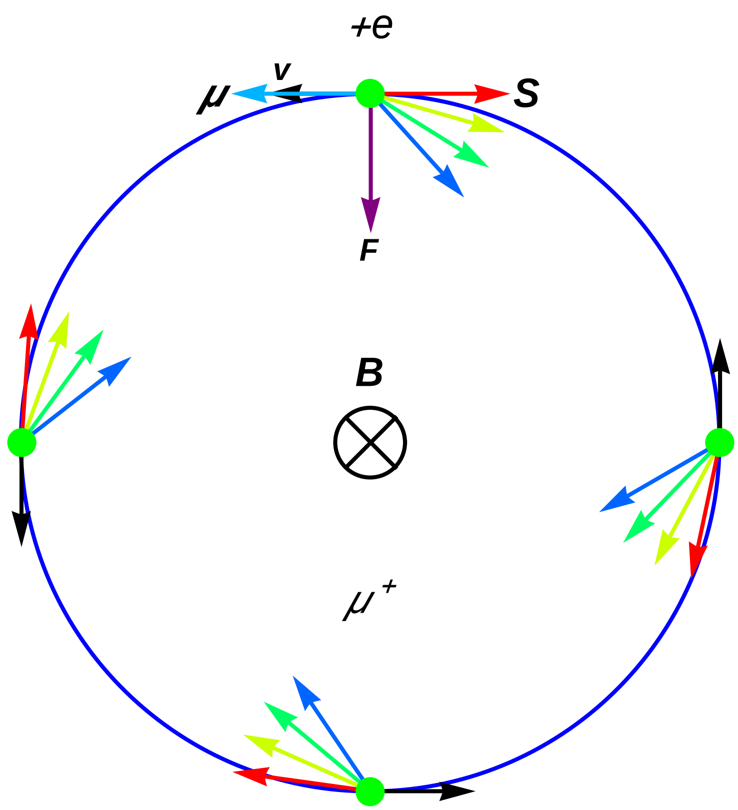


Fig.

(ii) Negative muon
Spin rotation (C.C.W. around the magnetic field)
Orbital rotation (C.C.W. around the magnetic field)

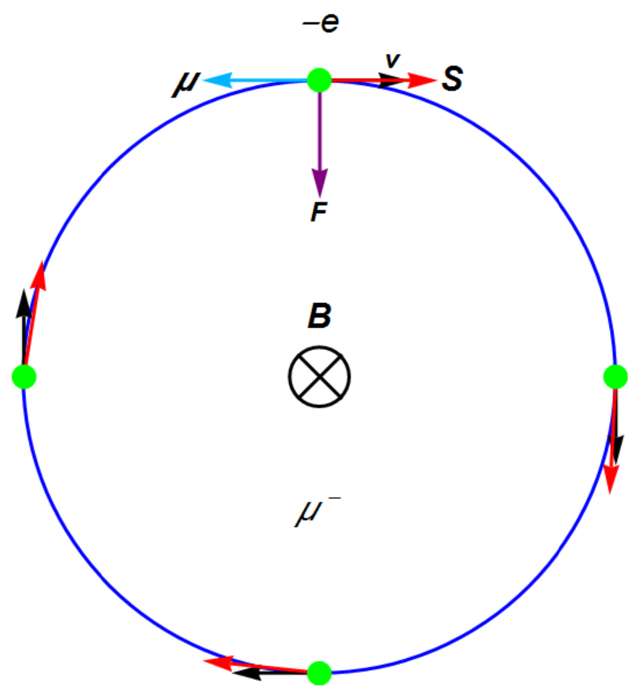


Fig. negative muon μ^- (-*e* charge , orbital motion: C.C.W)

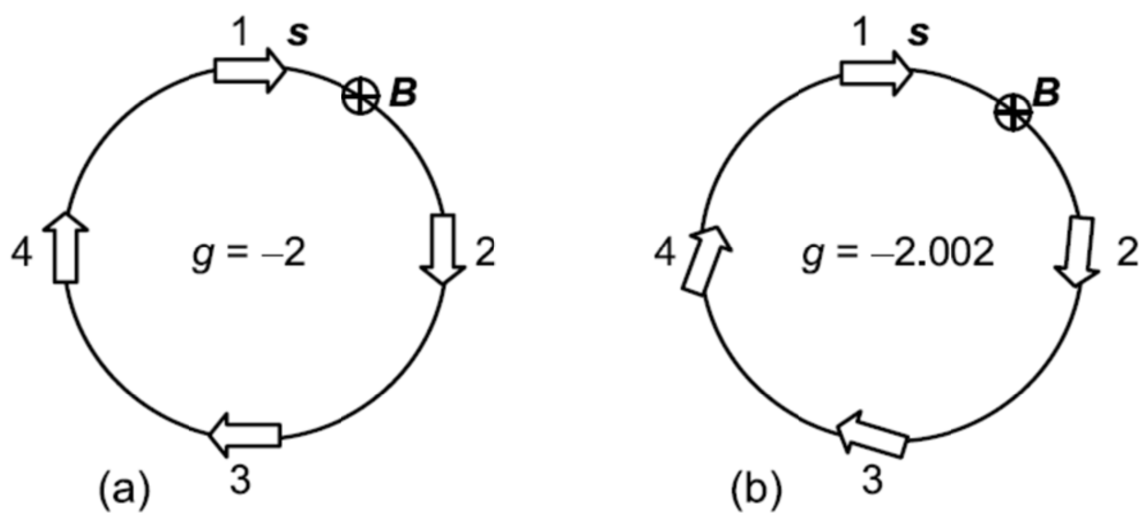


Fig. Explanation for the (*g*-2) experiment (**Dubbers and Stockmann**) for negative muon μ^-

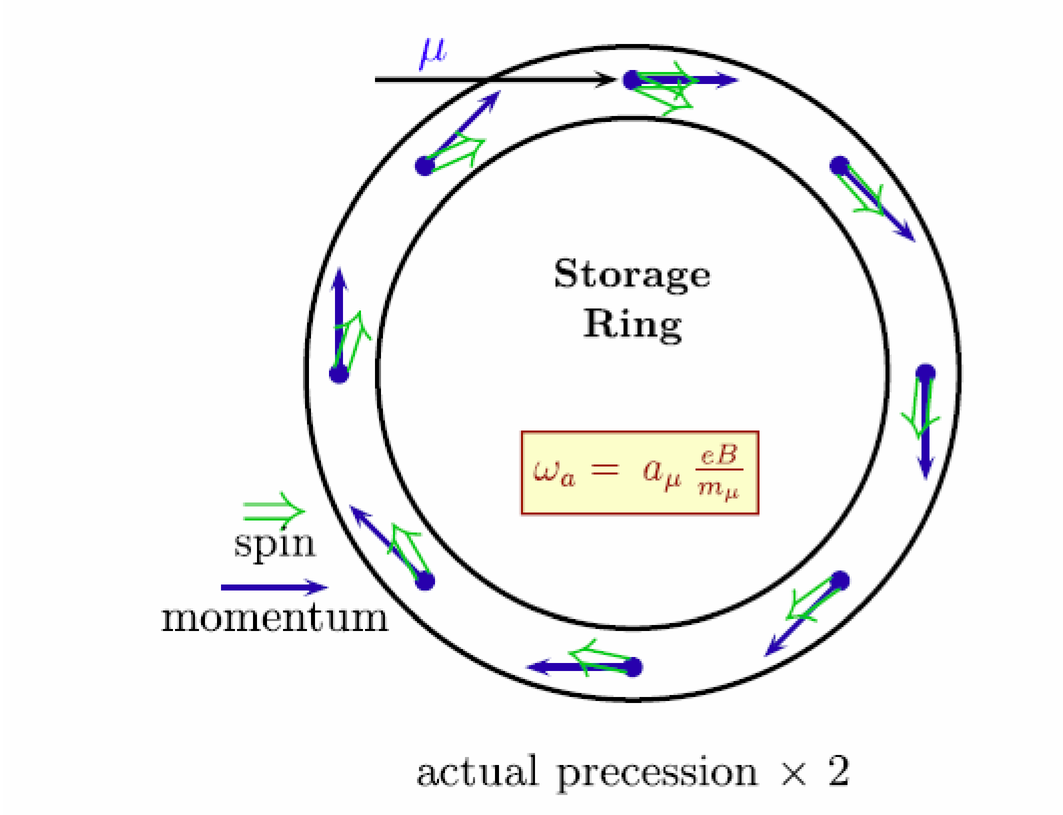


Fig. Explanation for the (*g*-2) experiment (**Jegerlehner and Nyffeler**).

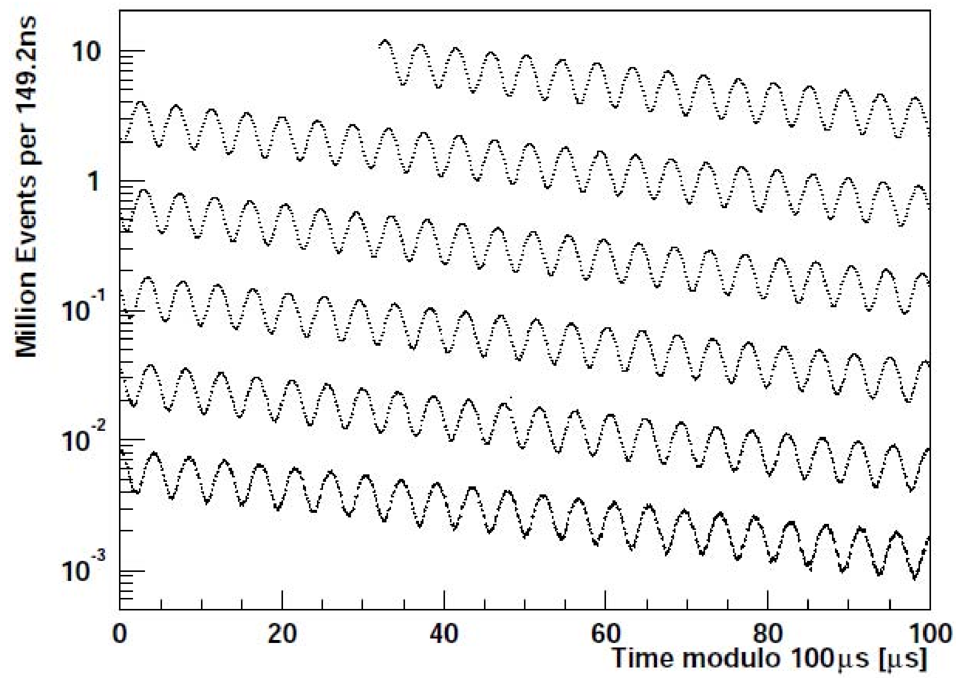


Figure 26. Histogram of the total number of electrons above 1.8 GeV versus time (modulo 100 μ s) from the 2001 μ^- data set. The bin size is the cyclotron period, $\approx$ 149.2 ns, and the total number of electrons is 3.6 billion.

7. Classical theory for muon μ^+ (charge, e>0) (classical)

We start with

$$\frac{d}{dt}\boldsymbol{S} = \boldsymbol{\mu} \times \boldsymbol{B} = -\frac{eg\mu_0}{\hbar}(\boldsymbol{S} \times \boldsymbol{B})$$

When $\boldsymbol{B}$ is directed along the z axis, we get

$$-\frac{eg\mu_0}{\hbar}(\boldsymbol{S} \times \boldsymbol{B}) = -\frac{eg\mu_0}{\hbar}\begin{vmatrix} \boldsymbol{e}_x & \boldsymbol{e}_y & \boldsymbol{e}_z \\ S_x & S_y & S_z \\ 0 & 0 & B \end{vmatrix} = -\frac{eg\mu_0 B}{\hbar}(S_y, -S_x, 0)$$

Then the equation of motion is obtained as

$$\frac{d}{dt}S_x = -\frac{eg\mu_0 B}{\hbar}S_y, \qquad \frac{d}{dt}S_y = \frac{eg\mu_0 B}{\hbar}S_x$$

$$\frac{d}{dt}(S_x + iS_y) = i\frac{eg\mu_0 B}{\hbar}(S_x + iS_y),$$

or

$$S_x + iS_y = (S_x^0 + iS_y^0)\exp(i\frac{eg\mu_0 B}{\hbar}t)$$

8. Spin precession of positive muon (quantum mechanics)

The spin magnetic moment for the muon μ^+ (charge +*e*, spin 1/2, γ<0)

$$\hat{\boldsymbol{\mu}} = -\frac{g\mu_0}{\hbar}\hat{\boldsymbol{S}},$$

with

$$\mu_0 = \frac{e\hbar}{2m_\mu c}.$$

In the presence of an external magnetic field (***B***) along the *z* axis, the spin Hamiltonian is given by

$$\hat{H} = -\hat{\boldsymbol{\mu}} \cdot \boldsymbol{B} = \frac{g\mu_0 B}{\hbar}\hat{S}_z = \omega_0 \hat{S}_z$$

where

$$\omega_0 = \frac{geB}{2m_\mu c} \qquad (e>0)$$

Spin precession:

$$\begin{aligned} |\psi(t)\rangle &= \exp(-\frac{i}{\hbar}\hat{H}t)|\psi(t=0)\rangle \\ &= \exp(-\frac{i}{\hbar}\omega_0 \hat{S}_z t)|\psi(t=0)\rangle \\ &= \exp(-\frac{i}{2}\omega_0 \hat{\sigma}_z t)|\psi(t=0)\rangle \end{aligned}$$

The spin of μ^+ is rotated around the z axis in counter clock-wise way.

Suppose that the initial state

$$\left|\psi(t=0)\right\rangle = \left|+x\right\rangle = \frac{1}{\sqrt{2}}[\left|+z\right\rangle + \left|-z\right\rangle]$$

$$\begin{aligned}
\left|\psi(t)\right\rangle &= \frac{1}{\sqrt{2}}\exp(-\frac{i}{2}\omega_0\hat{\sigma}_z t)[\left|+z\right\rangle + \left|-z\right\rangle] \\
&= \frac{1}{\sqrt{2}}[\exp(-\frac{i}{2}\omega_0\hat{\sigma}_z t)\left|+z\right\rangle + \exp(-\frac{i}{2}\omega_0\hat{\sigma}_z t)\left|-z\right\rangle] \\
&= \frac{1}{\sqrt{2}}[e^{-i\omega_0 t/2}\left|+z\right\rangle + e^{i\omega_0 t/2}\left|-z\right\rangle] \\
&= \frac{1}{\sqrt{2}}e^{-i\omega_0 t/2}[\left|+z\right\rangle + e^{i\omega_0 t}\left|-z\right\rangle]
\end{aligned}$$

Then we have

$$\begin{aligned}
\left\langle S_x\right\rangle &= \left\langle\psi(t)\right|\hat{S}_x\left|\psi(t)\right\rangle \\
&= \frac{\hbar}{2}\frac{1}{\sqrt{2}}e^{i\omega_0 t/2}\begin{pmatrix}1 & e^{-i\omega_0 t}\end{pmatrix}\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\frac{1}{\sqrt{2}}e^{-i\omega_0 t/2}\begin{pmatrix}1 \\ e^{i\omega_0 t}\end{pmatrix} \\
&= \frac{\hbar}{4}\begin{pmatrix}1 & e^{-i\omega_0 t}\end{pmatrix}\begin{pmatrix}0 & 1 \\ 1 & 0\end{pmatrix}\begin{pmatrix}1 \\ e^{i\omega_0 t}\end{pmatrix} \\
&= \frac{\hbar}{2}\cos(\omega_0 t)
\end{aligned}$$

$$\begin{aligned}
\left\langle S_y\right\rangle &= \left\langle\psi(t)\right|\hat{S}_y\left|\psi(t)\right\rangle \\
&= \frac{\hbar}{2}\frac{1}{\sqrt{2}}e^{i\omega_0 t/2}\begin{pmatrix}1 & e^{-i\omega_0 t}\end{pmatrix}\begin{pmatrix}0 & -i \\ i & 0\end{pmatrix}\frac{1}{\sqrt{2}}e^{-i\omega_0 t/2}\begin{pmatrix}1 \\ e^{i\omega_0 t}\end{pmatrix} \\
&= \frac{\hbar}{4}\begin{pmatrix}1 & e^{-i\omega_0 t}\end{pmatrix}\begin{pmatrix}0 & -i \\ i & 0\end{pmatrix}\begin{pmatrix}1 \\ e^{i\omega_0 t}\end{pmatrix} \\
&= \frac{\hbar}{2}\sin(\omega_0 t)
\end{aligned}$$

The spin rotates around the z axis in counter clock wise. In the complex plane

$$\langle S_x \rangle + i\langle S_y \rangle = \frac{\hbar}{2} e^{i\omega_0 t}$$

9. How to detect

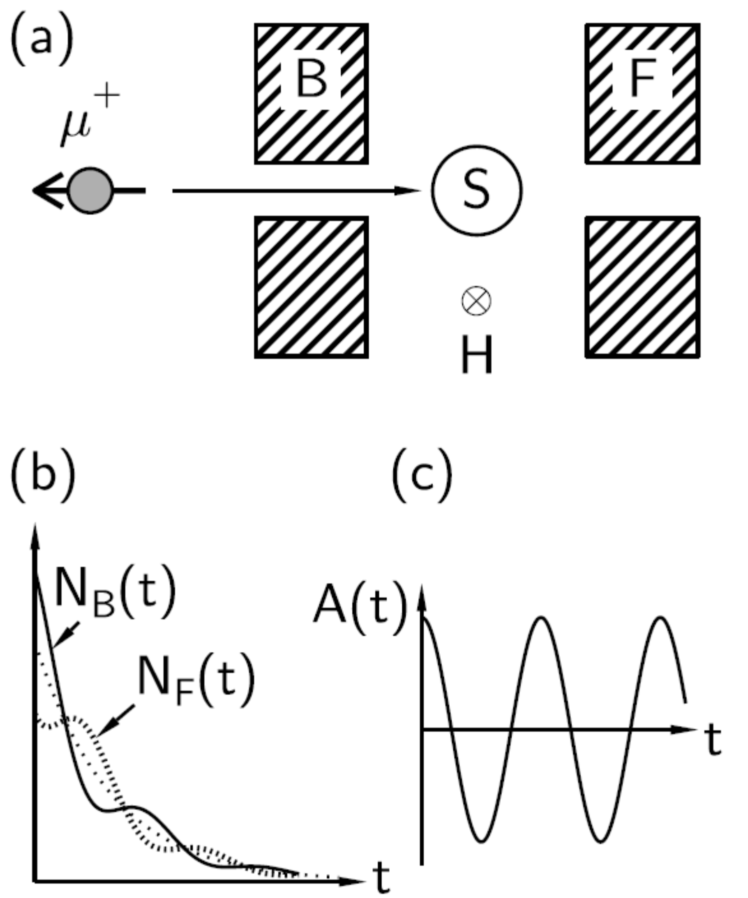


Fig. Schematic illustration of a μSR experiment. (a) A spin-polarized beam of muons is implanted in a sample S. Following decay, positrons are detected in either a forward detector F or a backward detector B. If a transverse magnetic field B is applied to the sample as shown then the muons will precess. (b) The number of positrons detected in the forward and backward detectors. (c) The asymmetry function. (**Blundell**).

$$\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu \qquad (\tau = 2.2\ \mu\text{s})$$

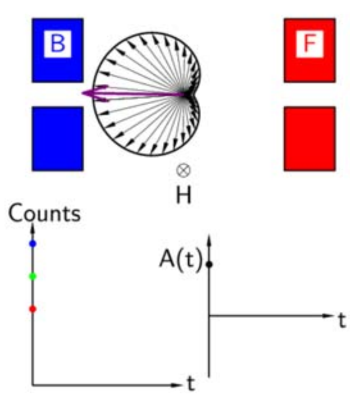
$$\begin{aligned}\langle +x|\psi(t)\rangle &= \frac{1}{2}e^{-i\omega_0 t/2}\begin{pmatrix}1 & 1\end{pmatrix}\begin{pmatrix}e^{i\omega_0 t}\\ 1\end{pmatrix}\\ &= \frac{1}{2}e^{-i\omega_0 t/2}(e^{i\omega_0 t}+1)\\ &= \frac{1}{2}(e^{i\omega_0 t/2}+e^{-i\omega_0 t/2})\\ &= \cos\left(\frac{\omega_0 t}{2}\right)\end{aligned}$$

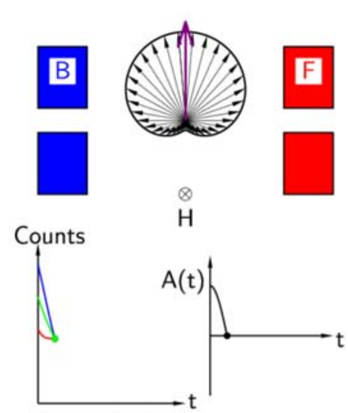
$$\begin{aligned}\langle -x|\psi(t)\rangle &= \frac{1}{2}e^{-i\omega_0 t/2}\begin{pmatrix}1 & -1\end{pmatrix}\begin{pmatrix}e^{i\omega_0 t}\\ 1\end{pmatrix}\\ &= \frac{1}{2}e^{-i\omega_0 t/2}(e^{i\omega_0 t}-1)\\ &= \frac{1}{2}(e^{i\omega_0 t/2}-e^{-i\omega_0 t/2})\\ &= \sin\left(\frac{\omega_0 t}{2}\right)\end{aligned}$$

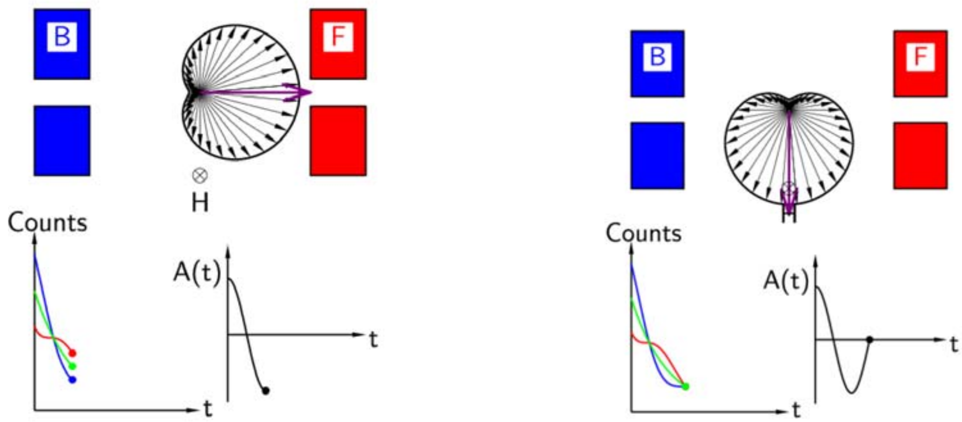
$$P(F) = |\langle +x|\psi(t)\rangle|^2 = \cos^2\left(\frac{\omega_0 t}{2}\right)$$

$$P(B) = |\langle -x|\psi(t)\rangle|^2 = \sin^2\left(\frac{\omega_0 t}{2}\right)$$

$$P(F)-P(B) = \cos^2\left(\frac{\omega_0 t}{2}\right)-\sin^2\left(\frac{\omega_0 t}{2}\right) = \cos(\omega_0 t)\,.$$







10. Spatial distribution of positrons from the decay of a polarized muon

The muon decay and its anisotropy

$$\mu^{+} \rightarrow e^{+} + \nu_{e} + \overline{\nu}_{\mu}$$

Only the positron e^{+} is detected.

The direction of its emission is anisotropic with respect to the spin vector $\boldsymbol{S}_{\mu}$. The probability of emission is given by

$$W(\theta) = 1 + a\cos\theta\,.$$

We make a polar plot of $W(\theta)$ where a is changed as a parameter (a = 0 – 1).
The angular distribution of e+ has maximum in muon spin direction.

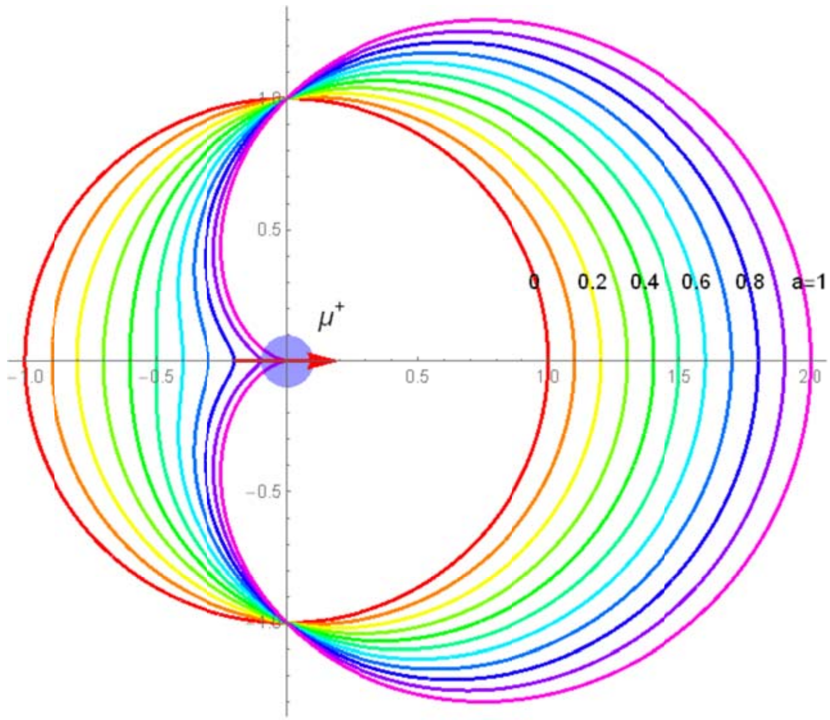


Fig. Plot of the probability $W(\theta) = 1 + a\cos\theta$ where a is changed as a parameter between a = 0 and a = 1.

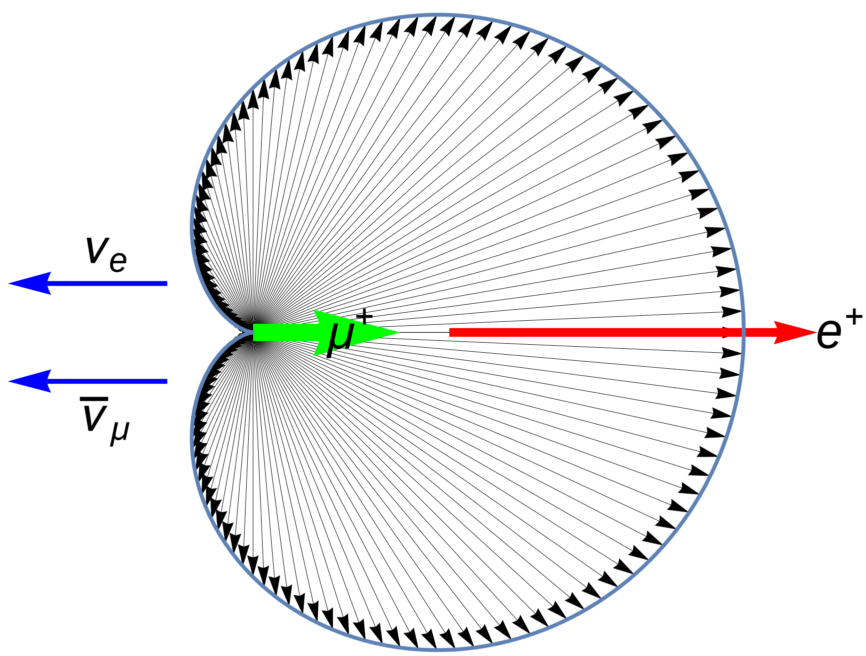


Fig. $\mu^+ \rightarrow e^+ + \nu_e + \overline{\nu}_\mu$. Plot of the probability $W(\theta) = 1 + a\cos\theta$ with a = 0.75.

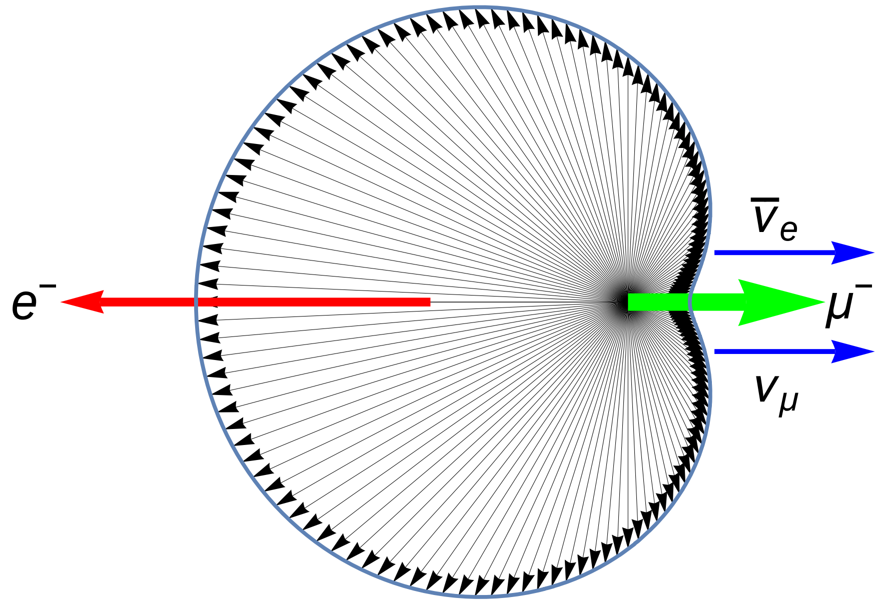


Fig. $\mu^- \rightarrow e^- + \overline{\nu}_e + \nu_\mu$. Plot of the probability $W(\theta) = 1 + a\cos\theta$ with $a = 0.75$.

11. Parity violation

The direction of the muon-spin is reversed in the mirror so that the positrons are emitted predominantly in a direction opposite to that of the muon-spin. The violation of parity means that in our universe only the process on the left-hand side of the diagram is ever observed.

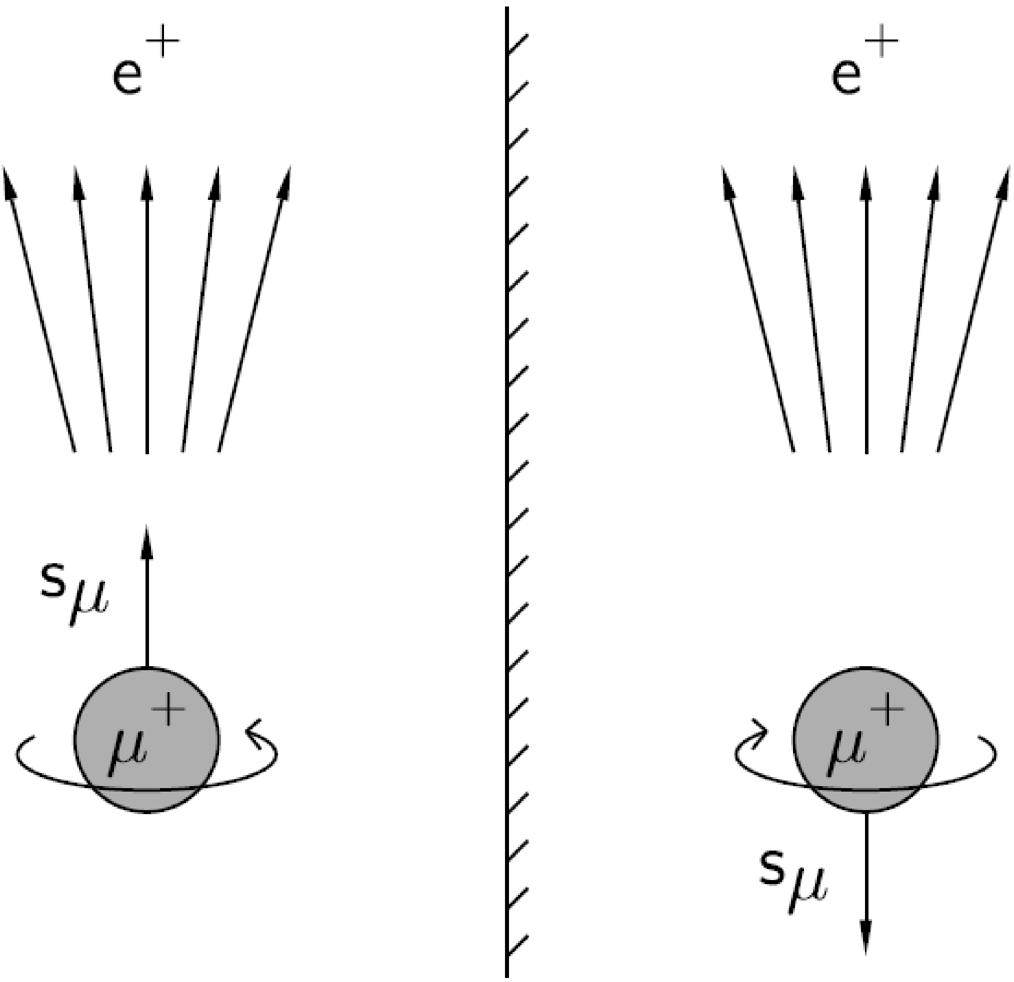


Fig. Parity-violating collinear decay of a pion π^+ at rest into a muon μ^+ and a muon neutrino ν_μ. (**Blundell**)

12. Experimental configuration

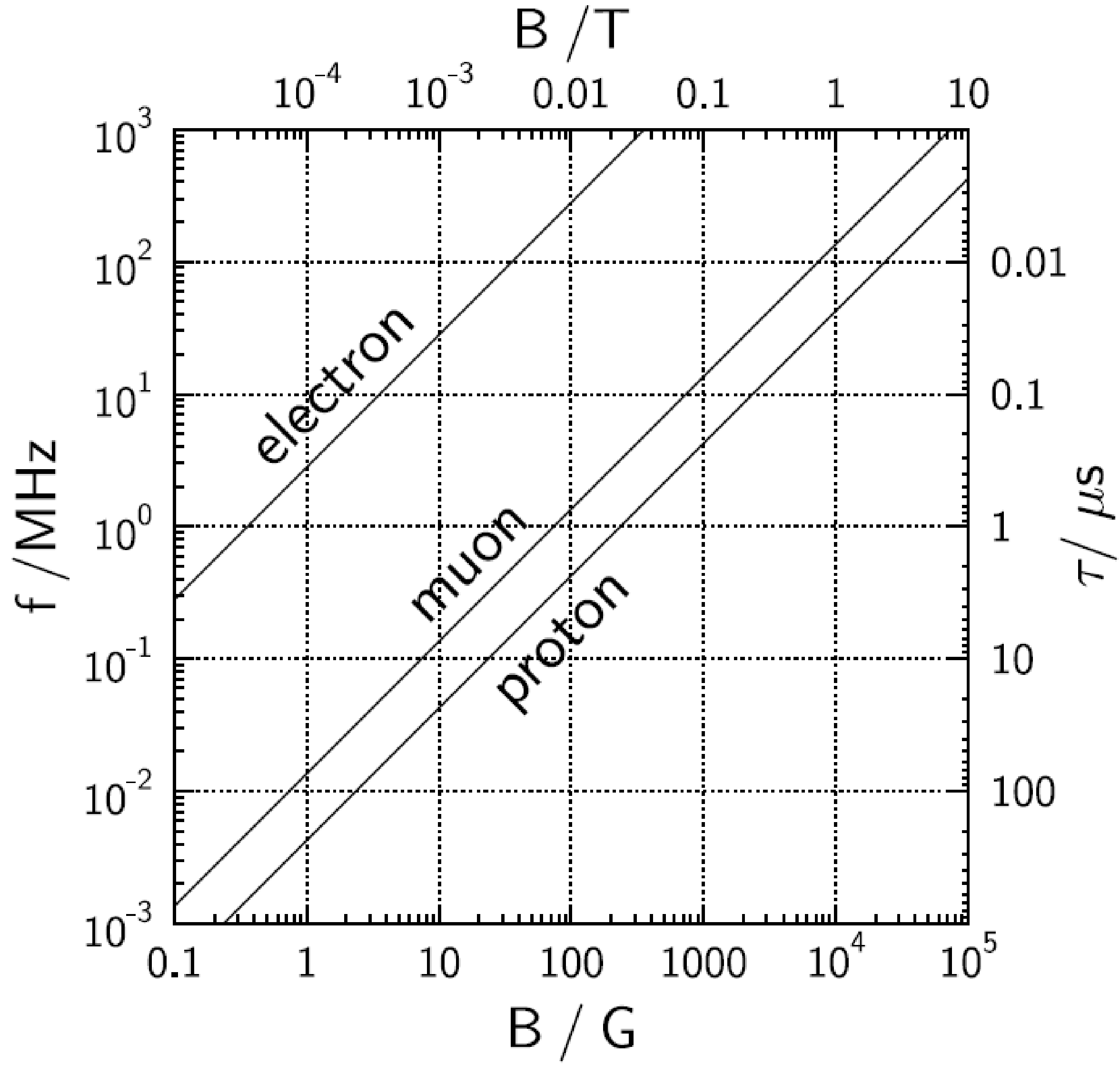
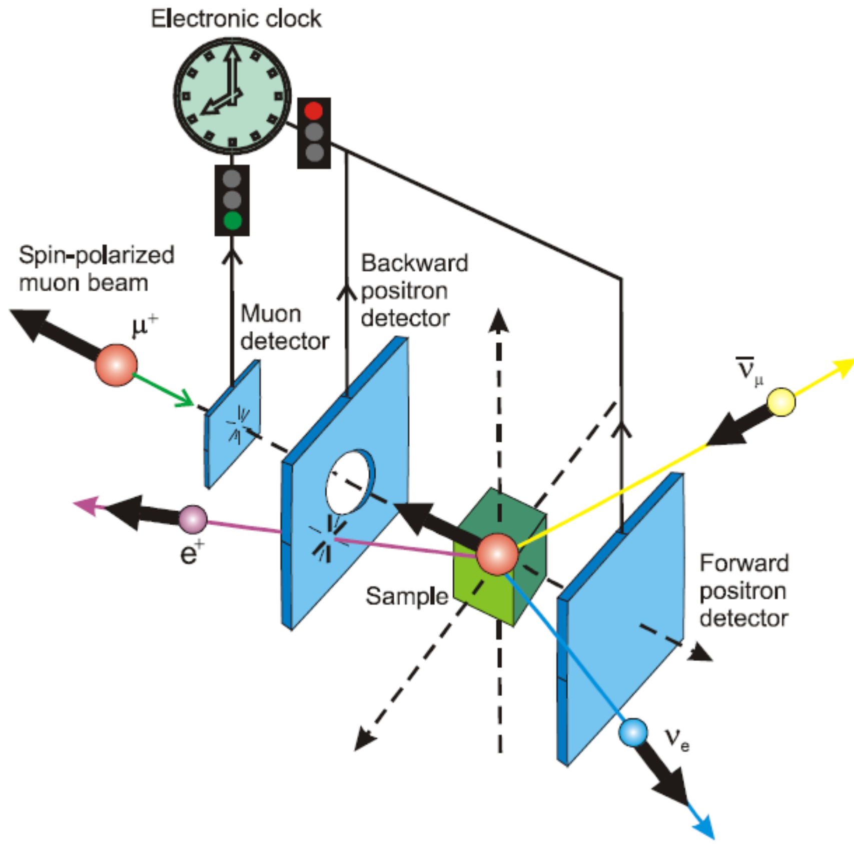


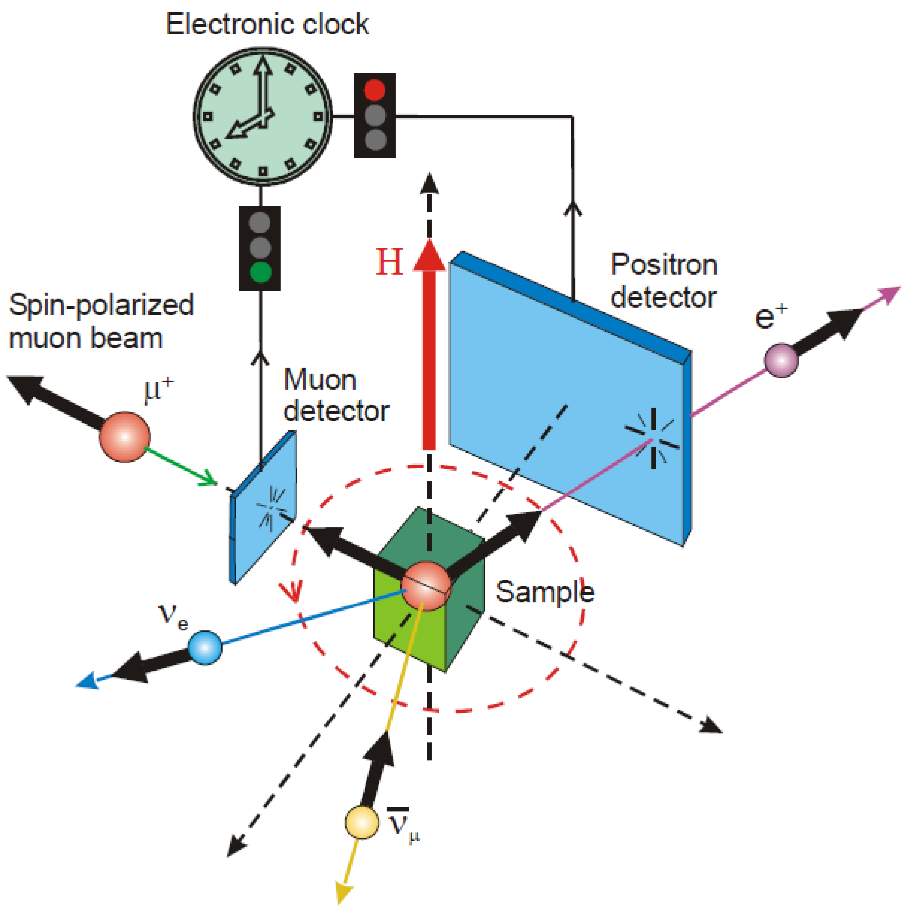
Fig. The Larmor precession frequency f in MHz (and the corresponding period $\tau = 1/f$) for the electron, muon and proton as a function of applied magnetic field B. (from Blundell, see Reference)

(i) Zero-field μSR



Schematic of a zero field (ZF) μSR setup.

(ii) Transverse field μSR



Schematic of a transverse field (TF) μSR setup.

13. Example-1

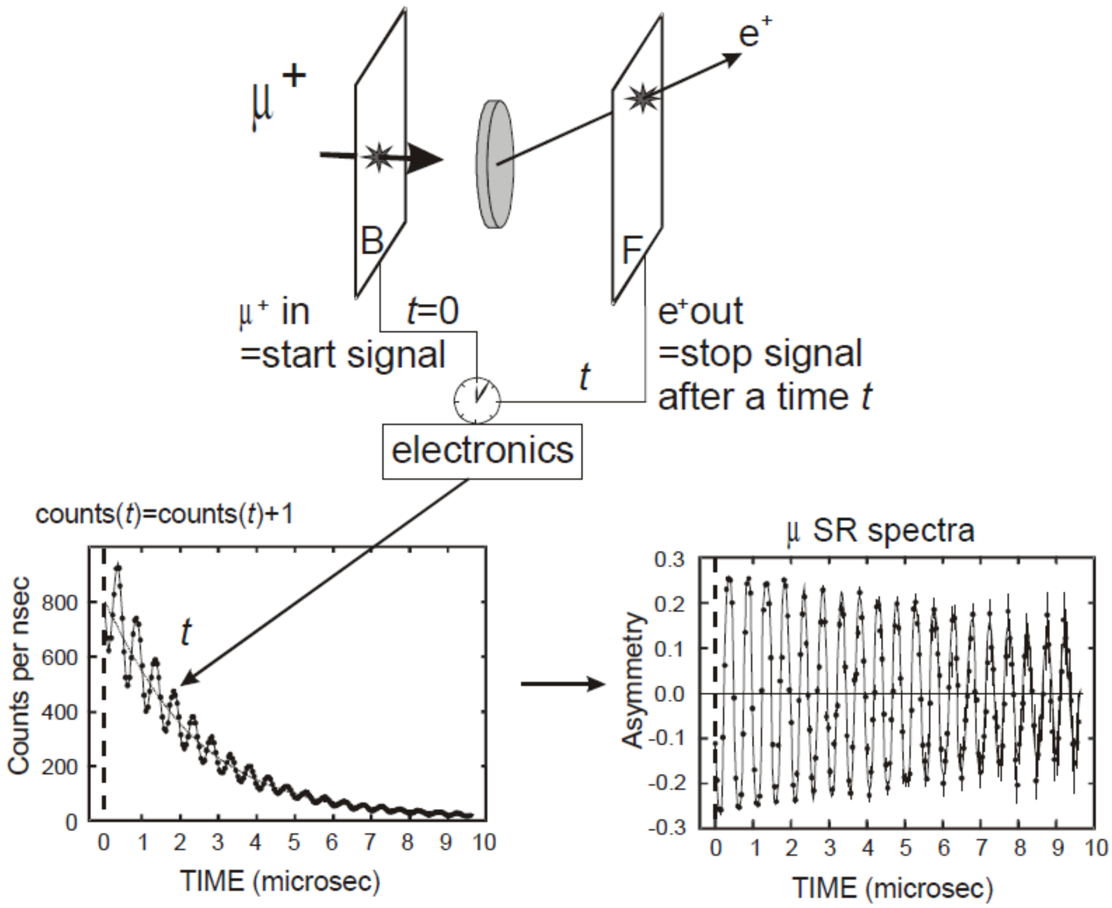


Fig. Schematic illustration of a μSR experiment. The arrival time t_0 of the muon is either given by a special detector (continuous beam, see figure) or by the arrival of a muon pulse (pulsed beams, not shown). The time between the muon arrival and the subsequent decay recorded by a positron emission at time t is used to build a rate vs. time histogram. By removing the exponential decay due to the muon lifetime and a possible background, one obtains the μSR signal reflecting the time dependence of the muon polarization.

14. Example Fourier transform analysis

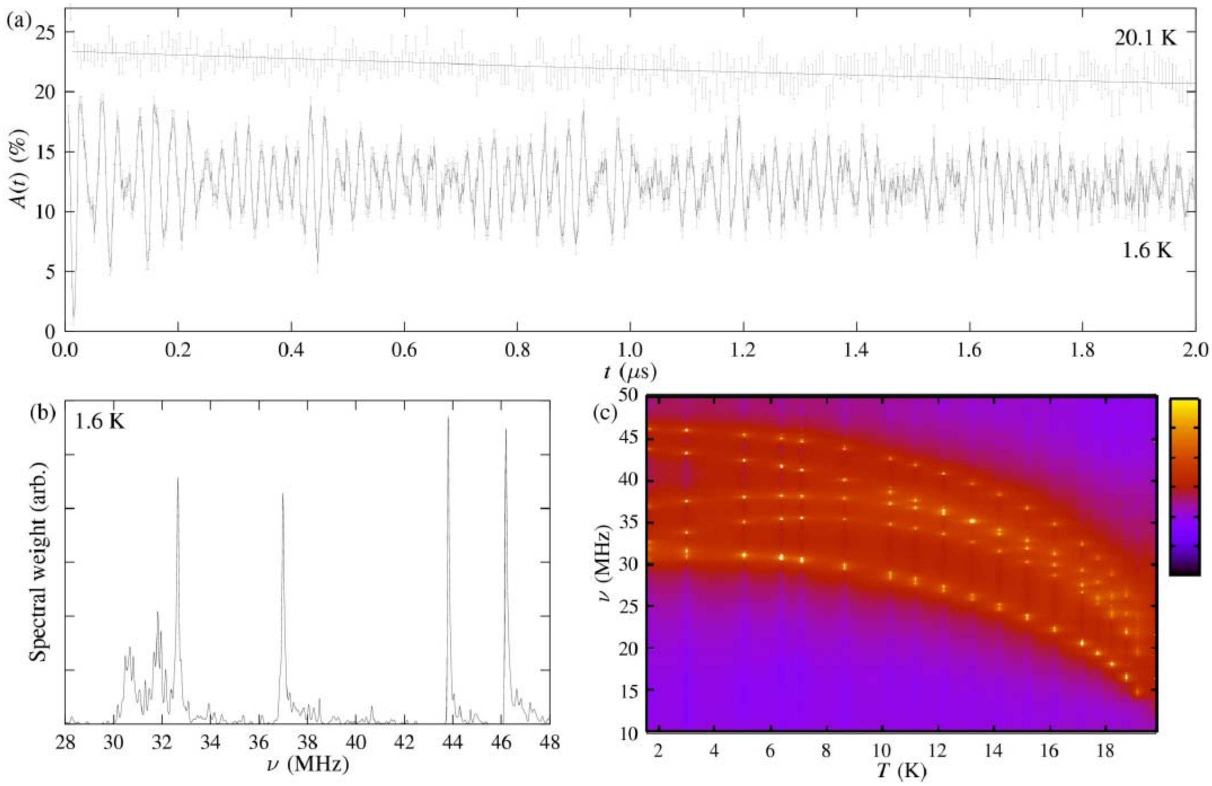


Fig. Example of experimental results.(a) A(t). (b) Fourier transform spectrum. (c) Magnetic phase transition.
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APPENDIX-I **Formula**

$$\exp[-\frac{i}{2}(\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{n})\theta]=\hat{1}\cos\frac{\theta}{2}-i(\hat{\boldsymbol{\sigma}}\cdot\boldsymbol{n})\sin\frac{\theta}{2}$$

APPENDIX-II Physical properties of muon

Muon μ± (lepton, charge ±1, matter μ⁻, antimatter μ⁺, spin 1/2, fermion)				
Muon rest energy (mc^2)	1.692 833 667[86] e-11	$kg.m^2.s^{-2}$	J	**105.658 3715[35]** MeV
Muon rest mass	1.883 531 475[96] e-28	kg		0.113 428 9267[29] u
Muon magnetic moment	**- 4.490 448 07[15] e-26**	$m^2.A$	J/T	
Muon g-factor g_μ	- 2.002 331 8418[13]	Dimensionless		$(\mu / \mu_B) * (m / m_e)$ / spin
Muon magnetic moment anomaly	1.165 920 91[63] e-3	Dimensionless		$(abs(g_\mu) - 2) / 2$
Muon gyromagnetic ratio	135.538 817[12] e+6	$kg^{-1}.s.A$	Hz/T	$= \mu_n / h\, S_n$
Muon half-life time	**1.52 e-6**	s		

APPENDIX-III Comment on **(g-2) experiment**

We consider the motion of spin *S* (=1/2) in the presence of an external magnetic field *B* along the *z* axis. The magnetic moment of spin ***S*** is given by

$$\hat{\mu}_z=-\frac{g\mu_B\hat{S}_z}{\hbar}.$$

Note that here we consider the spin of electron. The direction of the magnetic moment vector is antiparallel to that of spin. Then the spin Hamiltonian (Zeeman energy) is described by

$$\hat{H}=-\hat{\mu}_z B=-(-\frac{g\mu_B\hat{S}_z}{\hbar})B=g\frac{\mu_B B}{\hbar}\hat{S}_z$$

Since the Bohr magneton μ_B is given by $\mu_B=\frac{e\hbar}{2mc}$,

$$\frac{\mu_B B}{\hbar} = \frac{eB}{2mc} = \frac{1}{2}\omega_c \qquad (e>0).$$

or

$$\omega_c = \frac{eB}{mc} \qquad \text{(angular frequency of the Larmor precession)}$$

Thus the Hamiltonian can be rewritten as

$$\hat{H} = \frac{g}{2}\omega_0 \hat{S}_z .$$

Thus the Schrödinger equation is obtained as

$$\begin{aligned} |\psi(t)\rangle &= \exp[-\frac{i}{\hbar}\hat{H}t]|\psi(t=0)\rangle \\ &= \exp[-\frac{i}{\hbar}(\frac{g}{2}\omega_0 t)\hat{S}_z]|\psi(t=0)\rangle \end{aligned} .$$

We note that

$$\exp[-\frac{i}{\hbar}(\frac{g}{2}\omega_0 t)\hat{S}_z]$$

is the rotation operator around the z axis with the rotation angle $\Omega t = \frac{g}{2}\omega_c t$, where Ω is the angular frequency, where

$$\Omega = \frac{g}{2}\omega_c$$

The difference of the angular frequency is given by

$$\Delta\Omega = (\frac{g}{2} - 1)\omega_c = \frac{1}{2}(g-2)\omega_c$$

where

electron $g = 2.00231930436182$

muon	$g = 2.0023318414$